1) p 20, Example, *resistência* (resistance)

$$Q = RI^2 t$$

Modulus for *worst case* (always *adding*):

$$\Delta Q = \left|I^2 t\right|\Delta R + \left|2RIt\right|\Delta I + \left|RI^2\right|\Delta t$$

$$\frac{\Delta Q}{Q} = \frac{\Delta R}{R} + 2\frac{\Delta I}{I} + \frac{\Delta t}{t}$$

$$R = 100 \pm 1\ \Omega$$

$$I = 1{,}00 \pm 0{,}01\ \mathrm{A}$$

$$t = 100 \pm 1\ \mathrm{s}$$

$$Q = 100 \times 1^2 \times 100 = 10\,000\ \mathrm{J}$$

$$\Delta Q/Q \leq 1/100 + 2 \times 0{,}01/1 + 1/100 = 4/100 = 4\ \%$$

2) http://en.wikipedia.org/wiki/Error_propagation

$$\Delta X = \left|\frac{\partial f}{\partial A}\right| \cdot \Delta A + \left|\frac{\partial f}{\partial B}\right| \cdot \Delta B + \left|\frac{\partial f}{\partial C}\right| \cdot \Delta C + \cdots$$

$$\sigma_X^2 = \left(\frac{\partial f}{\partial A}\sigma_A\right)^2 + \left(\frac{\partial f}{\partial B}\sigma_B\right)^2 + \left(\frac{\partial f}{\partial C}\sigma_C\right)^2 + \cdots$$

Example 1: $y = f(\theta) = \arctan(\theta)$

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,\theta} = \frac{1}{1+\theta^2}$$

$$\Delta y = \frac{1}{1+\theta^2}\Delta\theta$$

Example 2: $R = V / I$

$$(\Delta R)^2 = \left(\frac{\partial R}{\partial V}\Delta V\right)^2 + \left(\frac{\partial R}{\partial I}\Delta I\right)^2$$

$$(\Delta R)^2 = \left(\frac{1}{I}\Delta V\right)^2 + \left(-\frac{V}{I^2}\Delta I\right)^2$$

This can be *simplified* by dividing both terms by R^2:

$$\left(\frac{\Delta R}{R}\right)^2 = \left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta I}{I}\right)^2$$

3) p 22, Example, *resistência* (resistance)

$$Q = RI^2 t$$

$$(\Delta Q)^2 = \left(\frac{\partial Q}{\partial R}\Delta R\right)^2 + \left(\frac{\partial Q}{\partial I}\Delta I\right)^2 + \left(\frac{\partial Q}{\partial t}\Delta t\right)^2$$

$$(\Delta Q)^2 = \left(I^2 t\ \Delta R\right)^2 + \left(2RIt\ \Delta I\right)^2 + \left(RI^2\ \Delta t\right)^2$$

This can be simplified (as before) by dividing by Q^2:

$$\left(\frac{\Delta Q}{Q}\right)^2 = \left(\frac{\Delta R}{R}\right)^2 + 2^2\left(\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta t}{t}\right)^2$$

With the data given, Q = 10000 J:

$$\left(\frac{\Delta Q}{Q}\right)^2 = \left(\frac{1}{100}\right)^2 + 2^2\left(\frac{0.01}{1}\right)^2 + \left(\frac{1}{100}\right)^2 =$$

$$= 10^{-4} + 4\times 10^{-4} + 10^{-4} = 6\times 10^{-4}$$

$$\Delta Q = 10000\times\sqrt{6}\times 10^{-2} = 10^4\times 2{,}45\times 10^{-2} = 245\ \text{J}$$

Suppose we adopt

$$\boxed{\Delta X = 2\sigma_X}$$

$$\sigma_Q = \frac{\Delta Q}{2} = \frac{245}{2} = 122\ \text{J}$$

Working with ΔX or σ_X gives the same conclusions, as they are related by a factor [in this case 2, which *comes* from Φ^{inv}(95%), i.e., NORMSINV((1+**0,95**)/2), or other].

❖