Governance of risky public goods under graduated punishment

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ABSTRACT

Ensuring global cooperation often poses governance problems shadowed by the tragedy of the commons, as wrong-doers enjoy the benefits set up by right-doers at no cost. Institutional punishment of wrong-doers is well-known to curtail their impetus as free-riders. However, institutions often have limited scope in imposing sanctions, more so when these are strict and potentially viewed as disproportionate. Inspired by the design principles proposed by the late Nobel Prize Elinor Ostrom, here we study the evolution and impact of a new form of institutional sanctioning, where punishment is graduated, growing with the incidence of free-riding. We develop an analytical model capable of identifying the conditions under which this design principle is conducive to the self-organization of stable institutions and cooperation. We employ evolutionary game theory in finite populations and non-linear public goods dilemmas in the presence of risk of global losses whose solution requires the self-organization of decision makers into an overall cooperative state. We show that graduated punishment is more effective in promoting widespread cooperation than conventional forms of punishment studied to date, being also less severe and thus, presumably, easier to implement. This effect is enhanced whenever the costs of its implementation are positively correlated with the severity of punishment. We frame our model within the context of the global reduction of carbon emissions, but the results are shown to be general enough to be applicable to other collective action problems, shedding further light into the origins of Human institutions.

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1. Introduction

Cooperation remains one of the major scientific challenges of our century (Pennisi, 2005). It is not only challenging to understand the mechanisms underlying the emergence of cooperation in Nature and societies (Axelrod, 1984), but also daring to apply our current understanding of Human cooperation (Rand and Nowak, 2013; Ostrom, 1990) to foster pro-sociality in situations in which cooperation remains astray. The mitigation of the dangerous effects of climate change provides a pressing instance of such challenges (Barrett, 2005; Barrett, 2007; Tavoni and Levin, 2014; Scheffer, 2009; Levin, 2006; Carattini et al., 2019), though several other examples abound, from overuse of antibiotics to the control of global pandemics (Smith et al., 2005; Levin et al., 2013; Levin, 2006; Wang et al., 2016; Anderson et al., 2020). As long as parties involved behave solely according to their self-interest — that is, they are not willing to cooperate — collectives are unlikely to pre-

serve their common goods, heading into the doomsday scenario of the *tragedy of the commons* (Hardin, 1968).

One of the predicaments of global cooperation — e.g., as in the case of environmental governance or managing of socio-technical systems, among others — is related to the difficulty in the implementation of monitoring and sanctioning mechanisms to be imposed on those wrong-doers who free-ride on others. Punishment, despite its shortfalls, is an ubiquitous tool in Humans' (Axelrod, 1986; Boyd and Richerson, 1992; Fehr and Gächter, 2002) and other species' lives (Clutton-Brock and Parker, 1995; Goto et al., 2010). However, identifying (and agreeing on) the best way to distribute (costly) penalties remains an open question, given the difficulty in assessing the advantages and disadvantages of each possibility (Bowles, 2009; Boyd et al., 2010; Nakamaru and Dieckmann, 2009; Szolnoki et al., 2011; Szolnoki et al., 2011; Perc and Szolnoki, 2012; Ohdaira, 2016; Iwasa and Lee, 2013; Chen et al., 2015; Sigmund et al., 2010; Tavoni et al., 2012; Pitt et al., 2012; Pitt et al., 2018; Sasaki and Uchida, 2013; Sasaki and Uchida, 2014; Perc, 2012; Hilbe and Traulsen, 2012; Schoenmakers et al., 2014; Perc et al., 2017; Góis et al., 2019; García and Traulsen, 2019; Dong et al., 2019).

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Inspired by the work of the late Nobel Prize Elinor Ostrom (Ostrom, 1990), here we study the behavioural ecology created by a new form of institutional sanctioning. Ostrom reported that many successful communities facing collective dilemmas apply graduated sanctions — a punishment whose severity is gradually adjusted to the harm caused by the defector — proposing it as one of the key design principles for managing common-pool resources (Ostrom, 1990). We develop an analytical model capable of identifying the conditions under which this design principle is conducive to the self-organization of stable institutions and the emergence of cooperation. We do so in the context of climate agreements, but our approach is general enough to be extended to other non-linear public goods problems with uncertain returns (Santos and Pacheco, 2011; Pacheco et al., 2015; Souza et al., 2009; Perc et al., 2017).

We model climate action as a public goods game — known as the *collective risk dilemma* (**CRD**) — in which groups of N individuals aim at averting an uncertain collective loss (Milinski et al., 2008). Each player starts with an initial endowment b (deemed as the asset value at stake) that may be used to mitigate the effects of climate change. A successful collective action will ensue if a minimum number of contributions by group members is achieved. If this threshold (n_{PG}) is not attained or surpassed, everyone will lose their remaining endowment with a probability r, which measures the risk of disaster in the absence of an agreement. Otherwise, everyone will keep whatever they have left. We consider a finite population of $Z > N$ individuals, who may each behave as a cooperator (**C**), defector (**D**) or punisher (**P**). **C**s and **P**s contribute a fraction of their endowment to reach a common goal, whereas **D**s do not contribute. Furthermore, **P**s contribute an additional cost to set up a sanctioning institution. Thus, **C**s and **P**s are the right-doers, whereas **D**s are the wrong-doers, as will become apparent below. We explore the similarities between behavioural and evolutionary dynamics by assuming that individuals revise their choices based on the decisions and success of their peers, allowing errors and other stochastic effects to influence and affect this social learning process. Despite its simplicity, this approach builds on a set of previous models that have been shown to lead to predictions that match the results of economic experiments (Milinski et al., 2008; Tavoni et al., 2011; Barrett, 2011; Milinski et al., 2011; Barrett and Dannenberg, 2012). Examples are the individual perception of risk, wealth inequality, or collective targets or requirements, among others (Wang et al., 2009; Santos and Pacheco, 2011; Santos et al., 2012; Vasconcelos et al., 2013; Vasconcelos et al., 2014; Hilbe et al., 2013; Chakra and Traulsen, 2014).

An important aspect of punishment that cannot be disregarded is that it is costly (Axelrod, 1986; Ostrom, 1990; Fehr and Gächter, 2002; Perc and Szolnoki, 2012; Skyrms, 2014). This creates a second-order cooperation dilemma or public good (Sigmund, 2010; Sasaki et al., 2012; Oya and Ohtsuki, 2017) that may be as difficult to solve as the original dilemma we aim at solving. As a result, sanctioning may become inherently unstable, irrespective of whether sanctions are applied directly by peers (known as peer-punishment) (Fehr and Gächter, 2002; Sigmund, 2010; Hilbe and Traulsen, 2012) or through a sanctioning institution sustained by taxes (known as pool-punishment) (Sigmund et al., 2010; Traulsen et al., 2012; Vasconcelos et al., 2013; Schoenmakers et al., 2014), the paradigm we adopt here. Institutions can work efficiently by applying fines to those who do not contribute to the collective goals, helping increasing cooperation in several scenarios (Ostrom, 1990; Yamagishi, 1986; Sigmund et al., 2010; Traulsen et al., 2012; Chen et al., 2015; Vasconcelos et al., 2013; Perc, 2012). However, as it represents a relatively high cost to those who voluntarily support the institution, the abovementioned second-order dilemma arises.

We model graduated (pool) punishment as a sanction that is conditional to the group losses. Thus, instead of assuming a strict (fixed) sanction that is applied irrespectively of the current state of the collective, graduated punishment depends on the harm caused by wrong-doers in refusing to engage pro-socially in a group endeavor. Furthermore, the costs to maintain a sanctioning institution are not necessarily constant over time — in states where defection is less prevalent, the institution may function at lower expenses. With this reasoning, we also analyzed the impact of assuming graduated costs, proportional to the severity of punishment. Notably, Iwasa and Lee (Iwasa and Lee, 2013) have shown that fines that grow with one's offence may maximize the sum of the payoffs of all individuals. This result was obtained in a static model, in the absence of punishment costs, and an ecology of strategies. Differently, other works (Shimao and Nakamaru, 2013) showed that stricter forms of punishment are more effective than graduated forms when individuals are embedded in a network, and face an N-person Prisoner's dilemma. These seemingly contradictory results illustrate the subtlety of the problem at stake, which may depend on the nature of the specific dilemma.

In the next section, we provide details of the analytical model we develop, which combines ideas and tools stemming from apparently unrelated areas, from non-equilibrium statistical physics combined with the mathematics of stochastic processes to economics and theoretical ecology. We analyze three sanctioning paradigms, and study, for each case, the evolutionary dynamics of each strategy, and the overall prevalence of cooperation and sanctioning institutions. Our results suggest that graduated punishment (and costs) is more effective in promoting cooperation than strict punishment, being both less costly (more efficient) and likely simpler to implement.
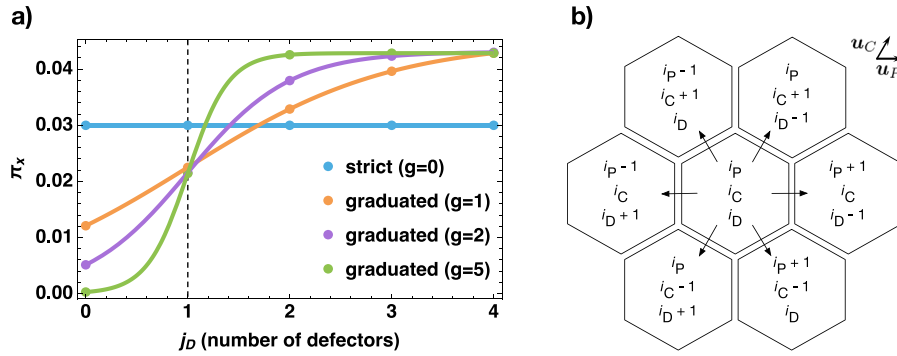
2. Model

We consider a finite population of size Z which participates in a **CRD** in which a sanctioning mechanism is allowed, and where individuals interact in groups of size N. Each individual may act as a **C**, a **P** or as a **D**. Every individual starts with an initial endowment b. **C**s and **P**s will contribute a fraction c of their endowment to a common pool, while **D**s do not contribute. If the total number of contributors in a group (the sum of the number of **C**s, j_C, and the number of **P**s, j_P) is below a certain threshold n_{PG} ($0 \leqslant j_C + j_P < n_{PG} \leqslant N$), everyone in the group will lose their remaining endowments with a probability r, the so-called risk of collective failure (Milinski et al., 2008; Santos and Pacheco, 2011; Vasconcelos et al., 2013). **P**s further contribute a punishment tax π_t to a sanctioning institution that effectively punishes **D**s with a fine π_f, provided there are enough funds available, that is, the institution will only be in place if at least n_P punishers contribute to it. We distinguish the type of institution by the scale at which it is formed and upon which it acts (Vasconcelos et al., 2013) — a global one (e.g., the United Nations) spanning the entire population (being supported by all **P**s and punishing all **D**s), as well as a local one acting at the group level (supported by all j_P **P**s in the group and punishing all j_D **D**s in that same group).

The payoff functions for **C**s, **P**s, and **D**s in a group where there are j_C **C**s, j_P **P**s, and $j_D = N - j_P - j_C$ **D**s can be written as

$$
\begin{aligned}
\Pi_C &= -cb + b\,\Theta(j_C + j_P - n_{PG}) + (1-r)b[1 - \Theta(j_C + j_P - n_{PG})] \\
\Pi_P &= \Pi_C - \pi_t \\
\Pi_D &= \Pi_C + cb - \pi_f\,\Theta(j_P - n_P)
\end{aligned}
\tag{1}
$$

respectively, where $\Theta(k)$ is the Heaviside function (being zero for $k < 0$ and one for $k \geqslant 0$), n_{PG} is a positive integer ($0 < n_{PG} < N$), and r ($0 \leqslant r \leqslant 1$), b, c, π_t (*tax*), and π_f (*fine*) are positive real num-

Fig. 1. Model details. **(a)** Tax (or fine) π_x versus the number of defectors j_D in a group, plotted for the different types of policies (see Eq. 2). The parameter g defines the steepness of the curves. The average $\overline{\pi_x}$ (over the number of defectors j_D) is the same for all values of g to enable direct comparison. Figure parameters: $N = 4$, $n_{PG} = 3$, and $\overline{\pi_x} = 0.03$. **(b)** Local illustration of the 2-dimensional phase space and of possible transitions from a given state $\boldsymbol{i} = \{i_P, i_C, i_D\}$ (each state $\boldsymbol{i}$ is represented by a hexagonal cell).

bers. Note that Eqs. (1) are defined for local institutions. For a global institution, punishment acts at the population level. In this case, we substitute j_P in Π_D, referring to group members, by i_P referring to the total number of punishers in the entire population (see Supplementary Data — **SD**).

Under strict punishment (cost), π_f (π_t) is a positive constant (Vasconcelos et al., 2013). Here we make π_x (with x a placeholder for $\{t, f\}$) depend on the number of **D**s by introducing a Fermi-type dependence (see Fig. 1a),

$$\pi_x \rightarrow \pi_x(j_D) \propto \frac{1}{1 + e^{-g[j_D - (N - n_{PG})]}} \qquad (2)$$

where g defines the steepness of the functions π_x centered at $N - n_{PG}$, the number of **D**s above which there is a risk r of losing everything. Thus, in the following, we investigate three different sanctioning policy combinations — **Strict fines and Strict taxes** (**SS**), **Graduated fines and Strict taxes** (**GS**), and **Graduated fines and Graduated taxes** (**GG**).

Every state of the population of size Z is defined by a vector $\boldsymbol{i} = \{i_C, i_P, i_D\}$, where i_K is the number of individuals in the population with strategy K. Similarly, we define the configuration of a group as $\boldsymbol{j} = \{j_C, j_P, j_D\}$, where j_K is the number of individuals within the group with strategy K. Naturally, given that we are assuming three strategies, the phase space simplex is a triangle where, at each point, the sum of individuals in the population adds up to Z. The *fitness* of a strategy corresponds to the average payoff of an individual playing that strategy in a population with configuration $\boldsymbol{i}$. Since all individuals are equally likely to interact in groups of size N (i.e., assuming a well-mixed population), we may write the fitness function $f_K(\boldsymbol{i})$ of a strategy K as (Hauert et al., 2007; Pacheco et al., 2008; Gokhale and Traulsen, 2010; Santos and Pacheco, 2011 and Van Segbroeck et al., 2012)

$$f_K(\boldsymbol{i}) = \binom{Z-1}{N-1}^{-1} \times \sum_{(\boldsymbol{j}:j_K=0)}^{(\boldsymbol{j}:j_K=N-1)} \Pi_K(\boldsymbol{j}) \binom{i_K-1}{j_K} \prod_{L=1(L\neq K)}^{3} \binom{i_L}{j_L} \qquad (3)$$

where $\Pi_K(\boldsymbol{j})$ is the payoff of a strategy K in a group with composition $\boldsymbol{j}$, and where $(\boldsymbol{j}:j_K=q)$ designates any group configuration in which there are specifically q players with strategy K.

Time evolution proceeds via a birth-death process (Karlin & Taylor, 1975; Taylor et al., 2004; Traulsen et al., 2006). Individuals revise their strategy through a social learning process equivalent to natural selection (with mutation or random exploration of strategies). At each time step, a random individual with strategy X is selected; with probability μ, she will mutate to a randomly chosen strategy from the space of available strategies; with probability $1 - \mu$, another individual with strategy Y is randomly selected;

subsequently, the individual with strategy X will update her strategy to Y with probability given by (Traulsen et al., 2006)

$$\varphi = \frac{1}{1 + e^{\beta(f_X - f_Y)}} \qquad (4)$$

where the "inverse temperature" $\beta(\geqslant 0)$ controls the intensity of natural selection, and f_X and f_Y are the fitness values of strategies X and Y, respectively. Because the update process only depends on the current state of the system (Markov process), the associated evolutionary dynamics of the vector $\boldsymbol{i}(t)$, whose probability density function we designate by $p_i(t)$, satisfies the *Master Equation* (van Kampen, 2007)

$$p_i(t + \tau) - p_i(t) = \sum_{\boldsymbol{i}'} \{T_{\boldsymbol{ii}'} p_{i'}(t) - T_{\boldsymbol{i}'\boldsymbol{i}} p_i(t)\} \qquad (5)$$

where $T_{\boldsymbol{ii}'}$ and $T_{\boldsymbol{i}'\boldsymbol{i}}$ are the transition rates between configurations $\boldsymbol{i}$ and $\boldsymbol{i}'$. In the following, we will make use of the stationary distribution ($\overline{p_i}$), which gives the fraction of time that the population spends in each possible configuration, and that will prove useful in computing average properties of this population dynamics. $\overline{p_i}$ can be easily computed in a semi-analytic way via an eigenvector search problem (van Kampen, 2007; Hindersin et al., 2019). Specifically, it is given by the eigenvector associated with the eigenvalue 1 of the transition matrix $\Lambda = [T_{ij}]^T$ whose elements are defined below.

The birth-death nature of the update process greatly restricts the number of possible transitions that can occur starting from a given configuration. This is illustrated in Fig. 1b, which also reflects the two dimensional layout of the phase space simplex. As a result, the matrix elements $T_{\boldsymbol{i}'\boldsymbol{i}}$ ($\boldsymbol{i}'\neq\boldsymbol{i}$) of Λ are associated with events in which an individual in state $\boldsymbol{i}$ with a given strategy L changes into another specific strategy K which, for the selection rule described above, may be written as follows
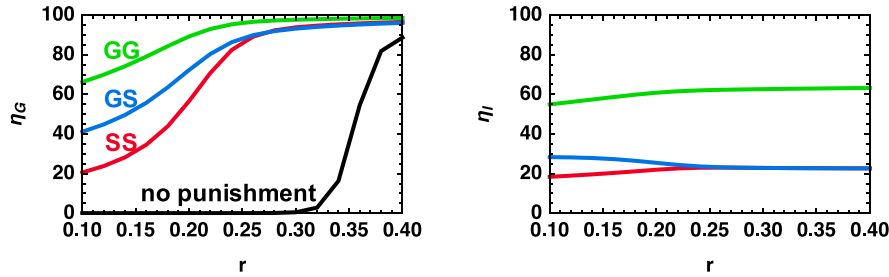
$$T_{L\rightarrow K}(\boldsymbol{i}) = (1 - \mu)\left[\frac{i_L}{Z}\frac{i_K}{Z-1}\left(1 + e^{\beta(f_L - f_K)}\right)^{-1}\right] + \mu\frac{i_L}{2Z}. \qquad (6)$$

Thus, the probability that the population remains in the same state is

$$T_{\boldsymbol{ii}} = 1 - \sum_{\boldsymbol{i}'\neq\boldsymbol{i}} T_{\boldsymbol{i}'\boldsymbol{i}}. \qquad (7)$$

Moreover, the probability to increase (decrease) by one the number of individuals with strategy K, $T_{\boldsymbol{i}}^{K^+}$ ($T_{\boldsymbol{i}}^{K^-}$) is given by

$$T_{\boldsymbol{i}}^{K^+} = \sum_{L\neq K} T_{L\rightarrow K}(\boldsymbol{i})$$
$$T_{\boldsymbol{i}}^{K^-} = \sum_{L\neq K} T_{K\rightarrow L}(\boldsymbol{i}). \qquad (8)$$

Fig. 2. Average group achievement η_G (left) and average institution prevalence η_I (right), in percentage, are plotted as a function of risk r, for different (local) sanctioning policies: Strict fines and taxes (**SS**), Graduated fines and Strict taxes (**GS**), and Graduated fines and taxes (**GG**). For global institutions, please see **SD**. Increasing graduation in punishment leads to increasing effectiveness in promoting overall success, mostly at lower risk. Model parameters: $Z = 100, b = 1, c = 0.1, N = 4$, $n_{PG} = 0.75 \times N, n_P = 0.25 \times N, \mu = 1/Z, \beta = 5, g = 2.5, \overline{\pi_f} = 0.3$, and $\overline{\pi_t} = 0.03$.

These quantities are useful to compute the gradient of selection ∇_i, which indicates the most likely direction of evolution of the population in phase space, and reads

$$\nabla_i = \left(T_i^{C^+} - T_i^{C^-} \right) u_C + \left(T_i^{P^+} - T_i^{P^-} \right) u_P \tag{9}$$

where we choose the unit vectors u_C and u_P as basis vectors (see Fig. 1b).

3. Results and discussion

In order to investigate the role of different sanctioning policies, we shall start by calculating some quantities of interest, such as the average fraction of groups that succeed in attaining a public good — the **average group achievement** η_G — as well as the average fraction of groups that succeed in setting up sanctioning institutions — the **average institution prevalence** η_I. They can be easily computed in terms of the stationary distribution $\overline{p_i}$ of the Markov process (Vasconcelos et al., 2013)

$$\eta_G = \sum_i \overline{p_i} a_G(i)$$
$$\eta_I = \sum_i \overline{p_i} a_I(i) \tag{10}$$

where

$$a_G(i) = \binom{Z}{N}^{-1} \times \sum_{\{j_P, j_C\}} \Theta(j_C + j_P - n_{PG}) \binom{i_P}{j_P} \binom{i_C}{j_C} \binom{i_D}{j_D}$$
$$a_I(i) = \binom{Z}{N}^{-1} \times \sum_{\{j_P, j_C\}} \Theta(j_P - n_P) \binom{i_P}{j_P} \binom{i_C}{j_C} \binom{i_D}{j_D} \tag{11}$$

Similarly, using the stationary distribution we can also obtain the average population configuration, computing the weighted average of all possible configurations of the population, and also the average fitness function (which denotes the overall welfare of the population).

In Fig. 2, we show the dependence of η_G and η_I on the risk r, for different sanctioning policies employing local institutions (for global institutions, see **SD**). Clearly, any type of institutional policy not only succeeds in creating sanctioning institutions but also improves the overall success of generating a public good. However, the introduction of graduation, in both punishment and institutional costs (**GG**) greatly improves the odds of success, compared with strict punishment (**SS**). In particular at low-risk, **GG**-policy more than triples the generation of public goods compared to **SS**-policy.

To probe deeper into the underlying mechanisms responsible for such a significant improvement stemming from graduated poli-

cies we show, in Fig. 3, the overall population dynamics for a fixed (and low) value of risk ($r = 0.15$). The arrows represent the gradient of selection, indicating the most likely direction of evolution (see Eq. 9). The gray shading in each simplex portrays the associated stationary distribution ($\overline{p_i}$) of the evolutionary dynamics.

From Fig. 3, as we move from left to right (**SS** → **GS** → **GG**), it is clear that the fraction of time the populations spend in the vicinity of a monomorphic state composed by defectors (around vertex **D**) is reduced, given the fact that, under graduated punishment, **D**s are effectively punished, even if sometimes by smaller amounts compared to punishment under **SS**-policy. This effect becomes most prominent under the **GG**-policy, i.e., when both sanctions and taxes become graduated and, as a result, institutions become easier to form. This trend is also corroborated by inspection of the gradients of selection in Fig. 3, where the magnitude of the transition from **D**s to **P**s increases as we move from left to right.

Interestingly, the adoption of graduated fines (by moving from **SS** to **GS**) leads to a significant increase of **C**s in the population, having however a limited impact on the fraction of individuals contributing to institutions (**P**s). Thus, while fostering cooperation, graduated fines, alone, cannot solve the second-order free-riding problem associated with institutional sanctioning. Differently, when taxes are also gradual (i.e., when we move from **GS** to **GG**), one observes a notable increase in the prevalence of **P**s at the expense of, mostly, a corresponding reduction of **D**s. This is achieved through the creation of a new coexistence state between **C**s and **P**s in the vicinity of the **C-P** edge of the simplex, as **P**s pay little costs in the absence of wrong-doers (**D**s). Thus, **GG**-policy, which combines the advantage of graduated punishment in surpassing the coordination points near vertex **D** and the benefit of graduated costs in keeping sustainable institutions (most evident in **C-P** edge), emerges as a promising ingredient to overcome such second order free-rider problems.

Finally, in Fig. 4 we investigate in more detail the role of $\overline{\pi_f}$ and $\overline{\pi_t}$ in the overall success in producing a public good (η_G) and in creating sanctioning institutions (η_I), by comparing the **SS**-policy with the **GG**-policy. Clearly, increasing $\overline{\pi_f}$ generally enhances cooperation, whereas $\overline{\pi_t}$ inhibits it. When directly comparing policies, in turn, we see that, once again, graduated punishment and costs greatly increase the range of parameter values (for both $\overline{\pi_f}$ and $\overline{\pi_t}$) for which the values of η_G and η_I are conducive to a solution of the **CRD** we are studying. Plus, we verify that results for the average fitness of the population are qualitatively similar to η_G, showing that the problem of overall welfare decrease due to punishment (Dreber et al., 2008) can be solved as well.

At this point it is relevant to ask what is the role of the parameter g, that is, the rate of change of the punishment and costs in the vicinity of $N - n_{PG}$. In the **SD**, we show that increasing g leads to a further increase in both η_G and η_I, mostly under the **GG**-policy.
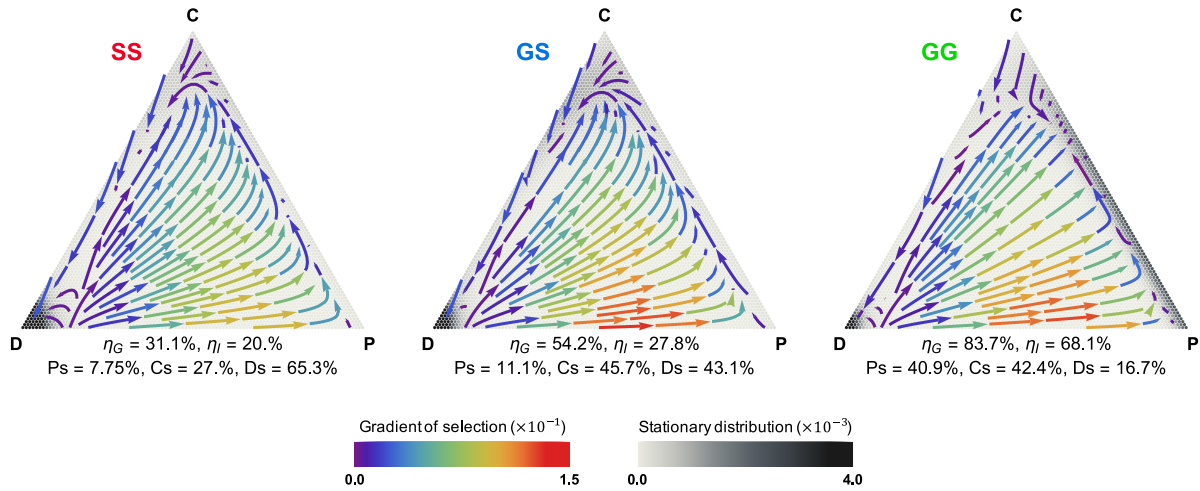
Fig. 3. Gradient of selection at each point of the state space i (colored vectors) and stationary distribution (gray shading) for local institutions and different sanctioning policies — Strict fines and Strict taxes (**SS**, left panel), Graduated fines and Strict taxes (**GS**, middle panel), and Graduated fines and Graduated taxes (**GG**, right panel). Below each simplex, the corresponding values of η_G, η_I, and the average population configuration are displayed. Model parameters: $Z = 100, b = 1, c = 0.1, r = 0.15, N = 4, n_{PG} = 0.75 \times N, n_P = 0.25 \times N, \mu = 1/Z, \beta = 5, g = 5$ (when graduated), $\overline{\pi_f} = 0.3$, and $\overline{\pi_t} = 0.03$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
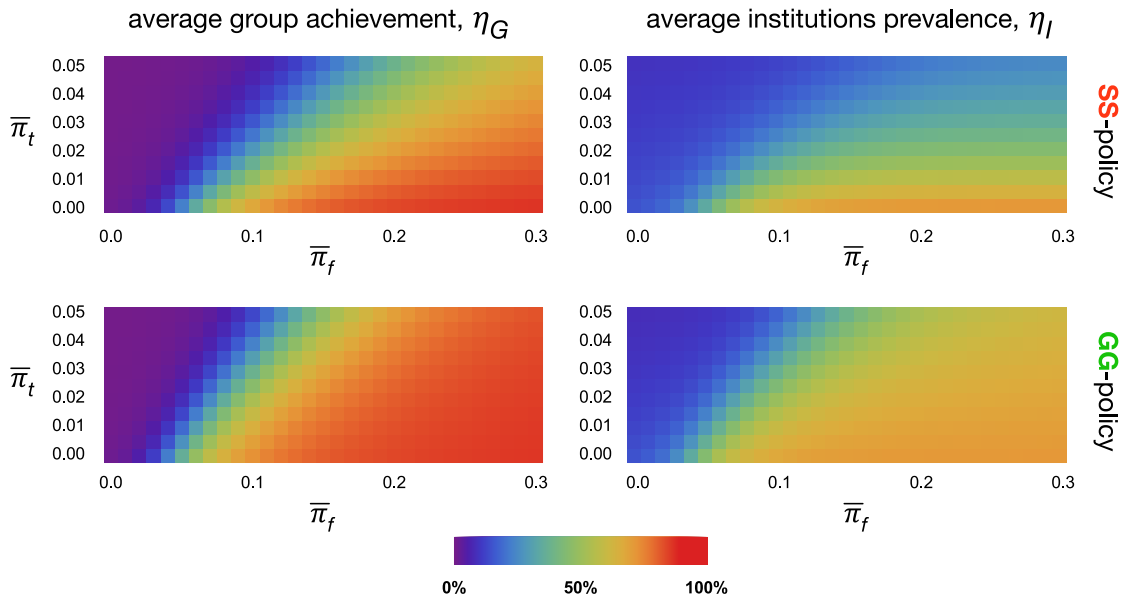
Fig. 4. Average group achievement η_G (left) and average institution prevalence η_I (right) for different values of $\overline{\pi_f}$ and $\overline{\pi_t}$, for the **SS**-policy (top) and the **GG**-policy (bottom). The range of variation of $\overline{\pi_f}$ and $\overline{\pi_t}$ is the same in all panels (as indicated). Model parameters: $Z = 50, b = 1, c = 0.1, r = 0.2, N = 4, n_{PG} = 0.75 \times N, n_P = 0.25 \times N, \mu = 1/Z, \beta = 5$, and $g = 5$ (when graduated).

Moreover, we also show how our present results on graduated punishment and costs are robust to variations of the other parameters — Z, c, n_{PG}, n_P, β, and μ — and even to changes in the nature of the dilemma, namely for linear public goods (see Section 3 in the **SD**).

4. Conclusions

To summarize, graduated punishment is shown to be more effective than strict punishment at preventing defection in a collective risk dilemma when the joint effort needed to maintain the public good is high. This is especially so for low levels of risk.

Furthermore, the **GG**-policy provides a solution to the problem of second-order free-riding, suggesting a new route to the origin of institutions among self-regarding individuals. The present approach overcomes most of the problems encountered in previous implementations of punishment, namely, those associated with the artificiality of assuming unconditional and uncoordinated punishment, as noticed in Boyd et al. (2010) and Hilbe and Sigmund (2010). Although we do not include explicitly communication between **P**s, their action is somewhat coordinated indirectly via the graduated costs — though the institution threshold (n_P) remains constant, the cost of punishment depends sensitively on the number of individuals amenable to be punished (**D**s).

Whenever this number is small, so are the costs incurred by **P**s, which renders them viable in the overall evolutionary dynamics. In the **SD**, we show that the conclusions obtained for the collective-risk dilemma remain valid in the context of public goods games with linear returns. This further emphasizes how our conclusions are general enough to be applicable to other collective action problems beyond climate action, offering a broader insight into the evolutionary origins of this kind of human institutions.

Finally, also in the **SD**, we further show that cooperation and the success of graduated punishment are significantly enhanced when multiple small institutions are considered when compared with a single global institution. These results are in line with the polycentric approach to environmental governance (also) proposed by Elinor Ostrom (Ostrom, 2010). Indeed, many small-scale societies uphold the common interest through rudimentary bottom-up institutions (Ostrom, 1990; Sigmund et al., 2010), which suggests that negotiations on climate change should benefit from decentralized and graduated institutions to mitigate possible dangerous effects, eventually coordinated with top-down regulative approaches.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.jtbi.2020.110423.

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Electronic Supplementary Material:
Governance of Risky Public Goods under Graduated Punishment

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In this Electronic supplementary material, we show that the results on the main text are robust to variations of the parameters, as well as to changes in the nature of the dilemma, namely when linear public goods are considered.

1. Global vs Local institutions

In Fig. 1, we show the dependence of the average group achievement (η_G, left panels) and of the average institutions prevalence (η_I, right panels) on the risk r, for local and global institutions, and for Strict fines and taxes (**SS**, in red), Graduated fines and Strict taxes (**GS**, in blue), and Graduated fines and taxes (**GG**, in green). Black lines indicate the reference scenario for the absence of any institution. It is clear that global institutions fail in the low risk region ($r \lesssim 0.3$) for all three sanctioning policies, which supports the bottom-up institutions philosophy proposed in Ostrom (2010) and analyzed in Santos & Pacheco (2011); Vasconcelos et al. (2013, 2015). For $r > 0.3$, cooperation emerges easily and there is no big variation between the different sanctioning policies (all η_G curves fairly overlap) – the risk plays the lead role. Since in

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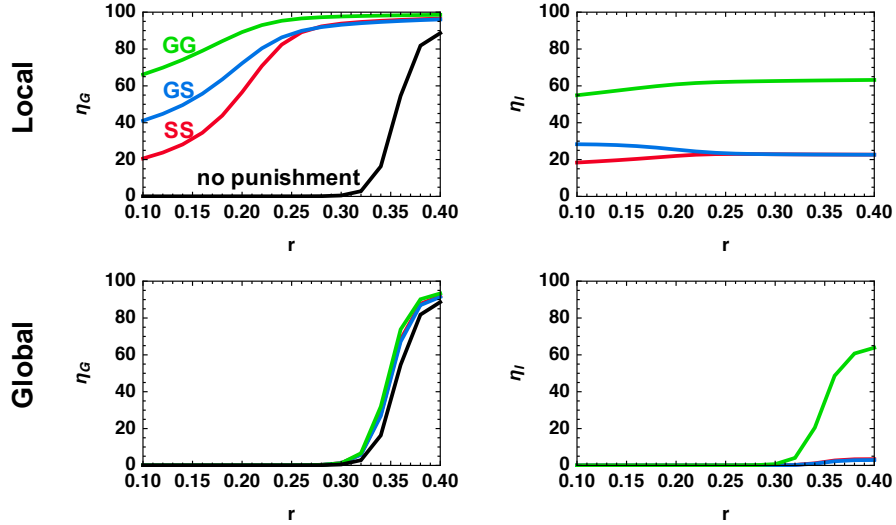*Email address:* `franciscocsantos@tecnico.ulisboa.pt` (Francisco C. Santos)

Figure 1: Average group achievement η_G (left) and average institution prevalence η_I (right) are plotted as a function of risk r, for different sanctioning policies, for local (above) and global (below) institutions. Model parameters: $Z = 100$, $b = 1$, $c = 0.1$, $N = 4$, $n_{PG} = 0.75 \times N$, $n_P = 0.25 \times N$ (local), $n_P = 0.25 \times Z$ (global), $\mu = 1/Z$, $\beta = 5$, $g = 2.5$ (local), $g = 0.25$ (global), $\overline{\pi_f} = 0.3$, $\overline{\pi_t} = 0.03$.

the case of graduated costs (**GG**-policy), the taxes are low when there is little defection, the institution is maintained with minor costs, that is, **C**s and **P**s are virtually the same, which leads to the rise of η_I (green curve in the bottom right figure).

2. Dependence on other parameters

To assess how graduate (or steep) should the punishment and costs be, we explore the dependence on parameter g. In Fig. 2, we can see that the higher this parameter, the more cooperation and institutions are enhanced. This means that the variation of fines and taxes (according to the number of defectors in the group) should be abrupt around $N - n_{PG}$ (see Fig. 1a in the main text).

In Fig. 3, we study the importance of the group size N and conclude that small groups are more favourable to a cooperative behaviour (as in Vasconcelos et al. (2013)) and also to sanctioning institutions prevalence.
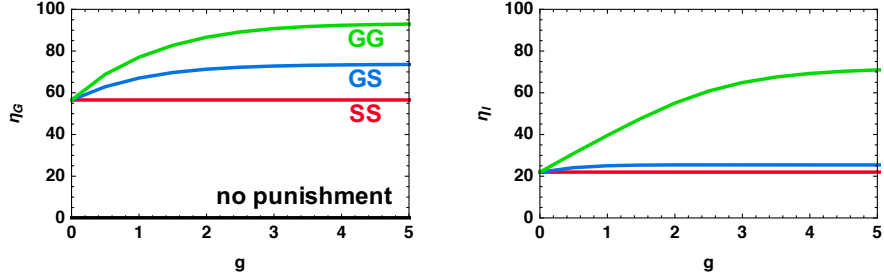
Figure 2: Average group achievement η_G (left) and average institution prevalence η_I (right) versus the steepness of graduated punishment/cost g, for local institutions and different sanctioning policies. For $g = 0$, SS-policy is recovered. Model parameters: $Z = 100$, $b = 1$, $c = 0.1$, $r = 0.2$, $N = 4$, $n_{PG} = 0.75 \times N$, $n_P = 0.25 \times N$, $\mu = 1/Z$, $\beta = 5$, $\overline{\pi_f} = 0.3$, $\overline{\pi_t} = 0.03$
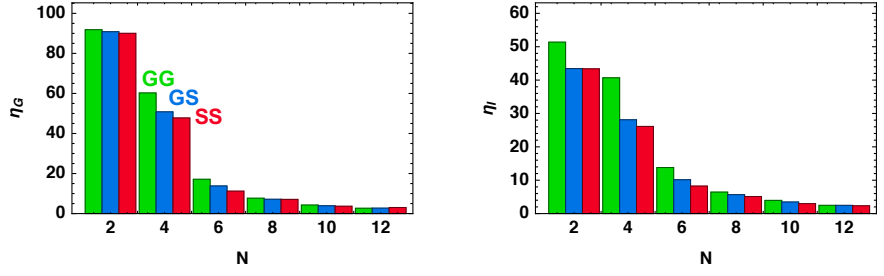

Figure 3: Average group achievement η_G (left) and average institution prevalence η_I (right) versus the size of the groups N, for local institutions and different sanctioning policies. Model parameters: $Z = 50$, $b = 1$, $c = 0.1$, $r = 0.2$, $n_{PG} = 0.75 \times N$ (note that since n_{PG} it is not always an integer, under this definition and for the chosen values of N, the value considered is the first integer above), $n_P = 0.5 \times N$, $\mu = 1/Z$, $\beta = 3$, $g = 5$, $\overline{\pi_f} = 0.3$, $\overline{\pi_t} = 0.02$.

The results about graduated punishment and costs are robust to variation of other parameters – Z, c, n_{PG}, n_P, β, μ, $\overline{\pi_f}$, and $\overline{\pi_t}$. Naturally, if c is high, fewer individuals will be willing to contribute, decreasing overall cooperation. Moreover, increasing n_P also hinders cooperation, as intuitively expected. It is also noteworthy that for low n_{PG} (compared to N), graduated punishment and costs do not always enhance cooperation more than strict punishment. However, as long as n_{PG} remains close to N, that is, as long as one requires a high consensus in the group before generating a public good, graduated policies

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will always be preferable to strict policies. Despite all dependencies, graduated

punishment and costs (**GG**-policy) works better than strict (**SS**-policy) for a

broader set of parameters.

3. Linear Public Goods Game

Until now, our discussion has been framed within the context of climate

action and the collective-risk dilemma. However, our conclusions are general

enough to be applicable to other collective action problems. As an example,

in this section, we address the impact of graduated sanctions in a linear pub-

lic goods game (LPGG), also known as the N-person Prisoner's dilemma, and

confirm the robustness of previous results.

In this case, the payoff functions for **C**s, **P**s, and **D**s in a group of j_C **C**s, j_P

Ps, and $j_D = N - j_P - j_C$ **D**s playing a LPGG can be written as

$$
\begin{aligned}
\Pi_C &= \left[\frac{F}{N}(j_C + j_P) - 1 \right] c \\
\Pi_P &= \Pi_C - \pi_t \\
\Pi_D &= \Pi_C + c - \pi_f\, \Theta(j_P - n_P)
\end{aligned}
\tag{1}
$$

where F is the multiplication or synergy factor and all other parameters had been introduced before. Under strict punishment (cost), π_f (π_t) is a positive constant. Here we make π_x (with x a placeholder for $\{t, f\}$) depend linearly on the number of **D**s,

$$
\pi_x \rightarrow \pi_x(j_D) = \overline{\pi_x}\,[1 + g\,(j_D - N/2)]
\tag{2}
$$

where g defines the slope of the functions π_x.

In the LPGG, no threshold (n_{PG}) exists. Consequently, in the following we

introduce a natural counterpart of the average group achievement (η_G) that we

designate by χ_G, defined as the weighted average of the fraction of **C**s and **P**s

in the population,

$$
\chi_G = \sum_i \overline{p_i} \left(\frac{i_C + i_P}{Z} \right) \times 100
\tag{3}
$$

54 Naturally, in what concerns the average institution prevalence (η_I) nothing is
55 changed.

56 In Fig. 4, we display two interesting dependencies, on g and F, while showing
57 that **GG**-policy remains as the most effective one in achieving cooperation and
58 sanctioning institutions, followed by **GS**-policy, and then **SS**-policy. The same
59 result is confirmed for variations in the other parameters. As expected, the
60 larger the multiplication factor F, the easier is to sustain cooperation, since the
61 public good is more profitable. Interestingly, the effect of g is similar to the
62 non-linear case (see Fig. 2) — the greater the slope of the punishment or cost
63 functions, the better the outcomes.

64 Overall, our conclusions remain valid under linear public goods games, being
65 general enough to be applicable to different collective action problems, and
66 offering a broader view into the evolutionary origins of this class of institutions.
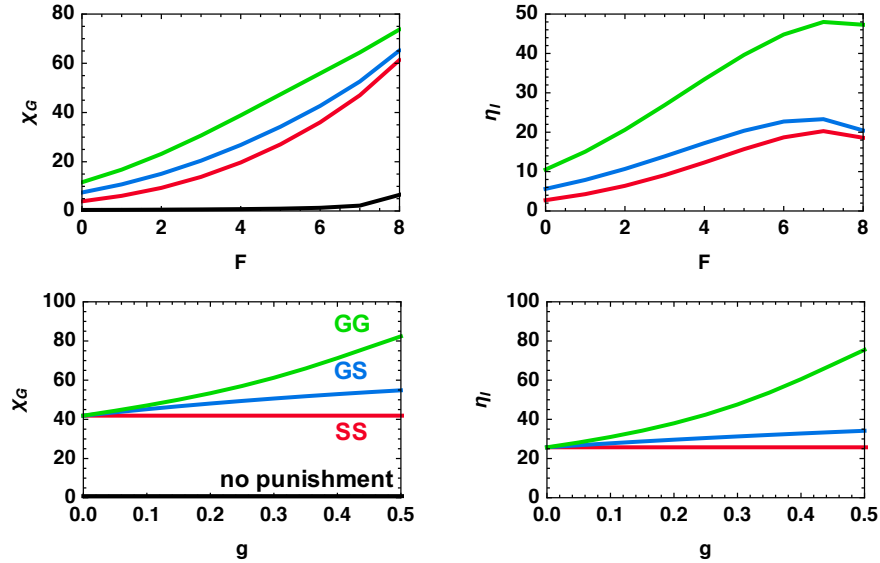

Figure 4: χ_G (left) and η_I (right) versus the multiplication or synergy factor F (above) and
the slope of graduated punishment/cost g (below), for different sanctioning policies (same
colors as before). Model parameters: $Z = 50$, $N = 8$ (above), $N = 4$ (below), $n_P = 0.25 \times N$,
$F = 2$ (below), $c = 0.1$, $\mu = 0.001$, $\beta = 5$, $g = 0.2$ (above), $\overline{\pi_f} = 0.3$, $\overline{\pi_t} = 0.03$

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