matrix by a new nonnegative matrix such that (i) the element c itself is replaced by a zero, (ii) those old zeros covered by a single line are retained, and (iii) the rest of the old zeros are replaced by c. But since these operations are equivalent to subtracting $c/2$ from each uncovered row and each uncovered column, and adding $c/2$ to each covered row and each covered column, Problem 8.14 once more guarantees that the optimal assignment is unaltered.

9.6 Xanadu National Airlines offers an excursion at one low price that allows a person to cover its entire service route. The ticket, which is valid for two weeks from the date of purchase, carries the following restriction: No city on the route can be revisited except the city of origin, which can be the last stop on the excursion. A foreign tourist, presently in city 1 (the capital), wishes to see provincial cities 2, 3, and 4, before returning to the capital; she decides to travel on the airlines. Flight times (in minutes) between the cities of interest are given in the table below, where dotted entries signify that service between corresponding locations is not available. Determine an acceptable itinerary which will minimize her total flight time.

Cities	1	2	3	4
1	···	65	53	37
2	65	···	95	···
3	53	95	···	81
4	37	···	81	···

	1	2	3	4
1	10 000	65	53	37*
2	65	10 000	95*	10 000
3	53	95*	10 000	81
4	37*	10 000	81	10 000

Tableau 6A

We begin by replacing each dotted entry in the timetable by an exorbitant number of prohibit assignments to those links under an optimal itinerary. The result is Tableau 6A. Applying the Hungarian method to this tableau, we obtain (on the second application of Step 2) the assignment indicated by the starred elements; namely, 1→4, 4→1, 2→3, 3→2. This is *not* a valid itinerary, for it returns the tourist to city 1 immediately after her first stop in city 4.

	1	2	3	4
1	10 000	65*	53	10 000
2	65	10 000	95*	10 000
3	53	95	10 000	81*
4	37*	10 000	81	10 000

Tableau 6B

	1	2	3	4
1	10 000	10 000	10 000	37*
2	65*	10 000	95	10 000
3	53	95*	10 000	10 000
4	10 000	10 000	81*	10 000

Tableau 6C

We now branch on the starred element $c_{14} = 37$ of Tableau 6A. The first branch is effected by replacing c_{14} by a prohibitively large number, as shown in Tableau 6B. The second branch is effected by replacing c_{41}, the transposed element, as well as all elements in the fourth row or first column except c_{14} itself, by a prohibitively large number. This is done in Tableau 6C.

Applying the Hungarian method to each of these two new cost matrices separately, we obtain valid itineraries for both: 1→2, 2→3, 3→4, 4→1, with a cost of 278 min, for Tableau 6B; and 1→4, 4→3, 3→2, 2→1, with a cost of 278 min, for Tableau 6C. Both solutions are optimal. Indeed, whenever the cost matrix is symmetric, an optimal circuit remains optimal when described in the opposite sense.

9.7 Develop a "near-optimal" algorithm for the traveling salesman problem.

We develop the *nearest-neighbor method*, based on the principle of sequentially selecting the cheapest remaining link such that its inclusion does not complete a circuit too soon.

STEP 1 Locate the smallest element in the cost matrix (break ties arbitrarily), circle it, and include the corresponding link in the itinerary.