COMPLEX #'S:

$z = a+bi$ | $z = r(\cos\theta + i\sin\theta)$ | $1/z = 1/r(\cos\theta - i\sin\theta)$ | $\bar{z} = a-bi$ | $\overline{z+w} = \bar{z}+\bar{w}$ | $\overline{zw} = \bar{z}\,\bar{w}$ | $\overline{z^n} = \bar{z}^n$ | $|z| = \sqrt{a^2+b^2}$ | $z\bar{z} = |z|^2$

DE MOIVRE'S THEOREM: n-pos. integer
$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

EULER'S FORMULA: $e^{iy} = \cos y + i\sin y$

ROOTS OF A COMPLEX #: $k = 0,1,2,\ldots$
$w_k = r^{1/n}\left[\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right]$

ALTERNATE FORM: $z = r^{1/n} e^{\left(\frac{i\theta}{n} + \frac{2\pi k}{n}\right)}$

$z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$ | $z_1/z_2 = r_1/r_2[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)],\ z_2 \neq 0$ | $z = re^{i\theta}$ | $e^{i\pi} = -1$ | $e^z = \sum \frac{z^n}{n!}$

TRIG. INTEGRALS $A = mx$ $B = nx$

$\int \sin A\cos B = \frac{1}{2}\int \sin(A-B) + \sin(A+B)$
$\int \sin A\sin B = \frac{1}{2}\int \cos(A-B) - \cos(A+B)$
$\int \cos A\cos B = \frac{1}{2}\int \cos(A-B) + \cos(A+B)$

TRIGONOMETRIC INTEGRALS: $\int \sin^m x\cos^n x\,dx$ (1) $\int \tan^m x\sec^n x\,dx$ (2) $\Rightarrow \int u^n \cdot u' = u^{n+1}/n+1 + C$

(1) **n odd**: save($\cos x$), express remaining factors in terms of $\sin x$ w/ $\cos^2 x = 1-\sin^2 x$
m odd: save($\sin x$), express remaining factors in terms of $\cos x$ w/ $\sin^2 x = 1-\cos^2 x$
(2) **n even**: save($\sec^2 x$), express remaining factors in terms of $\tan x$ w/ $\sec^2 x = 1+\tan^2 x$
m odd: save($\sec x\tan x$), express rem. factors in terms of $\sec x$ w/ $\tan^2 x = \sec^2 x - 1$

(*) If m,n even use $\frac{1}{2}$ angle (1) identities
$\sin x\cos x = \frac{1}{2}\sin 2x$

APPROXIMATE INTEGRATION: Riemann Sums with finite n Accuracy: $S_n > M_n > T_n > R_n \approx L_n$

$L_n = \frac{b-a}{n}(f(x_0) + f(x_1) + \cdots + f(x_{n-1}))$ | $R_n = \frac{b-a}{n}(f(x_1) + f(x_2) + \cdots + f(x_n))$ | $M_n = \frac{b-a}{n}\left(f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right)$ | Approx for $\int_a^b f(x)\,dx$

$T_n = \frac{b-a}{2n}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n))$ | $S_n = \frac{b-a}{3n}(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n))$ | max $f''(x), f^{4}(x)$ on $[a,b]$

$|E_{T_n}| \leq \frac{|\max f''(x)|(b-a)^3}{12n^2}$ | $|E_{M_n}| \leq \frac{|\max f''(x)|(b-a)^3}{24n^2}$ | $|E_{S_n}| \leq \frac{|\max f^4(x)|(b-a)^5}{180n^4}$ | $|E_{S_n}|$ for a cubic function is always 0

SERIES:

A series is the sum of the terms of a sequence. $\sum a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots = S$, if $\sum a_n$ is CONVERGENT

A. PARTIAL SUMS: S_n = the nth partial sum of $\sum a_n$ $S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$ If $\lim_{n\to\infty} S_n = S$ $\sum a_n$ is CONV. (as long as a_n CONV.)

GEOMETRIC SERIES
$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ CONV. if $|r|<1$

P-SERIES
$\sum_{n=1}^{\infty} \frac{1}{n^p}$ CONV. if $p>1$ DIV. if $p \leq 1$

TELESCOPING SERIES
$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}\left(\frac{1}{b_n} - \frac{1}{c_n}\right) = \frac{1}{b_1} - \frac{1}{c_n}$
USE PARTIAL FRACTIONS

If $\sum a_n$ and $\sum b_n$ are CONV. series, so are these
$\sum_{n=1}^{\infty} ca_n = c\sum_{n=1}^{\infty} a_n$ $\sum_{n=1}^{\infty}(a_n+b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$ $\sum_{n=1}^{\infty}(a_n-b_n) = \sum_{n=1}^{\infty} a_n - \sum_{n=1}^{\infty} b_n$

DIVERGENCE TEST: If $\lim_{n\to\infty} a_n \neq 0$ or DNE, $\sum a_n$ is DIV. **CONTRAPOSITIVE**: If $\sum a_n$ is CONV. then $\lim_{n\to\infty} a_n = 0$ necessary but not sufficient

THE INTEGRAL TEST: If f is a positive, decreasing, and continuous function on $[1,\infty)$, and Let $a_n = f(n)$, Then $\sum_{n=1}^{\infty} a_n$ is CONV. Iff the improper integral $\int_1^{\infty} f(x)\,dx$ is CONV. (*) Use only if the Integral is easy to Evaluate
(*) CONVERSE: $\int_1^{\infty} f(x)dx$ CONV. Iff $\sum_{n=1}^{\infty} a_n$ CONV. Goes Both ways

REMAINDER ESTIMATE FOR THE INTEGRAL TEST: If $\sum a_n$ CONV. and $R_n = S - S_n$, then $\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_n^{\infty} f(x)\,dx$
Also $S_n + \int_{n+1}^{\infty} f(x)\,dx \leq S \leq S_n + \int_n^{\infty} f(x)\,dx$ (*) $\sum a_n$ must be CONV. By the Integral Test for these to be applicable

COMPARISON TEST: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms (*) INCONCLUSIVE - try other Methods (3)(4)
(1) If $\sum a_n \geq \sum b_n$ for all n and $\sum a_n$ CONV., $\sum b_n$ also CONVERGES (3) If $\sum a_n \geq \sum b_n$ for all n and $\sum b_n$ CONV. - Inconclusive for $\sum a_n$
(2) If $\sum a_n \geq \sum b_n$ for all n and $\sum b_n$ DIV., $\sum a_n$ also DIVERGES (4) If $\sum a_n \geq \sum b_n$ for all n and $\sum a_n$ DIV. - Inconclusive for $\sum b_n$

LIMIT COMPARISON TEST: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms (*) (b) and (c) - Apply COMPARISON TEST
(a) If $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ Then either $\sum a_n$ and $\sum b_n$ Both CONV. or Both DIV. (b) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ Then $\sum b_n \geq \sum a_n$ and if $\sum b_n$ CONV., $\sum a_n$ also CONV. (c) If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ Then $\sum a_n \geq \sum b_n$ and if $\sum b_n$ DIV., $\sum a_n$ also DIV.

THE ALTERNATING SERIES TEST: The alt. series $\sum(-1)^n b_n$ is CONV if (a) $b_{n+1} \leq b_n$ for all n and (b) $\lim_{n\to\infty} b_n = 0$ Failing Alt. Series Test - Inconclusive
ERROR ESTIMATE FOR ALTERNATING SERIES: If $\sum(-1)^n b_n$ CONVERGES then $|R_n| = |S - S_n| \leq b_{n+1}$ Error Less than Next Term

ABSOLUTE CONVERGENCE: A series $\sum a_n$ is ABS. CONV. if $\sum |a_n|$ CONV. If $\sum a_n$ is ABS. CONV., $\sum a_n$ is CONV.
If $\sum a_n$ CONV. but $\sum |a_n|$ DIV., $\sum a_n$ is CONDITIONALLY CONV. (C.C.) (*) If $\sum a_n$ has some (−) terms, use $\sum |a_n|$ for comparison tests

RATIO AND Nth ROOT TESTS FOR ABS. CONV.: If these tests fail, $\sum a_n$ is at most C.C. and possibly DIV. * Use to Find ROC
RATIO TEST: If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$ then $\sum a_n$ is ABS. CONV. If $L > 1$, $\sum a_n$ is DIV.
Nth ROOT TEST: If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$ then $\sum a_n$ is ABS. CONV. If $L > 1$ or $L = \infty$, $\sum a_n$ DIV.
(*) If $L = 1$ For Either Test, Both are INCONCL. $L = 0$ ROC $= \infty$ $L = \infty$ ROC $= 0$

POWER SERIES: $\sum_{n=0}^{\infty} C_n(x-a)^n = C_0 + C_1x_1 + C_2x_2^2 + \cdots$

$\boxed{ROC = \frac{1}{\lim_{n\to\infty} |C_n^{1/n}|}}$ If limit exists or limsup $\boxed{IOC = (a-R, a+R)\ [(\text{Check Endpoints})]}$
Term By Term Diff. and Integ. are Applicable (*)

TAYLOR SERIES about a: $\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$

MACLAURIN SERIES about a=0: $\sum_{n=0}^{\infty} \frac{f^n(0)}{n!}x^n = f(0) + \frac{f'(0)}{1!}(x) + \frac{f''(0)}{2!}(x)^2 + \cdots$
f(x) = these formulas if it's analytic

To Find the TΣ or MaCΣ for f(x), find the first few derivatives and plug in to those formulas

ANALYTIC: If $f(x) = T_n(x) + R_n(x)$ where T_n is the nth degree Taylor polynomial of f at a, and $\lim_{n\to\infty} R_n(x) = 0$ on IOC then f(x) = The sum of its Taylor Series and is ANALYTIC * $|R_n(x)| = |f(x) - T_n(x)|$ For Error Approximation $|R_n(x)|$ = Error

TAYLOR'S FORMULA $|R_n(x)| = \left|\frac{f^{n+1}(z)}{(n+1)!}(x-a)^{n+1}\right|$ | f smooth on IOC, z between x and a | * Use Taylor's Formula to show ANALYTICITY by showing $\lim_{n\to\infty} R_n(x) = 0$ * Use it for Error Approximation w/ either n given or $R_n(x)$ given

USEFUL MACLAURIN SERIES: f(x), $T_n(x)$, IOC

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$ $(-1,1)$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ $\mathbb{R}$

$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\mathbb{R}$

$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ $\mathbb{R}$

$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$ $[-1,1]$

BINOMIAL SERIES: If k is any Real # and $|x|<1$ then
$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots = \sum_{n=0}^{\infty}\binom{k}{n}x^n$
where $\binom{k}{n} = \frac{k(k-1)\cdots(k-n+1)}{n!}$, $(n \geq 1)$ and $\binom{k}{0} = 1$ check n=0, n=1 | what n = matters

To find $f^n(a)$, $f^n(a) = n!\,C_n$

$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$ Bessel Function $\mathbb{R}$ = IOC

e^{-1/x^2} for $\mathbb{R}-\{0\}$, 0 if $x=0$ is smooth on $\mathbb{R}$ but not ANALYTIC

IMPROPER INTEGRALS: CONV. Lim Exists as a #, DIV. Lim DNE or $= \pm\infty$

Limits of Int. @ $\pm\infty$		Discontinuities @ a or b	
$\int_a^{\infty} f(x)dx = \lim_{t\to\infty}\int_a^t f(x)dx$	If $\int$ exists for $t \geq a$	$\int_a^b f(x)dx = \lim_{t\to b^-}\int_a^t f(x)dx$	f(x) cont. on $[a,b)$ Disc. @ b
$\int_{-\infty}^b f(x)dx = \lim_{t\to-\infty}\int_t^b f(x)dx$	If $\int$ exists for $t \leq b$	$\int_a^b f(x)dx = \lim_{t\to a^+}\int_t^b f(x)dx$	f(x) cont. on $(a,b]$ Disc @ a

BY PARTS: $\int uv' = uv - \int vu'$
$u =$ $v' =$
$u' =$ $v =$
Pick u to simplify
Pick complex v' you can integrate