Topics on OR: TSP
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TSP examples

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Some examples of the *travelling salesman problem* mainly from Bronson [1982] are presented and solved.

Key words: travelling salesman problem.

1. Fundamentals and scope

Some examples of the *traveling salesman problem* are presented and solved by the technique of *branch-and-bound* applied to successive *assignment problem* relaxations. This algorithm is suitable for didactical purposes. The MTZ [Miller *et al.*, 1960] formulation (also shown by Wagner [1972]) is a way to convert a TSP into a Linear Programming problem, susceptible of resolution (not shown here) through any available MILP (mixed integer Linear Programming) procedure (such as Lindo).

The TSP is so complex that many algorithms have been developed during the last half century and continue to appear, along with many others of heuristic type.

2. Examples of TSP

MTZ ([Miller *et al.*, 1960])

-1	20	23	4
30	-1	7	27
25	5	-1	25
3	21	26	-1

∞	20	23	4
30	∞	7	27
25	5	∞	25
3	21	26	∞

$z^* = 55$

(0)	(1) Forbid 1-4	(2) Forbid 4-1 Impose 1-4
-1 20 23 4	-1 20 23 ***-1***	-1 -1 -1 ***4***
30 -1 7 27	30 -1 7 27	30 -1 7 -1
25 5 -1 25	25 5 -1 25	25 5 -1 -1
3 21 26 -1	3 21 26 -1	***-1*** 21 26 -1
Saturated	Incumbent (OPT after 2)	Saturated
z = 19 1-4-1 2-3-2	z = 55 1-2-3-4-1	z = 57 1-4-2-3-1

In this and the following examples, a TSP program is used to get the solution,
http://alfa.ist.utl.pt/~mcasquil/calcul/F-tsp.html
and an AP program is used to calculate the successive relaxations [Casquilho, 2007],
http://alfa.ist.utl.pt/~mcasquil/calcul/F-assign.html

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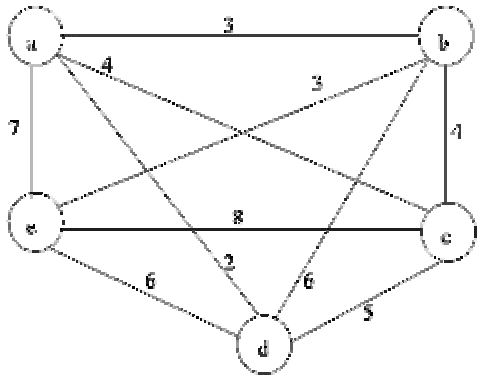


-1	3	4	2	7
3	-1	4	6	3
4	4	-1	5	8
2	6	5	-1	6
7	3	8	6	-1

∞	3	4	2	7
3	∞	4	6	3
4	4	∞	5	8
2	6	5	∞	6
7	3	8	6	∞

z^* = 19 1-3-2-5-4-1

(0)	(1) N **2-5**	(2) N **5-2** Y **2-5**
-1 3 4 2 7	-1 3 4 2 7	-1 3 4 2 -1
3 -1 4 6 3	3 -1 4 6 ***-1***	-1 -1 -1 -1 ***3***
4 4 -1 5 8	4 4 -1 5 8	4 4 -1 5 -1
2 6 5 -1 6	2 6 5 -1 6	2 6 5 -1 -1
7 3 8 6 -1	7 3 8 6 -1	7 ***-1*** 8 6 -1
Saturated	Incumbent (OPT after 3)	Saturated
z = 17 1-4-3-1, 2-5-2	z = 19 1-4-5-2-3-1	z = 19 1-4-1, 2-5-3-2

(2-3) N 5-2 **1-4** Y 2-5	(2-4) N 5-2 **4-1** Y 2-5 **1-4**
-1 3 4 ***-1*** -1	-1 -1 -1 ***2*** -1
-1 -1 -1 -1 3	-1 -1 -1 -1 3
4 4 -1 5 -1	4 4 -1 -1 -1
2 6 5 -1 -1	***-1*** 6 5 -1 -1
7 -1 8 6 -1	7 -1 8 -1 -1
Incumbent (OPT after 4)	Saturated
z = 19 1-3-2-5-4-1	z = 21 1-4-3-2-5-1

Bronson, 9.17 [Bronson, 1982, p 96 (Pr. 9.8, p 92)]

-1	35	80	105	165
35	-1	45	20	80
80	45	-1	30	75
105	20	30	-1	60
165	80	75	60	-1

∞	35	80	105	165
35	∞	45	20	80
80	45	∞	30	75
105	20	30	∞	60
165	80	75	60	∞

z^* = 270 1-2-4-5-3-1

(0)	(1) N **1-2**	(2) N **2-1**, Y **1-2**
-1 35 80 105 165	-1 ***-1*** 80 105 165	-1 ***35*** -1 -1 -1
35 -1 45 20 80	35 -1 45 20 80	***-1*** -1 45 20 80
80 45 -1 30 75	80 45 -1 30 75	80 -1 -1 30 75
105 20 30 -1 60	105 20 30 -1 60	105 -1 30 -1 60
165 80 75 60 -1	165 80 75 60 -1	165 -1 75 60 -1
Saturated	Incumbent	Incumbent (OPT also, after 1)
z = 235 1-2-1, 3-4-5-3	z = 270 1-3-5-4-2-1	z = 270 1-2-4-5-3-1

Bronson, 9.18 [Bronson, 1982, p 96]

-1	1	8	3	4
1	-1	8	2	3
1	3	-1	5	1
2	5	6	-1	5
5	3	7	6	-1

∞	1	8	3	4
1	∞	8	2	3
1	3	∞	5	1
2	5	6	∞	5
5	3	7	6	∞

z^* = 14 1-4-3-5-2-1

(0)				
-1	1	8	3	4
1	-1	8	2	3
1	3	-1	5	1
2	5	6	-1	5
5	3	7	6	-1
Saturated				
z = 13 1-2-4-1, 3-5-3				

(1) N **3-5**				
-1	1	8	3	4
1	-1	8	2	3
1	3	-1	5	***-1***
2	5	6	-1	5
5	3	7	6	-1
Incumbent				
z = 16 1-2-4-5-3-1				

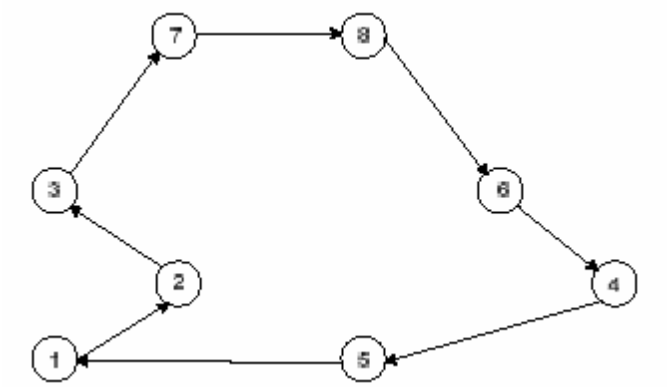
(2) N **5-3**, Y **3-5**				
-1	1	8	3	-1
1	-1	8	2	-1
-1	-1	-1	-1	***1***
-1	5	6	-1	-1
5	3	***-1***	6	-1
Incumbent (OPT after 1)				
z = 14 1-4-3-5-2-1				

Ramires & Soares [Ramires *et al.*, 2005]

-1	2	11	10	8	7	6	5
6	-1	1	8	8	4	6	7
5	12	-1	11	8	12	3	11
11	9	10	-1	1	9	8	10
11	11	9	4	-1	2	10	9
12	8	5	2	11	-1	11	9
10	11	12	10	9	12	-1	3
7	10	10	10	6	3	1	-1

∞	2	11	10	8	7	6	5
6	∞	1	8	8	4	6	7
5	12	∞	11	8	12	3	11
11	9	10	∞	1	9	8	10
11	11	9	4	∞	2	10	9
12	8	5	2	11	∞	11	9
10	11	12	10	9	12	∞	3
7	10	10	10	6	3	1	∞

z^* = 26 1-2-3-7-8-6-4-5-1



(0)							
-1	2	11	10	8	7	6	5
6	-1	1	8	8	4	6	7
5	12	-1	11	8	12	3	11
11	9	10	-1	1	9	8	10
11	11	9	4	-1	2	10	9
12	8	5	2	11	-1	11	9
10	11	12	10	9	12	-1	3
7	10	10	10	6	3	1	-1
Saturated							
z = 17 1-2-3-1, 4-5-6-4, 7-8-7							

(1) N **7-8**

-1	2	11	10	8	7	6	5
6	-1	1	8	8	4	6	7
5	12	-1	11	8	12	3	11
11	9	10	-1	1	9	8	10
11	11	9	4	-1	2	10	9
12	8	5	2	11	-1	11	9
10	11	12	10	9	12	-1	***-1***
7	10	10	10	6	3	1	-1

Saturated after 5

z = 28 1-8-7-2-3-1, 4-5-6-4

(2) N **8-7**, Y **7-8**

-1	2	11	10	8	7	6	-1
6	-1	1	8	8	4	6	-1
5	12	-1	11	8	12	3	-1
11	9	10	-1	1	9	8	-1
11	11	9	4	-1	2	10	-1
12	8	5	2	11	-1	11	-1
-1	-1	-1	-1	-1	-1	-1	***3***
7	10	10	10	6	3	***-1***	-1

Saturated

z = 21 1-2-3-7-8-1, 4-5-6-4

(2-3) N 8-7 **4-5**, Y 7-8

-1	2	11	10	8	7	6	-1
6	-1	1	8	8	4	6	-1
5	12	-1	11	8	12	3	-1
11	9	10	-1	***-1***	9	8	-1
11	11	9	4	-1	2	10	-1
12	8	5	2	11	-1	11	-1
-1	-1	-1	-1	-1	-1	-1	3
7	10	10	10	6	3	-1	-1

Saturated after 5

z = 29 1-2-3-1, 4-7-8-5-6-4

(2-4) N 8-7 **5-4**, Y 7-8, **4-5**

-1	2	11	10	-1	7	6	-1
6	-1	1	8	-1	4	6	-1
5	12	-1	11	-1	12	3	-1
-1	-1	-1	-1	***1***	-1	-1	-1
11	11	9	***-1***	-1	2	10	-1
12	8	5	2	-1	-1	11	-1
-1	-1	-1	-1	-1	-1	-1	3
7	10	10	10	-1	3	-1	-1

Saturated

z = 21 1-2-3-7-8-1, 4-**5-6**-4

(4-5) N 8-7 5-4 **5-6**, Y 7-8 4-5

-1	2	11	10	-1	7	6	-1
6	-1	1	8	-1	4	6	-1
5	12	-1	11	-1	12	3	-1
-1	-1	-1	-1	1	-1	-1	-1
11	11	9	-1	-1	***-1***	10	-1
12	8	5	2	-1	-1	11	-1
-1	-1	-1	-1	-1	-1	-1	3
7	10	10	10	-1	3	-1	-1

Incumbent, OPT after 14

z = 26 1-2-3-7-8-6-4-5-1

(4-6) N 8-7 5-4 **6-5** Y 7-8 4-5 **5-6**

-1	2	11	10	-1	-1	6	-1
6	-1	1	8	-1	-1	6	-1
5	12	-1	11	-1	-1	3	-1
-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	***2***	-1	-1
12	8	5	2	***-1***	-1	11	-1
-1	-1	-1	-1	-1	-1	-1	3
7	10	10	10	-1	-1	-1	-1

Saturated

z = 21 1-2-3-7-8-1, 4-5-**6-4**

(6-7) N 8-7 5-4 6-5 **6-4** Y 7-8 4-5 5-6

-1	2	11	10	-1	-1	6	-1
6	-1	1	8	-1	-1	6	-1
5	12	-1	11	-1	-1	3	-1
-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	2	-1	-1
12	8	5	***-1***	-1	-1	11	-1
-1	-1	-1	-1	-1	-1	-1	3
7	10	10	10	-1	-1	-1	-1

Saturated (by 5)

z = 31 1-2-4-5-6-3-7-8-1

(6-8) N 8-7 5-4 6-5 **4-6** Y 7-8 4-5 5-6 **6-4**

-1	2	11	-1	-1	-1	6	-1
6	-1	1	-1	-1	-1	6	-1
5	12	-1	-1	-1	-1	3	-1
-1	-1	-1	-1	1	***-1***	-1	-1
-1	-1	-1	-1	-1	2	-1	-1
-1	-1	-1	***2***	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	3
7	10	10	-1	-1	-1	-1	-1

Saturated

z = 21 1-2-3-7-8-1, 4-5-6-4

(8-9) N 8-7 5-4 6-5 4-6 **1-2**
Y 7-8 4-5 5-6 6-4

-1	***-1***	11	-1	-1	-1	6	-1
6	-1	1	-1	-1	-1	6	-1
5	12	-1	-1	-1	-1	3	-1
-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	2	-1	-1
-1	-1	-1	2	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	3
7	10	10	-1	-1	-1	-1	-1

Saturated (by 5)

z = 30 (2 cycles)

(8-10) N 8-7 5-4 6-5 4-6 **2-1**
Y 7-8 4-5 5-6 6-4 **1-2**

-1	***2***	-1	-1	-1	-1	-1	-1
-1	-1	1	-1	-1	-1	6	-1
5	-1	-1	-1	-1	-1	3	-1
-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	2	-1	-1
-1	-1	-1	2	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	3
7	-1	10	-1	-1	-1	-1	-1

Saturated

z = 21 1-2-3-7-8-1 4-5-6-4

(10-11) N 8-7 5-4 6-5 4-6 2-1 **2-3** Y 7-8 4-5 5-6 6-4 1-2	(10-12) N 8-7 5-4 6-5 4-6 2-1 **3-2** Y 7-8 4-5 5-6 6-4 1-2 **2-3**
-1 2 -1 -1 -1 -1 -1 -1 -1 -1 ***-1*** -1 -1 -1 6 -1 5 -1 -1 -1 -1 -1 3 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 3 7 -1 10 -1 -1 -1 -1 -1	-1 2 -1 -1 -1 -1 -1 -1 -1 -1 ***1*** -1 -1 -1 -1 -1 5 ***-1*** -1 -1 -1 -1 3 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 3 7 -1 -1 -1 -1 -1 -1 -1
Saturated (by 5) z = 31 (2 cycles)	Saturated z = 21 1-2-3-7-8-1 4-5-6-4

(12-13) N 8-7 5-4 6-5 4-6 2-1 2-3 **3-7** Y 7-8 4-5 5-6 6-4 1-2	(12-14) N 8-7 5-4 6-5 4-6 2-1 3-2 **7-3** Y 7-8 4-5 5-6 6-4 1-2 2-3 **3-7**
-1 2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 6 -1 5 -1 -1 -1 -1 -1 ***-1*** -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 3 7 -1 10 -1 -1 -1 -1 -1	-1 2 -1 -1 -1 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 5 -1 -1 -1 -1 -1 ***3*** -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 -1 ***-1*** -1 -1 -1 -1 3 7 -1 -1 -1 -1 -1 -1 -1
Saturated z = 31 (2 cycles)	Saturated z = 21 1-2-3-7-8-1 4-5-6-4

(14-15) N 8-7 5-4 6-5 4-6 2-1 3-2 7-3 **8-1** Y 7-8 4-5 5-6 6-4 1-2 2-3 3-7	(12-14) N 8-7 5-4 6-5 4-6 2-1 3-2 7-3 **1-8** Y 7-8 4-5 5-6 6-4 1-2 2-3 3-7 **8-1**
-1 2 -1 -1 -1 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 5 -1 -1 -1 -1 -1 3 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 ***-1*** -1 -1 -1 -1 -1 -1 -1	-1 2 -1 -1 -1 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 5 -1 -1 -1 -1 -1 3 -1 -1 -1 -1 -1 1 -1 -1 -1 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 2 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 **3** 7 -1 -1 -1 -1 -1 ***-1*** -1
Saturated (infeasible) z = ∞	Saturated (no further nodes feasible) z = 21 1-2-3-7-8-1 4-5-6-4

3. Epilogue

Some examples of TSP were presented by B& B applied to successive AP relaxations. Comments were made to other methods.

4. Acknowledgements

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