Application of Quantum-Like Bayesian Networks in Social Sciences

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Motivation

Quantum probability was created in order to **explain paradoxical findings** that could not be addressed through classical probability theory.

Predictions concerned with human decision making **tend to violate the laws of classical probability**, since inferences are performed using limited data coupled with several heuristics.

Recent literature suggests that quantum probability can be used as a mathematical alternative to the classical theory and it **is able to accommodate** these violations, **improving** the probabilistic inferences

Goal: Develop a probabilistic graphical model that can accommodate violations of the law of total probability and predict human decisions!

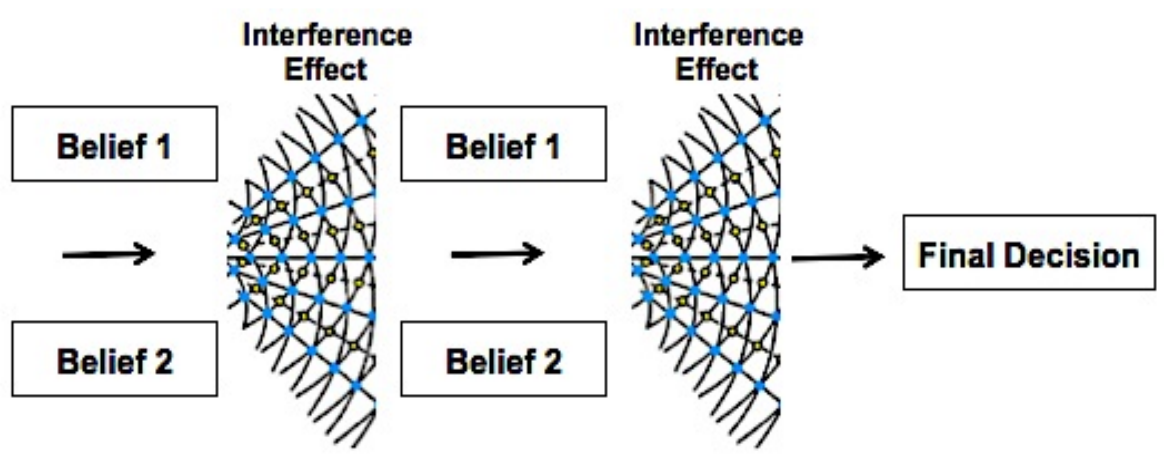
Quantum Cognition

Quantum cognition is a research field that aims at using the **mathematical principles of quantum mechanics** to model **cognitive** systems for human decision making.

Classical probability theory is very **rigid** in the sense that it poses many constraints and assumptions (single trajectory principle, obeys set theory, etc.), it becomes too limited to provide simple models that can capture human judgments and decisions

Quantum theory models information via wave functions that can be in different states at the same time (**superposition**).

These waves can **crash** and **interfere** with each other, influencing the final probabilities in a decision problem.



The Sure Thing Principle

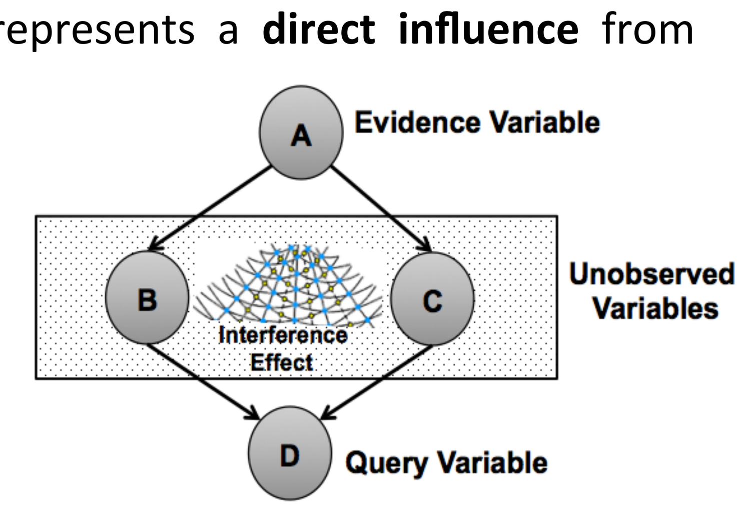
Several experiments in the literature show violations of the Sure Thing Principle: "If one chooses **action *A*** over some other **action B** under the state of the **world X**, and if one also chooses **action A** over **B** under the complementary state of the **world ¬X**, then one should always choose **action A** over **action B** even when the state of the **world in unknown**." (Savage, 1954)

Quantum-Like Bayesian Networks

Bayesian Networks are **directed acyclic graph** structures in which each **node** represents a **random variable** and each **edge** represents a **direct influence** from source node to the target node.

In a Quantum-Like Bayesian Network, classical probabilities are replaced by quantum probability amplitudes, through **Born's rule**:

$$Pr(A) = | e^{i\theta_A} \psi_A |^2$$



Inferences are computed through the computation of the **full joint probability distribution** (1) and **marginalization** (2).

(1) $Pr(X_1, \ldots, X_n) = \prod_{i=1}^{n} Pr(X_i | Parents(X_i)) \longrightarrow Pr_q(X_1, \ldots, X_n) = \left| \prod_{i=1}^{n} QPr(X_i | Parents(X_i)) \right|^2$

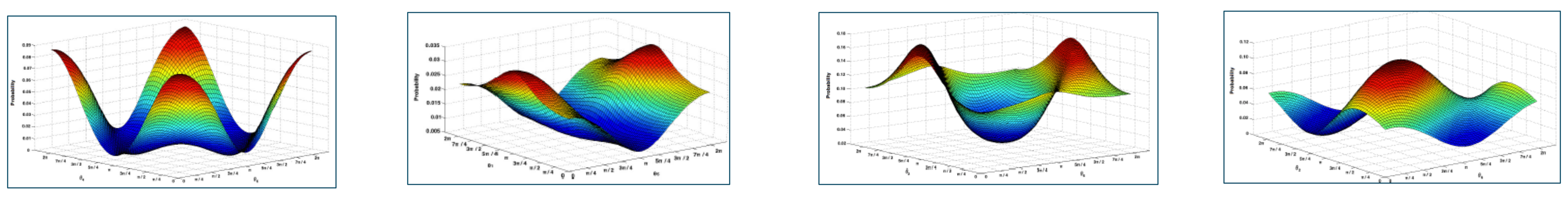
(2) $Pr_c(X|e) = \alpha Pr_c(X, e) = \alpha \left[\sum_{y \in Y} Pr_c(X, e, y) \right] \longrightarrow Pr_q(X|e) = \alpha \left| \sum_{y} \prod_{x=1}^{N} QPr(X_x | Parents(X_x), e, y) \right|^2$

If we expand the quantum marginal probability formula (2), we obtain:

$$Pr_q(X|e) = \alpha \left[\sum_{i=1}^{|Y|} \left| \prod_{x}^{N} QPr(X_x | Parents(X_x), e, y = i) \right|^2 + 2 \sum_{i=1}^{|Y|-1} \sum_{j=i+1}^{|Y|} |QPr(X_x | Parents(X_x), e, y = i)| \, |QPr(X_x | Parents(X_x), e, y = j)| \cos(\theta_i - \theta_j) \right]$$

Quantum Interference Effects!

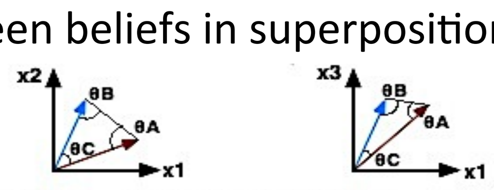
Problems:
Exponential growth of quantum parameters!
Final probabilities can be **ANYTHING** in a given **range**!



How to find these quantum parameters in order to accommodate violations to the Sure Thing Principle?

Estimating Quantum Parameters

We propose a similarity **heuristic** that is able to compute the quantum parameters **through vector similarities** between beliefs in superposition.



$$Pr(B) = \alpha [\sum_{i=1}^{N} |\psi_i|^2 + 2 \cdot |\psi_1| \cdot |\psi_2| \cdot \cos(\theta_1 - \theta_2) + 2 \cdot |\psi_1| \cdot |\psi_3| \cdot \cos(\theta_1 - \theta_3) + \cdots$$

In quantum cognition, the quantum parameters are seen as inner products, we represent each pair of random variables in 2-dimenional vectors.

By computing the similarity between belief vectors, additional information is gained. A heuristic function can be constructed with the relationships between belief vectors.

$$h(a, b) = \begin{cases} \pi & \text{if } \phi < 0 \\ \pi - \frac{\theta C_{a,b}}{2} & \text{if } \phi > 0.2 \\ \pi - \theta C_{a,b} & \text{otherwise} \end{cases}$$

Prisoner's Dilemma Game

Two prisoners, who are in separate cells, are each given an opportunity to betray the other (**defect**), or to remain silent (**cooperate** with the other).

The Prisoner's Dilemma Game is an example where people tend to violate the Sure Thing Principle. When given information, people tend to **defect**. Under uncertainty, people tend to **cooperate**.

		Prisoner A Choices	
		Stay Silent	*Confess and Betray*
Prisoner B Choices	*Stay Silent*	Each serves one month in jail	Prisoner A goes free Prisoner B serves full year in jail
	Confess and Betray	Prisoner A serves full year in jail Prisoner B goes free	Each serves three months in jail

Several works of the literature simulated this game with different payoff matrices.

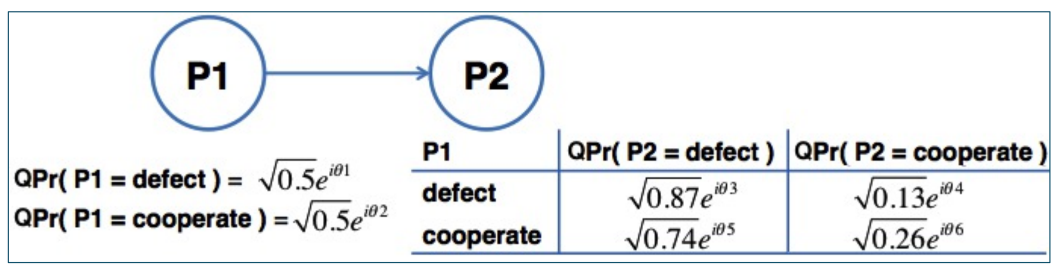
Three conditions were tested:

1. Participants were **informed** that the other participant chose ***defect.***
2. Participants were **informed** that the other participant chose ***cooperate.***
3. Participants **were not informed** about the other participant's decision.

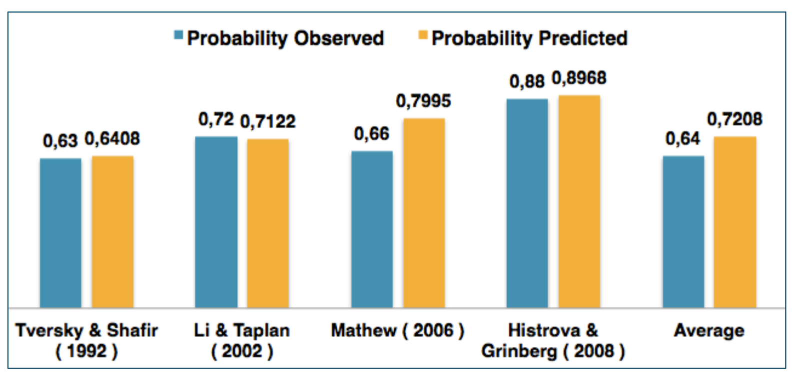
Literature	Known to Defect	Known to Collaborate	Unknown	Classical Probability
Shafir and Tversky (1992)	0.9700	0.8400	0.6300	0.9050
Crosson (1999)[a]	0.6700	0.3200	0.3000	0.4950
Li and Taplin (2002)[b]	0.8200	0.7700	0.7200	0.7950
Busemeyer et al. (2006a)	0.9100	0.8400	0.6600	0.8750
Hristova and Grinberg (2008)	0.9700	0.9300	0.8800	0.9500
Average	0.8700	0.7400	0.6400	0.8050

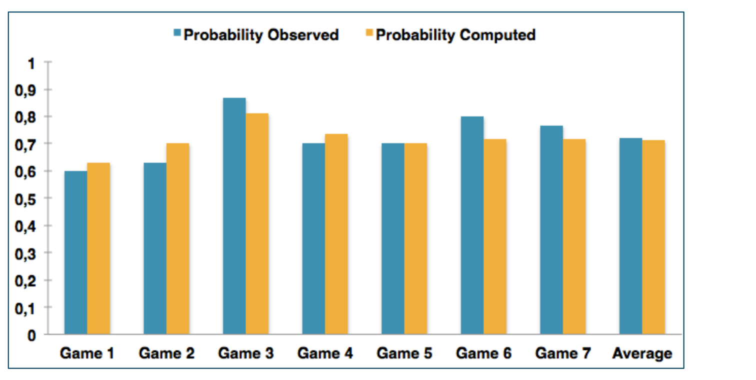
Results

We modeled each work reported in the literature with the proposed Quantum-Like Bayesian Network and the suggested heuristic.



The results obtained showed that the proposed model was able to accommodate violations to the Sure Thing Principle in the Prisoner's Dilemma Game.
The heuristic function also provided accurate predictions for several different games.





Conclusions

We propose a Quantum-Like Bayesian Network that uses quantum interference effects to accommodate violations to the Sure Thing Principle.

Quantum parameters are found based on a heuristic that measures the similarities that the belief vectors share between them.

References

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