03-2014 **Zionts, "An intuitive algebraic approach for solving LP problems"**

Solving impossible problems…

Original problem:

$$[\max] z = 0{,}56x_1 + 0{,}42x_2$$
$$\text{s. to } x_1 + 2x_2 \le 240$$
$$1{,}5x_1 + x_2 \le 180$$
$$x_1 \le 110$$

[max]	$z =$	$0{,}56\,x_1$	$+0{,}42\,x_2$		
	s. t.	x_1	$+2\,x_2$	$\le$	240
		$1{,}5\,x_1$	$+\,x_2$	$\le$	180
		x_1		$\le$	110 !
		x_1		$\ge$	130 !

Confusion (standard, canonical:, augmented)

http://en.wikipedia.org/wiki/Linear_programming

Converting to *standard form* (not necessary for us) [max] $z = c^T x$ s.t. $Ax \le b$

(Multiply by -1; replace = by ≤&≥; $x_{free}=x^+-x^-$.)

Converting to *canonical form* (following)

(Introduce slack var.s and artificial var.s.)

Introduce (one or more) slack variables (always: if necessary)

		structural	structural	slack	slack	slack	slack		
[max]	$z =$	$0{,}56\,x_1$	$+0{,}42\,x_2$	$+0\,x_3$	$+0\,x_4$	$+0\,x_5$	$+0\,x_6$		
	s. t.	x_1	$+2\,x_2$	$+\,x_3$	$+0\,x_4$	$+0\,x_5$	$+0\,x_6$	=	240
		$1{,}5\,x_1$	$+\,x_2$	$+0\,x_3$	$+\,x_4$	$+0\,x_5$	$+0\,x_6$	=	180
		x_1	$+0\,x_2$	$+0\,x_3$	$+0\,x_4$	$+\,x_5$	$+0\,x_6$	=	110
		x_1	$+0\,x_2$	$+0\,x_3$	$+0\,x_4$	$+0\,x_5$	$-\,x_6$	=	130

This is not the canonical form, because no immediate solution can be found: there is no identity matrix !

Introduce (one or more) artificial variables (always: if necessary) ***This is no longer our problem !! Is it useful ?*** (Big) M = 1,0E+02

		structural	structural	slack	slack	slack	slack	artificial		Infinity
[max]	$z =$	$0{,}56\,x_1$	$+0{,}42\,x_2$	$+0\,x_3$	$+0\,x_4$	$+0\,x_5$	$+0\,x_6$	$-M\,x_7$		
	s. t.	x_1	$+2\,x_2$	$+\,x_3$	$+0\,x_4$	$+0\,x_5$	$+0\,x_6$	$+0\,x_7$	=	240
		$1{,}5\,x_1$	$+\,x_2$	$+0\,x_3$	$+\,x_4$	$+0\,x_5$	$+0\,x_6$	$+0\,x_7$	=	180
		x_1	$+0\,x_2$	$+0\,x_3$	$+0\,x_4$	$+\,x_5$	$+0\,x_6$	$+0\,x_7$	=	110
	$(\times M)$	x_1	$+0\,x_2$	$+0\,x_3$	$+0\,x_4$	$+0\,x_5$	$-\,x_6$	$+\,x_7$	=	130

In order to follow the Simplex Method, the starting solution (here, x_3, x_4, x_5, x_7) must have 0 coefficients in z. Simply, subtract adequately (4.th constraint).

		structural	structural	slack	slack	slack	slack	artificial		
[max]	$z =$	$(0{,}56+M)\,x_1$	$+0{,}42\,x_2$	$+0\,x_3$	$+0\,x_4$	$+0\,x_5$	$-M\,x_6$	$+0\,x_7$		$-\,130\,M$
	s. t.	x_1	$+2\,x_2$	$+\,x_3$	$+0\,x_4$	$+0\,x_5$	$+0\,x_6$	$+0\,x_7$	=	240
		$1{,}5\,x_1$	$+\,x_2$	$+0\,x_3$	$+\,x_4$	$+0\,x_5$	$+0\,x_6$	$+0\,x_7$	=	180
		x_1	$+0\,x_2$	$+0\,x_3$	$+0\,x_4$	$+\,x_5$	$+0\,x_6$	$+0\,x_7$	=	110
		x_1	$+0\,x_2$	$+0\,x_3$	$+0\,x_4$	$+0\,x_5$	$-\,x_6$	$+\,x_7$	=	130

Giving the problem to Solver

Then, change 130 to 0.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7				Solver model
[max] z =	100,56	0,42	0	0	0	-100	0	**11067,9**			11067,9
s. t.	1	2	1	0	0	0	0	240	=	240	7
	1,5	1	0	1	0	0	0	180	=	180	TRUE
	1	0	0	0	1	0	0	110	=	110	100
	1	0	0	0	0	-1	1	130	=	**130**	100
X =	**110**	**15**	**100**	**0**	**0**	**0**	**20**				

We ALWAYS solve the problem — maybe not our problem.