

$$y = f(x) = \frac{1}{b-a}$$

$$\mu = E(x) = \int_{-\infty}^{+\infty} xf(x)dx =$$

$$= \int_a^b \frac{x}{b-a} dx = \frac{1}{b-a} \int_a^b x dx = \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b =$$

$$= \frac{1}{2} \frac{1}{b-a} (b^2 - a^2) = \frac{1}{2} \frac{(b+a)(b-a)}{b-a}$$

$$\boxed{\mu = \frac{a+b}{2}}$$

$$\sigma^2 = E[(x-\mu)^2] = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x) dx =$$

$$= \int_a^b \frac{\left(x - \frac{a+b}{2}\right)^2}{b-a} dx$$

Let $u = x - \frac{a+b}{2}$.

$$\sigma^2 = \frac{1}{b-a} \int_{-(b-a)/2}^{(b-a)/2} u^2 du = \frac{1}{b-a} \left[\frac{u^3}{3} \right]_{-(b-a)/2}^{(b-a)/2} =$$

$$= \frac{1}{3} \frac{1}{b-a} \left[\left(\frac{b-a}{2}\right)^3 + \left(\frac{b-a}{2}\right)^3 \right] = \frac{1}{3} \frac{1}{b-a} \frac{2}{8} (b-a)^3 =$$

$$= \frac{1}{12} (b-a)^2$$

$$\boxed{\sigma = \frac{b-a}{\sqrt{12}}}$$

Physical dimensions match, as expected !

