$$p(k;\mu) = \mathrm{e}^{-\mu}\,\frac{\mu^k}{k!}$$

t = "tiny" = $2^{2-2^{10}} = 2^{2-1024} = 2^{-1022} = 2{,}225 \times 10^{-308}$. Last good n ?

$$\mathrm{e}^{-\mu}\,\frac{\mu^n}{n!} > t$$

$$\frac{\mu^n}{n!} > \mathrm{e}^{\mu}\,t$$

$$n\ln\mu - \ln(n!) > \mu + \ln t$$

Stirling:

$$\left(n+\frac{1}{2}\right)\ln n - n + \left\{\frac{1}{12n+1}\right\} + \ln\left(\sqrt{2\pi}\right) < \ln(n!) < \left(n+\frac{1}{2}\right)\ln n - n + \left\{\frac{1}{12n}\right\} + \ln\left(\sqrt{2\pi}\right)$$

$$n\ln\mu - \left[\left(n+\frac{1}{2}\right)\ln n - n + \left\{\frac{1}{12n}\right\} + \ln\left(\sqrt{2\pi}\right)\right] > \mu + \ln t$$

$$\mu + \ln t < n\ln\mu - \left[\left(n+\frac{1}{2}\right)\ln n - n + \left\{\frac{1}{12n}\right\} + \ln\left(\sqrt{2\pi}\right)\right]$$

Consider fraction $\frac{1}{12n}$ (more prudent).

$$\mu + \ln\left(t\sqrt{2\pi}\right) < -\left(n+\frac{1}{2}\right)\ln n + (1+\ln\mu)n + \left\{\frac{1}{12n}\right\}$$

$$\boxed{\left(n+\frac{1}{2}\right)\ln n - (1+\ln\mu)n - \left\{\frac{1}{12n}\right\} < -\mu - \ln\left(t\sqrt{2\pi}\right)}$$

For Newton-Raphson,

$$f(n) = \left(n+\frac{1}{2}\right)\ln n - (1+\ln\mu)n - \left\{\frac{1}{12n}\right\} + \left[\mu + \ln\left(t\sqrt{2\pi}\right)\right] = 0$$

$$f'(n) = \ln n + \left(n+\frac{1}{2}\right)\frac{1}{n} - (1+\ln\mu) + \left\{\frac{1}{12n^2}\right\} =$$

$$= \ln n + 1 + \frac{1}{2n} - 1 - \ln\mu + \left\{\frac{1}{12n^2}\right\} = \ln\frac{n}{\mu} + \frac{1}{2n} + \left\{\frac{1}{12n^2}\right\}$$

etc.

❖