Dynamics and Control of a High Speed Train Pantograph System

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High speed trains permit faster travelling between long distance destinations, making it an easy and comfortable way of travelling. The pantograph is the element of the train that collects electrical current from the cable system above (the catenary), to the train motors. The contact force variation can cause contact losses, electric arc formations and sparking. This deteriorates the quality of current collection and increases the electrical related wear, therefor becoming a limiting factor for the maximum train speed. The increase of the static contact force is not an efficient way to deal with the problem, because it increases mechanical abrasive wear and produces an excessive uplift of the contact wire. Maintaining the contact force in an admissible region is crucial for high speed trains. In this work a model in SimMechanics is created for the pantograph and the catenary. The complexity of the contact interface between the pantograph and the catenary is studied. The control strategy is based on a PID controller, and robust $H_2$ and $H_\infty$ controllers. Both approaches are studied and compared. A virtual reality pantograph is created for better perception of the motion of the pantograph. The main targeted conclusion is to confirm that the usage of robust control is superior and more flexible then classical PID control strategies making the current collection constant, and therefore producing low wear of the registration strip. Furthermore, we verify that model approximations are very influential on contact force dynamics.

Keywords: Pantograph, Catenary, Robust controller, Modelling, Closed chain systems.

1. INTRODUCTION

High speed rail vehicles are a comfortable and fast way of transportation. Until today open loop pantograph systems proved to be a very robust solution for trains. Because of rail line degradation, train circulation with adverse weather conditions, like intense wind, and the desire to achieve higher speeds, can cause excessive wear in the pantograph contact interface. Improving high speed circulation on the railway isn’t the only goal in the presented study, low cost maintenance of the pantograph and the catenary is equally important, thus the ideal solution would be of having a system that has low maintenance comparing with present solutions, higher train circulation speeds and to avoid the necessity of changing the hole railway system, including the catenary. Train speed, and pantograph actuating force with the catenary, are the primary variables to maintain a stable current collection.

High speeds generally produce lower contact forces, a problem that arises from building faster trains is in the current collector of the train, the pantograph. In order to collect electric current from the cable network system above the train, the catenary, it is necessary for the pantograph to contact the catenary. Excessive contact force causes damage to the pantograph and the lack of contact force causes current perturbation and electric arcs which damages the pantograph.

The pantograph which will be studied is the SNCF train pantograph and it functions as follows: it is activated by a pneumatic device comprised of an air cushion controlled by an electro-pneumatic plate and assisted by an electronic board. The system adjusts the air cushion pressure, and at the same time, the force to apply on the catenary, in real time, to obtain the best possible electrical contact between the catenary and the pantograph, regardless of operating conditions. The applied pressure is calculated according to the speed of the train-set, taking the load bearing capacity of the pantograph itself, the movement direction and the composition of the train-set (single unit or multiple unit) into account.

2. MODEL CREATION

The Pantograph model used in this paper is based on the CX pantograph from SNCF(fig. 1), this pantograph has spring and damper elements in the registration strip.
Although the primary model has passive elements in the registration strip, variations in this model were studied to understand the importance of these elements in model dynamics and influence in control results.

A pantograph consists of a collection of bodies and mechanical elements attached to a railway carbody that is moving along the track. Due to their structural stiffness, the components that compose the pantograph are considered here as rigid bodies. These bodies are connected by a set of kinematic joints, responsible to control their relative motion and are used to model the relevant internal forces resulting from the interaction among bodies of the system (see Wood and Kennedy [2003]).

The dynamics of a pantograph is complex, simplifications have to be made in order to obtain a simple and easy model to study. The contact interface study was divided in various models organized by complexity, the reference model which compares to the SCNF pantograph model is a one degree of freedom movement registration strip with a catenary force directly applied to the center of the pantograph and is called the Type 1 model.

To obtain the nonlinear model, a SimMechanics pantograph was created, the formulation of the numeric equations of motion was automatic. Using SimMechanics simplifies the modelling process, specially closed chain systems. The modelling environment uses the Newton Euler equations for closed chain systems which is the most efficient numeric method for these structures.

A correct model of the pantograph means that the contact interface has to be the more realistic as possible (fig. 2). A progressive complexity approach was used to obtain the nonlinear model system and thus the pantograph dynamics. In order to use robust controllers, linearization of the nonlinear model had to be done, all linear models are obtained in the nominal conditions, the process can be seen in fig. 14.

<table>
<thead>
<tr>
<th>movement</th>
<th>Actuator</th>
<th>Passive elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Plane</td>
<td>1</td>
<td>not present</td>
</tr>
<tr>
<td>Spacial</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Plane</td>
<td>2</td>
<td>present</td>
</tr>
<tr>
<td>Spacial</td>
<td>3</td>
<td>present</td>
</tr>
</tbody>
</table>

Table 1. Pantograph model type considerations

3. VIRTUAL PANTOGRAPH

To better visualize the motion of the pantograph a 3D model was developed, this model was made with the help of SolidWorks, for the creation of individual elements. The assembly was done using VRML tools. In fig. 4 is a picture of the virtual world developed with the above tools referred.

4. CATENARY MODEL

The catenary model is a spring/damper system, these two elements are used in the $z$ direction and in the $y$ direction, both are placed in the center of the registration strip. This is a simplification of the system meaning that the pantograph only has one degree of freedom. The implementation is simple it uses a parallel association of a spring and a damper elements; to simulate catenary perturbation (force variations) passive element displacement is used that generate as a result force variations. The implementation follows in fig. 5 and it’s global positioning in the pantograph can be seen in fig. 5. Other authors instead of this approach use finite element developments to obtain the catenary system (see Pisano and Usai [2004], Jin et al. [2002], Seo et al. [2006]).
One of the performance objectives of the controller design is to keep the error between the controlled output and the set-point as small as possible, when the closed-loop system is affected by external signals. Thus, to be able to assess the performance of a particular control, we need to be able to quantify the relationship between this error, the process and the controller. The sensitivity function has an important role in robust control definitions (an introduction about the sensitivity function can be found in Botto [2006] on page 34).

5.1 Robust Control Definition

From Chandrasekharan [1996], "Robust control refers to the control of unknown plants with unknown dynamics subject to unknown disturbances". The key issue with robust control systems is uncertainty and how the control system can deal with this problem. Fig. 6 shows an expanded view of the simple closed-loop, uncertainty is shown entering the system in three places. There is uncertainty in the model of the plant. There are disturbances that occur in the plant system. Also there is noise which is read on the sensor inputs. Each of these uncertainties can have an additive or multiplicative component.

5.2 The sensitivity function

The sensitivity function that we will use is defined in the Laplace domain as:

\[ S(s) \triangleq \frac{E(s)}{R(s) - d(s)} \]

where the symbol “\( \triangleq \)" is used to denote ”definition”. Thus the sensitivity function, \( S(s) \), relates the external inputs, \( R(s) \) and \( d(s) \), to the feedback error \( E(s) \). Notice, however, that it does not take into account the effects caused by the noise, \( N(s) \).

\[ E(s) = R(s) - Y(s) = R(s) - [G_P(s)U(s) + d(s)] \quad (1) \]

But, \( U(s) = G_c(s)E(s) \)

i.e. \( E(s) = R(s) - G_c(s)G_P(s)E(s) - d(s) \)

Rearranging \( E(s)[1 + G_cG_P] = R(s) - d(s) \)

Hence, \( \frac{E(s)}{R(s) - d(s)} = \frac{1}{1 - G_c(s)G_P(s)} \)

Since \( \frac{Y(s)}{d(s)} = \frac{1}{1 + G_c(s)G_P(s)} \)
it follows that
\[ S(s) = \frac{Y(s)}{d(s)} = \frac{E(s)}{R(s) - d(s)} \] (2)

Thus, the sensitivity function has an important role to play in judging the performance of the controller because it also describes the effects of the disturbance, \( d(s) \), on the controlled output, \( Y(s) \). For the controller to achieve good disturbance rejection, \( S(s) \) should be made as small as possible by an appropriate design for the controller, \( G_c(s) \). In particular, \( S(s) = 0 \) if perfect control is achievable (Zhou et al. [1996]).

However, most physical systems are “strictly proper”. In terms of their transfer-function representation, this means that the denominator of the transfer function is always of higher order than the numerator. Thus,
\[ \lim_{s \to \infty} G_c(s)G_P(s) = 0 \]

In the frequency domain, this becomes
\[ \lim_{w \to \infty} G_c(jw)G_P(jw) = 0 \]

Hence,
\[ \lim_{w \to \infty} |S(jw)| = \lim_{w \to \infty} \left| \frac{1}{1 - G_c(jw)G_P(jw)} \right| \]

Thus, on the other hand, \( S(jw) \) has to be close to zero for ideal disturbance rejection, while on the other, at high frequencies, \( S(jw) \) is one! What the results are telling us is that perfect control cannot be achieved over the whole frequency range. Indeed, the analysis shows that perfect control can only be achieved over a small range of frequencies, at the low frequency end of the frequency response, i.e. near steady-state. This subject is thoroughly developed in Zhou et al. [1996].

Since
\[ S(s) = \frac{1}{1 + G_c(jw)G_P(jw)} \] (3)

5.3 The complementary sensitivity function

The complementary sensitivity function is, as suggested by the name, defined as:
\[ T(s) = 1 - S(s) \] (4)

If there is no measurement noise, i.e. \( N(s) = 0 \), then since
\[ S(s) = \frac{1}{1 + G_c(jw)G_P(jw)} \]

\[ T(s) = 1 - \frac{1}{1 + G_c(jw)G_P(jw)} \] (5)

\[ T(s) = \frac{G_c(jw)G_P(jw)}{1 + G_c(jw)G_P(jw)} = \frac{Y(s)}{R(s)} \] (6)

In this case, the complementary sensitivity function simply relates the controlled variable \( Y(s) \) to the desired output, \( R(s) \). Thus, it is clear that \( T(s) \) should be as close as possible to 1 by an appropriate choice of the controller. Again, since most physical processes are strictly proper in the open-loop, i.e.
\[ \lim_{s \to \infty} G_c(s)G_P(s) = 0 \]

this means that, in the frequency domain,
\[ \lim_{w \to \infty} |T(jw)| = \lim_{w \to \infty} \left| \frac{G_c(jw)G_P(jw)}{1 + G_c(jw)G_P(jw)} \right| = 0 \]

As in the case of the sensitivity function, \( S(jw) \), the desired value of the complementary sensitivity function, \( T(jw) \), can be achieved only near low frequencies. This subject is thoroughly developed in Skogestad and Postlewaite [2005] and Zhou et al. [1996].

5.4 The trade-off

When there is process noise, in terms of process inputs and outputs, \( T(s) \) is also affected by \( N(s) \). In this case, \( T(s) \) has to be made small so as reduce the influence of random inputs on system characteristics. In other words, we want \( T(s) \approx 0 \) or equivalently, \( S(s) \approx 1 \). Comparing this with the noise free situation where we require \( T(s) \approx 1 \) or \( S(s) \approx 0 \).

This illustrates the compromise that often has to be made in control systems design: good set-point tracking and disturbance rejection has to be traded off against suppression of the process noise.

5.5 Weighted sensitivity

The sensitivity function \( S \) is a very good indicator for closed-loop performance, both for \( SISO \) and \( MIMO \) systems. The main advantage of considering \( S \) is that because we ideally want \( S \) small, it is sufficient to consider just its magnitude. Typical specifications in terms of \( S \) include:

1. Minimum bandwidth frequency.
2. Maximum tracking error at selected frequencies.
3. System type, or alternatively the maximum steady-state tracking error, \( A \).
4. Shape of \( S \) over selected frequency ranges.
5. Maximum peak magnitude of \( S \).

Mathematically, these specifications may be captured simply by an upper bound, \( 1/|w_P(s)| \), on the magnitude of \( S \) where \( w_P(s) \) is a weight selected by the controller designer. The subscript \( P \) stands for performance since \( S \) is mainly used as a performance indicator, and the performance requirement becomes
\[ |L(jw)| < 1/|w_P(jw)|, \forall w \] (7)
\[ \Leftrightarrow |w_P(s)| < 1, \forall w \Leftrightarrow \] (8)
The last equivalence follows from the definition of the $H_\infty$ norm, and in other words the performance requirement is that the $H_\infty$ norm of the weighted sensitivity, $w_P S$, must be less than one.

5.6 State-space format for continuous time LTI systems

While $H_2$ and $H_\infty$ optimization can be applied to discrete-time ($DT$) and continuous-time ($CT$) systems, represented in many of possible formats (state-space, transfer matrix, zero-pole, etc.), the basic algorithms are traditionally developed and explained for $CT$ systems defined by state-space equations. A state-space model for a finite order $CT$ LTI system with input $u=u(t)$, output $y=y(t)$, and state $x=x(t)$ has the form:

$$
\dot{x}(t) = Ax(t) + Bu(t),
$$

(9)

$$
y(t) = Cx(t) + Du(t),
$$

(10)

When a continuous time LTI system describes the plant in a standard feedback optimization setup, its input is partitioned into the disturbance and actuator components. Similarly, the output is partitioned into the cost and measurement components.

$$
G_P = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}
$$

(11)

Fig. 8. Partitioned plant

6. PID CONTROLLER

The controller takes a measured value from a process and compares it with a reference setpoint value. The difference (or “error” signal) is then used to adjust some input to the process in order to bring the process measured value to its desired setpoint (fig. 9), see Botto [2006].

7. EXPERIMENTAL RESULTS

Experimental results from SNCF train circulation where used, and these results are presented in fig. 10 which can be found in the SNCF training session manuals Menuet [2006] and are presented also in fig. 11.

7.1 Model comparison with experimental results

The pantograph was modeled without any experimental data information and therefore the results presented aren’t tuned for the experimental data. Many information was assumed and may not represent the real experimental simulation. In fig. 12 and 13 it is shown that the models don’t approximate the experimental data. The frequency presented of 2 Hz follows from the knowledge of the assumed low frequency catenary influence. Assuming a train velocity of 350 Km/h and a catenary span of 60 m it was concluded that the low frequency would be of 1.76 Hz approximately 2 Hz. Experimental results contradict this assumption, for low frequencies it was verified that a 7 Hz perturbation would approximate more the experimental data. Not only in frequency there are differences but also in force amplitude, this happens because the catenary model constants are incorrect (spring and damper).

<table>
<thead>
<tr>
<th>$y_{cat}(cm)$</th>
<th>$y_{pant}(cm)$</th>
<th>$F(N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>170</td>
</tr>
<tr>
<td>-3.8</td>
<td>6</td>
<td>150</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
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<td>4</td>
<td>210</td>
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<tr>
<td>-3.8</td>
<td>6</td>
<td>170</td>
</tr>
<tr>
<td>-2.2</td>
<td>7.5</td>
<td>110</td>
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<tr>
<td>0</td>
<td>6</td>
<td>300</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>255</td>
</tr>
<tr>
<td>-3.8</td>
<td>5.8</td>
<td>180</td>
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<td>8</td>
<td>225</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>240</td>
</tr>
</tbody>
</table>

Table 2. Global SNCF pantograph experimental results

<table>
<thead>
<tr>
<th>$y_{cat}(cm)$</th>
<th>$y_{pant}(cm)$</th>
<th>$F(N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>cm</td>
</tr>
<tr>
<td>3.8</td>
<td>cm</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>$\Delta F$</th>
<th>190 N</th>
</tr>
</thead>
</table>

Table 3. Experimental results highlights
8. ROBUST CONTROLLER

The standard $H_\infty$ optimal control problem is to find all stabilizing controllers $G_c$ which minimize

$$||G(s)||_\infty = \max_{w} \sigma(F_l(G_P, G_c)(jw))$$

We minimize the peak of the singular value of $F_l(G_P(jw), G_c(jw))$.

The standard $H_2$ optimal control problem is to find a stabilizing controller $G_c$ which minimizes

$$||G(s)||_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{+\infty} G(jw)G(jw)^T dw;}$$

By minimizing the $H_2$ norm, the output (or error) power of the generalized system, due to a unit intensity white noise input, is minimized; we are minimizing the root-mean-square (rms).

For a particular problem the generalized plant $G_P$ will include the plant model, the interconnection structure uncertainty and the designer specified weighting functions. $T$ is a partitioned system, obtained by $G_P$ (Plant) and the weighting functions: $W_1$, $W_2$, $W_3$, the partitioned plant can be seen in equation 14.

$$G = \begin{bmatrix} W_1s & (W_2/G_P)T \\ W_3T \end{bmatrix}$$

9. RESULTS

In order to obtain a robust controller a linear model has to be developed.

The operating point is defined by the initial position and orientation of the pantograph. The expected dimension of the system is a MISO system, two inputs one output system, where in this case one of the input signals is a perturbation. Thus, the system we are analyzing is in fact a SISO system (equation 15 and equation 16), with perturbation added to it and therefore all the traditional SISO theory is applied directly.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1313 & -5.6 & -127.3 & 33.58 \\ 0 & 0 & 0 & 1 \\ 150.1 & 0.5427 & -7.1 & -11.07 \end{bmatrix} x + \begin{bmatrix} 0 \\ -0.02087 \\ 0.0062 \end{bmatrix} u$$

$$y = [-500 \ -10 \ -1676 \ -33.52] x + \begin{bmatrix} 0 \\ 500 \end{bmatrix} u$$
The Type 1 system can be seen in transfer function where $G_1$ is the relation between joint torque (Nm) and catenary force (N) and $G_2$ is the relation between catenary perturbation (m) and catenary force (N) (equation 17 linear version).

$$G_1 = \frac{-0.5702s^2 - 188s - 7973}{s^4 + 16.68s^3 + 1365s^2 + 9611s + 28470}$$

$$G_2 = \frac{500s^3 + 7812s^3 + 6.492e05s^2 + 4.198e06s + 8.094e05}{s^4 + 16.68s^3 + 1365s^2 + 9611s + 28470}$$

The controller which is placed in the closed-loop was obtained by loop shaping with trial and error to find the correct weighting functions (equation 18), this can be seen in fig. 15. The notation used here is different compared with previous sections $K = G_c$ and $G = G_p$.

$$w_1 = (\frac{w_0}{\tau s + 1})^n, w_2 = 0.1, w_3 = 10^{-3}$$

$$w_0 = 10; \quad \tau = 50; \quad n = 1$$

![Singular Values](image)

Fig. 15. Loop shaping

The controller was obtained with the help of the Robust Control Toolbox in Matlab and the expressions is as follows. The controller function used is presented in equation 19, where $K$ is the controller developed to be placed in the closed-loop (see Balas et al. [2006] for controller definition and usage).

$$[K] = \text{hinf}(\text{interconnected}_\text{plant}, 1, 1);$$

Where $K$ is the closed-loop controller. An $H_\infty$ control law has been designed which achieves an infinity norm of 0.9066 for the interconnection structure, the interconnection structure is a closed-loop representation of the controlled system in a special Matlab variable. The $H_\infty$ controller $K$ is in state space representation:

$$A = \begin{bmatrix}
-4545 & 1122 & 6655 & 5108 & 1.6e + 004 \\
146 & -27.5 & -172.9 & -132 & -398 \\
465.6 & -144.5 & -685 & -525.1 & -1639 \\
278 & -79.68 & -396.6 & -309.6 & -973 \\
-1494 & 379.9 & 2183 & 1677 & 5248
\end{bmatrix}$$

$$B = \begin{bmatrix}
-0.003671 \\
0.001235 \\
0.005232 \\
0.003393 \\
0.01355
\end{bmatrix}$$

$$C = \begin{bmatrix}
-1.359e + 005 & 3.503e + 004 & ... \\
1.993e + 005 & 1.531e + 005 & 4.799e + 005
\end{bmatrix}$$

$$D = [0]$$

The process used to obtain an $H_\infty$ robust controller is the same as $H_\infty$ with the only difference that we use another Matlab function. The controller obtained had the same global characteristics as the $H_\infty$ with one difference which was having less problems converging to a solution. The system behavior was observed for both systems, thus showing that the major differences between these methods occurred in the mathematical formulation not in the results.

$$[K] = \text{h2syn}(\text{interconnected}_\text{plant}, 1, 1);$$

The $PID$ controller used was tuned based on trial and error the weighting gains obtained are respectively:

$$P = 15, \quad I = 3, \quad D = 0.1$$

The controller above is relative to the lower joint. For the upper joint, which are the vertical translation joints that support the registration strip another set of $PID$ controller gains where used:

$$P = 45, \quad I = 3, \quad D = 0.1$$

The Type 1 model presented is very similar to a real pantograph except for the consideration of catenary movement along the registration strip frame. In this case no movement is considered, and thus the force sensed by the registration strip is the same sensed by the catenary. An illustration of the forces involved in the contact interface is presented in fig. 16.

![Free body diagram of Type 1 registration strip](image)

Fig. 16. Free body diagram of Type 1 registration strip

In order to simulate the catenary effect on the pantograph, the position of the spring damper system would have to be changed to produce an increase or decrease of contact force. The first type of perturbation studied was a step input displacement on the catenary of 0.1 meters. The results are presented in fig. 17.

Changing the step perturbation to a sine input signal to the catenary position, with a 2 Hz frequency and an amplitude of 0.1 meters of catenary displacement, offers interesting results which are presented in fig. 18.
The objective of this work is to model a real pantograph as more realistically as possible and to implement a robust closed-loop system. The Type 5 model is the pantograph system which models more closely a real system, here we have three inputs controlled separately with the same control objective. Because the catenary slides along the frame and because we have two upper controllers in the registration strip, one in the left and the other in the right there will be times where the cable will be totaly on the left or on the right side of the frame, this means that it will not be a good idea for the actuator (left or right) to follow the reference always. In summary the controller power is affected by the catenary motion, thus it will be maximum when the cable is on top of the actuator and minimum when at the most distant point. The results for a 20 Hz perturbation can be seen in fig. 19.

Finding the right sensitivity transfer functions was a wearisome ordeal, because the working frequencies of a pantograph can go up to 20 Hz (20 Hz of perturbation signal). Providing a controller that could be robust and have good performance, wasn’t possible for the Type 3 model, so a compromise had to be reached.

From the experiments made in control it was found that high input frequencies applied on the pantograph affect the control performance, this means that the controller must only work in specific frequency regions for safety reasons. To control the loop shaping process in order to get a correspondence with the nonlinear behavior was very difficult.

In general, the robust controller proved to be better than a normal PID controller with exception to the modelling steps which are more complex. It’s not elementary to find the right combination of weights that filter the objective designer frequency specifications for the robust controllers. Further more as expected the robust controller proved to have better performance to model parameter changes and signal noise than the PID controller.

REFERENCES
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