Truncated Gaussian distribution

$$x' = \frac{x-\mu}{\sigma} \qquad a' = \frac{a-\mu}{\sigma} \qquad b' = \frac{b-\mu}{\sigma}$$

$$a \le x \le b$$

$$\Delta\phi = \phi(b') - \phi(a') \tag{1}$$

$$\Delta\Phi = \Phi(b') - \Phi(a') \tag{2}$$

$$f(x;\mu,\sigma,a,b) = \frac{1}{\sigma\,\Delta\Phi}\phi(x') \tag{3}$$

$$\boxed{F(x;\mu,\sigma,a,b) = \frac{\Phi(x') - \Phi(a')}{\Delta\Phi}} \tag{4}$$

For the truncated, it is:

$$\mu' = \mu - \sigma\frac{\Delta\phi}{\Delta\Phi} \tag{5}$$

$$\frac{\sigma'^2}{\sigma^2} = 1 - \frac{b'\phi(b') - a'\phi(a')}{\Delta\Phi} - \left(\frac{\Delta\phi}{\Delta\Phi}\right)^2 \tag{6}$$

For simulation:

From {4}, it is

$$r = \frac{\Phi(x') - \Phi(a')}{\Delta\Phi} \tag{7}$$

$$\Phi(x') = \Phi(a') + r\Delta\Phi \tag{8}$$

$$x' = \Phi^{\text{inv}}(\Phi(a') + r\Delta\Phi) \tag{9}$$

and finally

$$\boxed{x_r = \mu + \sigma\,\Phi^{\text{inv}}\left(\Phi(a') + \underset{\text{variable}}{r}\,\Delta\Phi\right)} \tag{10}$$

Example: $\mu = 50$, $\sigma = 4$, $a = 40$, $b = 58$

$$\Delta\phi = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right) =$$

$$= \phi\left(\frac{58-50}{4}\right) - \phi\left(\frac{40-50}{4}\right) = \phi(2) - \phi(-2{,}5) =$$

$$= 0{,}053991 - 0{,}017528 = 0{,}036463$$

$$\Delta\Phi = \Phi(b') - \Phi(a') = \Phi(2) - \Phi(-2{,}5) =$$

$$= 0{,}97725 - 0{,}00621 = 0{,}97104$$

$$\boxed{x_r = 50 + 4\Phi^{\text{inv}}(0{,}00621 + r\,0{,}97104)}$$

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