Chen, Der-San, Robert G. Batson, Yu Dang,
"Applied Integer Programming: modeling and solution", Wiley, 2010

## Chapter 3 - Transformation using 0-1 variables

3.1 Transform Logical (Boolean) Expressions
3.2 Transform Nonbinary to 0-1 Variable
3.3 Transform Piecewise Linear Functions
3.4 Transform 0-1 Polynomial Functions
3.5 Transform Functions with Products of Binary and Continuous Variables

### 3.6 Transform Nonsimultaneous Constraints

### 3.6.1 Either/or constraints

Two disjunctive regions: $x$ is outside the interval $(3,10)$

$$
x \leq 3 \vee x \geq 10
$$

This becomes, exclusively,

$$
\left\{\begin{array}{c}
x-3 \leq 0 \\
-x+10 \leq 0
\end{array}\right.
$$

Let $M$ be a big (enough) number and $y$ binary.

$$
\left\{\begin{array}{c}
x-3 \leq M y \\
-x+10 \leq M(1-y)
\end{array}\right.
$$

Verify. If $y=1$, the $2 .{ }^{\text {nd }}$ constraint (only) applies:

$$
\left\{\begin{array}{c}
(x-3 \leq M \approx \infty) \\
-x+10 \leq 0
\end{array}\right.
$$

If $y=0$, the $1 .{ }^{\text {st }}$ constraint (only) applies:

$$
\left\{\begin{array}{c}
x-3 \leq 0 \\
(-x+10 \leq M \approx \infty)
\end{array}\right.
$$

### 3.6.2 $\boldsymbol{p}$ out of $\boldsymbol{m}$ constraints must hold

This case is a direct generalization of the previous one, where it was $m=2$ and $p=1$.

$$
\begin{array}{cr}
f_{i}(\mathbf{x}) \leq b_{i} & i=1 . . m \\
f_{i}(\mathbf{x})-b_{i} \leq 0 & i=1 . . m
\end{array}
$$

With vector $\mathbf{y}$ (i.e., $y_{i}, i=1 . . m$ ) binary,

$$
\begin{gathered}
f_{i}(\mathbf{x})-b_{i} \leq M \quad y_{i} \quad i=1 . . m \\
\sum_{i=1}^{m} y_{i}=m-p
\end{gathered}
$$

### 3.6.3 Disjunctive constraint sets

### 3.6.4 Negation of a constraint

(Obvious.)

### 3.6.5 If/then constraints

$$
\text { If } f_{1}(\mathbf{x})-b_{1} \leq 0 \text { then } f_{2}(\mathbf{x})-b_{2} \leq 0
$$

is equivalent to

$$
\text { Either }-f_{1}(\mathbf{x})+b_{1} \leq 0 \quad \text { or } \quad f_{2}(\mathbf{x})-b_{2} \leq 0
$$

By "either/or",

$$
\left\{\begin{array}{c}
-f_{1}(\mathbf{x})+b_{1} \leq M y \\
f_{2}(\mathbf{x})-b_{2} \leq M(1-y)
\end{array}\right.
$$

Solved (related) problems in the book

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