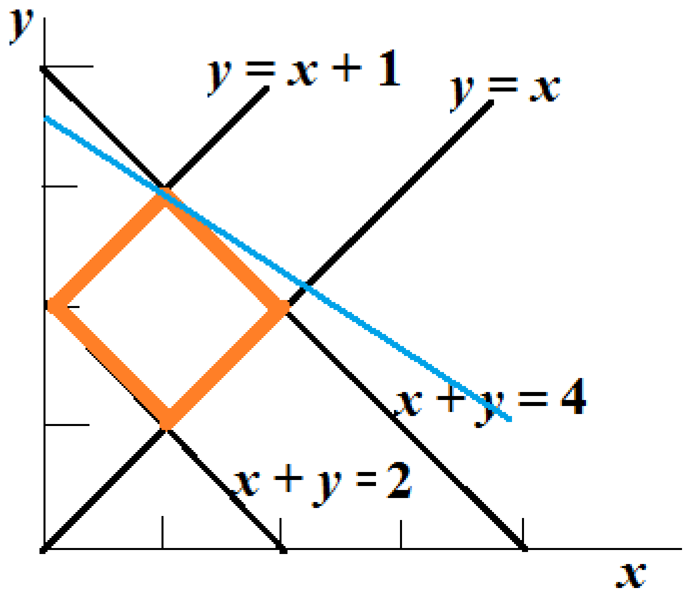
Linear Programming

Multiple solutions



1)

$$\begin{cases} [\max]\, z = \frac{2}{3}x + y \\ y \le x + 2 \\ y \ge x \\ x + y \ge 2 \\ x + y \le 4 \end{cases} \rightarrow \begin{cases} [\max]\, z = \frac{2}{3}x + y \\ -x + y \le 2 \\ x - y \le 0 \\ x + y \ge 2 \\ x + y \le 4 \end{cases} \rightarrow \begin{cases} [\max]\, z = \frac{2}{3}x_1 + x_2 + 0x_3 + \\ \quad 0x_4 + 0x_5 - Mx_6 + 0x_7 \\ -x_1 + x_2 + x_3 = 2 \\ x_1 - x_2 + x_4 = 0 \\ x_1 + x_2 - x_5 + \bar{x}_6 = 2 \\ x_1 + x_2 + x_7 = 4 \end{cases} \rightarrow$$

$$\begin{cases} [\max]\, z = \frac{2}{3}x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 - Mx_6 + 0x_7 \\ -x_1 + x_2 + x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 = 2 \\ x_1 - x_2 + 0x_3 + x_4 + 0x_5 + 0x_6 + 0x_7 = 0 \\ x_1 + x_2 + 0x_3 + 0x_4 - x_5 + \bar{x}_6 + 0x_7 = 2 \\ x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 + x_7 = 4 \end{cases}$$

Solution: $z^* = 3\frac{2}{3}$, X^* = (1, 3, 0, 2, 2, 0, **0**)

2) $[\max]\, z = 1{,}5x + y$

Solution: z^* = 5, X^* = (2, 2, 2, 0, 2, 0, **0**)

3) $[\max]\, z = x + y$

Solution: z^* = 4, X_1^* = (2, 2, 2, 0, 2, 0, **0**)
Solution: z^* = 4, X_2^* = (1, 3, 0, 2, 2, 0, **0**)

4) $[\max]\, z = x + y$ without (last) $x + y \le 4$

Solution: $z^* = \infty$ (unbounded, because: feasible region open)

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