Pneumatic conveying

Angle of repose[1]

Error: interval with probability p (relation between δ and σ)

$$\Phi\left[\frac{(\mu+\delta)-\mu}{\sigma}\right]-\Phi\left[\frac{(\mu-\delta)-\mu}{\sigma}\right]=p$$

$$\Phi\left(\frac{\delta}{\sigma}\right)-\Phi\left(-\frac{\delta}{\sigma}\right)=p$$

$$\Phi\left(\frac{\delta}{\sigma}\right)-\left[1-\Phi\left(\frac{\delta}{\sigma}\right)\right]=p$$

$$2\Phi\left(\frac{\delta}{\sigma}\right)-1=p$$

$$\Phi\left(\frac{\delta}{\sigma}\right)=\frac{1+p}{2}$$

$$\sigma=\frac{\delta}{\tilde{\Phi}\left(\frac{1+p}{2}\right)}$$

With $z_p \equiv \tilde{\Phi}\left(\frac{1+p}{2}\right)$,

$$\boxed{\sigma=\frac{\delta}{z_p}}$$

Angle of repose, β

$$Q \equiv \tan\beta=\frac{h}{b}$$

Now (Nepf[2], Rule 5, generally),

$$\delta Q=\delta(\tan\beta)=\sqrt{\left(\frac{\partial Q}{\partial h}\delta h\right)^2+\left(\frac{\partial Q}{\partial b}\delta b\right)^2}$$

$$\left(\sec^2\beta\right)\delta\beta=\sqrt{\left(\frac{1}{b}\delta h\right)^2+\left(-\frac{h}{b^2}\delta b\right)^2}=\sqrt{\frac{\delta^2 h}{b^2}+\frac{h^2\delta^2 b}{b^4}}=$$

$$=\frac{h}{b}\sqrt{\left(\frac{\delta h}{h}\right)^2+\left(\frac{\delta b}{b}\right)^2}$$

$$\left(\sec^2\beta\right)\delta\beta=(\tan\beta)\sqrt{\left(\frac{\delta h}{h}\right)^2+\left(\frac{\delta b}{b}\right)^2}$$

$$\frac{\cos\beta}{\sin\beta}\frac{1}{\cos^2\beta}\delta\beta=\sqrt{\left(\frac{\delta h}{h}\right)^2+\left(\frac{\delta b}{b}\right)^2}$$

$$\boxed{\delta\beta=(\sin\beta\cos\beta)\sqrt{\left(\frac{\delta h}{h}\right)^2+\left(\frac{\delta b}{b}\right)^2}}$$

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[1] Ângulo de talude.

[2] "Propagation of Uncertainty through Mathematical Operations", http://web.mit.edu/fluids-modules/www/exper_techniques/2.Propagation_of_Uncertaint.pdf

or

$$\delta\beta = \frac{1}{2}\sin 2\beta\sqrt{\left(\frac{\delta h}{h}\right)^2 + \left(\frac{\delta b}{b}\right)^2}$$

Alternatively

$$\beta = \arctan\frac{h}{b}$$

$$\delta\beta = \sqrt{\left(\frac{\partial\beta}{\partial h}\delta h\right)^2 + \left(\frac{\partial\beta}{\partial b}\delta b\right)^2} = \sqrt{\left[\frac{\partial \arctan(h/b)}{\partial h}\delta h\right]^2 + \left[\frac{\partial \arctan(h/b)}{\partial b}\delta b\right]^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\arctan x = \frac{1}{1+x^2}$$

$$\delta\beta = \sqrt{\left[\frac{1}{1+(h/b)^2}\frac{1}{b}\delta h\right]^2 + \left[\frac{1}{1+(h/b)^2}\left(-\frac{h}{b^2}\right)\delta b\right]^2}$$

$$\delta^2\beta = \left[\frac{1}{1+(h/b)^2}\frac{1}{b}\delta h\right]^2 + \left[\frac{1}{1+(h/b)^2}\left(-\frac{h}{b^2}\right)\delta b\right]^2 =$$

$$= \frac{h}{b}\left[\frac{1}{1+(h/b)^2}\frac{\delta h}{h}\right]^2 + \frac{h}{b}\left[\frac{1}{1+(h/b)^2}\frac{\delta b}{b}\right]^2$$

$$\delta^2\beta = \frac{h}{b}\frac{1}{1+(h/b)^2}\left[\left(\frac{\delta h}{h}\right)^2 + \left(\frac{\delta b}{b}\right)^2\right]$$

$$\delta^2\beta = \tan\beta\frac{1}{1+\tan^2\beta}\left[\left(\frac{\delta h}{h}\right)^2 + \left(\frac{\delta b}{b}\right)^2\right] =$$

$$= \tan\beta\frac{\cos^2\beta}{\cos^2\beta+\sin^2\beta}\left[\left(\frac{\delta h}{h}\right)^2 + \left(\frac{\delta b}{b}\right)^2\right]$$

$$\delta^2\beta = \sin\beta\cos\beta\left[\left(\frac{\delta h}{h}\right)^2 + \left(\frac{\delta b}{b}\right)^2\right]$$

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