Note that the primal, program (*1*), contains three variables and four constraints, while its dual, program (*2*), contains four variables and three constraints.

5.2 Determine the symmetric dual of the program

$$\begin{aligned} \text{maximize:} \quad & z = 2x_1 + x_2 \\ \text{subject to:} \quad & x_1 + 5x_2 \leq 10 \\ & x_1 + 3x_2 \leq 6 \\ & 2x_1 + 2x_2 \leq 8 \\ \text{with:} \quad & \text{all variables nonnegative} \end{aligned} \tag{1}$$

This program has the form (*5.2*), with x-variables replacing w-variables. Proceeding as in Problem 5.1, we generate its dual, (*5.1*), with w-variables replacing x-variables:

$$\begin{aligned} \text{minimize:} \quad & z = 10w_1 + 6w_2 + 8w_3 \\ \text{subject to:} \quad & w_1 + w_2 + 2w_3 \geq 2 \\ & 5w_1 + 3w_2 + 2w_3 \geq 1 \\ \text{with:} \quad & \text{all variables nonnegative} \end{aligned} \tag{2}$$

5.3 Show that both the primal and dual programs in Problem 5.2 have the same optimal value for z, and that the solution of each is imbedded in the final simplex tableau of the other.

Introducing slack variables x_3, x_4, and x_5, respectively, in the constraint inequalities of program (*1*) of Problem 5.2, and then applying the simplex method to the resulting program, we generate sequentially Tableaux 1 and 2.

		x_1	x_2	x_3	x_4	x_5	
		2	1	0	0	0	
x_3	0	1	5	1	0	0	10
x_4	0	1	3	0	1	0	6
x_5	0	2*	2	0	0	1	8
$(z_j - c_j)$:		−2	−1	0	0	0	0

Tableau 1

	x_1	x_2	x_3 (slack variables)	x_4	x_5	
x_3	0	4	1	0	−1/2	6
x_4	0	2	0	1	−1/2	2
x_1	1	1	0	0	1/2	4
	0	1	0 (solution to the dual)	0	1	8

Tableau 2

The solution to the primal is obtained from Tableau 2 as $x_1^* = 4$, $x_2^* = 0$, with $z^* = 8$. The solution to the dual program is found in the last row of this tableau, in those columns associated with the slack variables for the primal. Here, $w_1^* = 0$, $w_2^* = 0$, and $w_3^* = 1$.

We can solve the dual directly by introducing surplus variables w_4 and w_5, and artificial variables w_6 and w_7, to program (*2*) of Problem 5.2, and then applying the two-phase method, which generates Tableaux 1′, . . . , 4′.

		w_1	w_2	w_3	w_4	w_5	w_6	w_7	
		10	6	8	0	0	M	M	
w_6	M	1	1	2	−1	0	1	0	2
w_7	M	5*	3	2	0	−1	0	1	1
$(c_j - z_j)$:		10	6	8	0	0	0	0	0
		−6	−4	−4	1	1	0	0	−3

Tableau 1′

	w_1	w_2	w_3	w_4 (surplus variables)	w_5	
w_5	−4	−5	0	−1	1	1
w_3	1/2	1/2	1	−1/2	0	1
	6	2	0	4 (solution to the primal)	0	−8

Tableau 4′

Problem 5.02 R. Bronson, 1982, 0 46

```
Primal
max -------------- Solution: z* = 8, x = (4 0 6 2 0)
 2 1 0 0 0
subject to
 1 5 1 0 0 +10
 1 3 0 1 0 +6
 2 2 0 0 1 +8
Artificials
 0
BigM: ...
Initial basis
 3 4 5

Dual
min -------------- Solution: z* = 8, x = (0 0 1 0 1 0 0)
 10 6 8 0 0 0 0
subject to
 1 1 2 -1  0 1 0 +2
 1 3 2  0 -1 0 1 +1
Artificials
 6 7
BigM: 1+1
Initial basis
 6 7
```

```
nonbasics:            5          2     |
delta_vec:       -1.000     -1.000     |
                                       |
                                       |
SOLUTION:            max z,   8.0000   | at basis: 1
                                       |
  Variable  Value         Coefficient    Contribution
       1  4.00000               2.000  |   8.0000
       3  6.00000               0.000  |   0.0000
       4  2.00000               0.000  |   0.0000
```

```
nonbasics:            1          7          2          4          6
delta_vec:        6.000      10.00      2.000      4.000      6.000
                                       |
                                       |
SOLUTION:            min z,   8.0000   | at basis: 3
                                       |
  Variable  Value         Coefficient    Contribution
       3  1.00000               8.000  |   8.0000
       5  1.00000               0.000  |   0.0000
```