Standard deviation of a sample

Definition

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Basic **(classical) method, subject to rounding error**

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i^2 - 2\bar{x}x_i + \bar{x}^2\right)$$

$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}x_i^2 - 2\bar{x}\sum_{i=1}^{n}x_i + n\bar{x}^2\right)$$

$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}x_i^2 - n\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2\right]$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}x_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}x_i\right)^2$$

$$\boxed{(n-1)s^2 = \sum_{i=1}^{n}x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n}x_i\right)^2}$$

Improved **method, incurring smaller rounding error**

Same as before, but the following substitution is done previously, in order to reduce the orders of magnitude:

$$x_i' = x_i - \bar{x}$$

The average becomes 0, but the standard deviation remains the same.

Progressive **method**

With $a_0 = 0$ and $q_0 = 0$, for $k = 1..n$,

$$\begin{cases} a_k = a_{k-1} + \frac{1}{k}(x_k - a_{k-1}) \\ q_k = q_{k-1} + (x_k - a_{k-1})(x_k - a_k) \end{cases}$$

$$\boxed{(n-1)s^2 = q_n}$$

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