

SIMULATION routine ("Monte Carlo Method")

Simulation of random phenomena, based on (pseudo-)random numbers

- 1) Define the phenomenon to simulate (random variable, x) and (**a**) characterize it through an adequate probability function or (**b**) go to **2**.

DISCRETE VARIABLE

Let $f(x)$ be the (point) probability function.

CONTINUOUS VARIABLE

Let $f(x)$ be the probability density function.

- 2) Determine the cumulative (distribution) function.

$$F(x) = \sum_{t=x_{\min}}^x f(t)$$

(x_{\min} is the least value possible for x .)

$$F(x) = \int_{x_{\min}}^x f(t) dt$$

Whenever the analytical form of $F(x)$ is unknown or difficult to deduce, it is necessary to tabulate $F(x)$.

- 3) Adopt uniform random numbers of k digits, with k "compatible" with the values of $F(x)$ to use (same number of significant digits).

(The value of k depends of the circumstances of the problem.)

- 4) (*Simulation proper*) Extract a random number, N , and divide it by 10^k , to obtain the random number u , reduced to the interval $[0, 1)$.

$$u = N / 10^k$$

- 5) Solve the equation " $F(x) = u$ " for x , i.e.,

$$F(x-1) \leq u < F(x)$$

[with $F(x_{\min} - 1) = 0$] whence x is obtained.

$$F(x) = u$$

If the function F has a known inverse, \tilde{F} ,

$$x = \tilde{F}(u)$$

The value x is a simulated value of the random variable under consideration.

- 6) Return to the simulation procedure —steps **4** and **5**— until a "sufficient" number of values is calculated; or finish.

