Analytical Performance Evaluation of a Class of Receivers with Joint Equalization and Carrier Frequency Synchronization

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***Abstract*—In this paper we consider an SC-FDE (Single Carrier Frequency-Domain Equalization) block transmission with residual frequency errors. We employ frequency-domain iterative DFE (Decision Feedback Equalization) receivers denoted by Iterative Block-Decision Frequency Equalization (IB-DFE), which are combined with a Decision-directed (DD) Carrier Frequency Offset (CFO) estimator. This estimator produces a CFO estimate within each iteration of the equalizer. We develop a model to predict the performance of IB-DFE receivers with joint detection and carrier frequency synchronization[1].**

I. INTRODUCTION

Due to an increased demand for wireless services, future systems are required to support high quality of service at high data rates. For such high data rates, the time-dispersion effects associated to the multipath propagation can be severe. In this case, conventional time-domain equalization schemes are not practical. Alternative techniques employing block transmission, with appropriate cyclic extensions and employing FDE (Frequency-Domain Equalization) techniques have been shown to be suitable for high data rate transmission over severely time-dispersive channels without requiring complex receivers. The most popular modulations based on this latter concept are the OFDM (Orthogonal Frequency Division Multiplexing) modulations [1]. An alternative approach based on the same principle are block transmission SC modulations (Single Carrier) combined with FDE (also denoted SC-FDE) [2]. Both schemes have similar implementation complexities, nevertheless SC-FDE schemes require simpler receivers. In this paper we consider the SC-FDE approach.

A promising IFDE (Iterative FDE) technique for SC-FDE, denoted IB-DFE (Iterative Block-Decision Feedback Equalizer), was proposed in [3]. This technique was later extended to diversity scenarios and layered space-time schemes (see [4] and references therein.) These IFDE receivers can be regarded as iterative DFE (Decision Feedback equalizer) receivers with the feedforward and the feedback operations implemented in the frequency domain. An IB-DFE receiver with joint post-equalization carrier frequency synchronization was presented in [5]. This receiver can be regarded as a modified turbo equalization scheme where, for each iteration, we perform DD (Decision-directed) CFO (Carrier Frequency Offset) estimation.

In order to maintain high power and spectral efficiencies, the cyclic prefix, which is longer than the overall channel impulse response length, should be a small fraction of the block duration. As a consequence, we usually need large blocks for severely time-dispersive channels, with hundreds or even thousands of symbols and, typically the frequency errors cannot exceed a small fraction of the inverse of the block duration. This means that we have higher sensitivity to frequency errors for larger blocks, making accurate carrier synchronization mandatory. Frequency errors usually originate from the frequency mismatch between the oscillators at the transmitter and receiver. Another possible source of frequency errors is the Doppler frequency shift caused by relative motion between the transmitter and the receiver.

While discussing carrier synchronization, two synchronization levels have to be distinguished, namely, carrier phase and carrier frequency synchronization. Furthermore, depending on the magnitude of the carrier frequency offset two types of synchronization can be attained, namely, a coarse synchronization and a fine synchronization. Usually, algorithms for fine synchronization require some type of previous coarse frequency estimation. Typically, fine carrier frequency synchronization systems are designed to deal with frequency offsets less than 10% of the symbol duration. Also, based on the degree of knowledge that the receiver has on the transmitted signal, these systems can be separated into three categories: data-aided (DA); non-data aided (NDA); and decision-directed (DD) ([6], [7] and references therein.)

A DD method for the estimation of the carrier frequency offset was presented in [8]. There, a CFO estimator that relies on data decisions to produce an estimate of the CFO is analyzed. If the receiver had perfect knowledge on the transmitted data, the CFO estimate would be unbiased. However, since we rely on estimates of the data symbols, which contain errors, the CFO estimates are biased. Finally, the statistics of the bias were determined and a model for the

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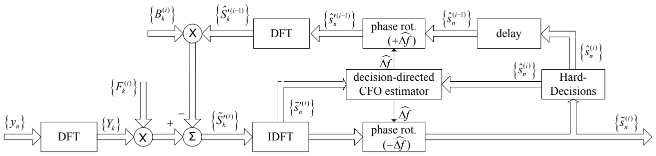

Fig. 1. IB-DFE with joint carrier frequency synchronization.

bias, as a function of the true CFO and of the signal-to-noise ratio (SNR), was established.

In this paper we consider a SC-FDE block transmission in the presence of residual frequency errors. In particular, we consider an IB-DFE receiver with joint carrier frequency synchronization as described in [5] (see Fig. 1.) Moreover, the output of the frequency-domain equalizer is approximated by an Gaussian distribution [9], [10], [11] allowing to conveniently describe the output of the equalizer by an equivalent Additive White Gaussian Noise (AWGN) channel. This approximation allows the post-equalization carrier frequency synchronization step to be described by the model introduced in [8]. That being so, we propose to model a receiver structure with joint equalization and carrier synchronization and eventually predict the performance of the real system.

This paper is organized as follows. In Sec. II we describe the output of the iterative FDE receiver by an equivalent AWGN channel. Sec. III describes the joint equalization and carrier synchronization, with subsections dedicated to CFO impact, the modeling of the CFO synchronization and the modeling of the equalization with joint carrier frequency synchronization. Sec. IV presents the performance results and Sec. V the conclusions.

II. ITERATIVE FDE RECEIVERS

In this section we see how the output of the IFDE presented in [5] can be described by an equivalent AWGN channel. Moreover, the BER performance of the IFDE can be predicted if a Gaussian distribution is assumed for the output [9], [10], [11]. In this case the time-domain output of the equalizer associated to each iteration, $\{\tilde{s}_n^{(i)};\, n = 0, 1, \ldots, N-1\}$, can be approximated by

$$\tilde{s}_n^{(i)} = s_n + \eta_n^{(i)}, \tag{1}$$

where $\{s_n;\, n = 0, 1, \ldots, N-1\}$ are the transmitted symbols selected form a given constellation, *e.g.*, QPSK, while $\eta_n^{(i)} = \tilde{s}_n^{(i)} - s_n$ is the overall error including both the channel noise and the residual ISI. The frequency-domain samples can be written in a similar fashion, $\tilde{S}_k^{(i)} = S_k + \eta_k^{(i)}$, where $\{\tilde{S}_k^{(i)};\, k = 0, 1, \ldots, N-1\} = \mathrm{DFT}\{\tilde{s}_n^{(i)};\, n = 0, 1, \ldots, N-1\}$, $\{S_k;\, k = 0, 1, \ldots, N-1\} = \mathrm{DFT}\{s_n;\, n = 0, 1, \ldots, N-1\}$ and $\{\eta_k;\, k = 0, 1, \ldots, N-1\} = \mathrm{DFT}\{\eta_n;\, n = 0, 1, \ldots, N-1\}$, with $\mathrm{DFT}\{\cdot\}$ denoting the Discrete Fourier Transform operation. Therefore, for the output of the IFDE a "signal-to-noise plus interference ratio" (SNIR) can be defined as

$$\mathrm{SNIR}^{(i)} = \mathrm{E}[|S_k|^2]/\mathrm{E}[|\eta_k^{(i)}|^2]. \tag{2}$$

In (2), $\mathrm{E}[|\eta_k^{(i)}|^2]$ is given by [10]

$$\begin{aligned} \mathrm{E}[|\eta_k^{(i)}|^2] = {} & \mathrm{E}[|F_k^{(i)}|^2]\mathrm{E}[|N_k|^2] + \\ & + \mathrm{E}[|F_k^{(i)} H_k - 1 - (\rho^{(i-1)})^2 B_k^{(i)}|]\mathrm{E}[|S_k|^2] + \\ & + \mathrm{E}[|B_k^{(i)}|^2](\rho^{(i-1)})^4 (1 - (\rho^{(i-1)})^2)\mathrm{E}[|S_k|^2]. \end{aligned} \tag{3}$$

In (3) we have that $\{H_k;\, k = 0, 1, \ldots, N-1\}$ is the Channel Frequency Response (CFR), and $\{N_k;\, k = 0, 1, \ldots, N-1\} = \mathrm{DFT}\{\nu_n;\, n = 0, 1, \ldots, N-1\}$ where $\{\nu_n;\, n = 0, 1, \ldots, N-1\}$ is a complex white Gaussian sequence with variance $2\sigma_\nu^2$.

Also in (3) B_k and F_k are the optimum feedback and feedforward equalizer coefficients, respectively, and are given by [10], $B_k^{(i)} = F_k^{(i)} H_k - 1$ and $F_k^{(i)} = \breve{F}_k^{(i)}/\gamma^{(i)}$, where $\breve{F}_k^{(i)}$ is given by $\breve{F}_k^{(i)} = H_k^*/[\alpha + (1 - (\rho^{(i-1)})^2)|H_k|^2]$, with $\alpha = E[|N_k|^2]/E[|S_k|^2]$ and $\gamma^{(i)} = \frac{1}{N}\sum_{k=0}^{N-1} \breve{F}_k^{(i)} H_k$.

From (1) results that (2) can be regarded as the SNR of an equivalent AWGN channel. Thus, for QPSK signaling with Gray coding, the BER of the IFDE can be approximated by

$$P_b^{(i)} = \mathcal{Q}\left(\sqrt{\mathrm{SNIR}^{(i)}}\right), \tag{4}$$

with $\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$. The correlation factor $\rho^{(i-1)}$ in (3) relates with the BER (4) through the expression [10]

$$\rho^{(i-1)} = 1 - 2P_b^{(i-1)}. \tag{5}$$

Equations (2)-(4) provides the means to predict the BER performance of the IFDE receiver for successive iteration orders. The quality of this approximation can be appreciated in Sec. IV.

III. JOINT EQUALIZATION AND CARRIER SYNCHRONIZATION

A. Impact of the Carrier Frequency Offset

Let us assume a slowly-varying scenario. In this case, as seen is sec. II, the overall channel at the output of the FDE can be regarded as a stationary flat fading channel (at least for the duration of a given block). Therefore, we describe the impact of the CFO assuming an ideal Gaussian channel.

Moreover, let us also assume that there is a residual carrier frequency offset between the transmitter and the receiver local oscillators. In this case the transmitted time-domain data symbols $\{s_n;\, n = 0, 1, \ldots, N-1\}$ undergo a phase rotation, $s_n' = s_n \exp(j2\pi\Delta f nT/N)$. For the sake of simplicity, it is assumed that the residual frequency offset results exclusively from the transmitter; we also assume that the phase rotation is zero for $n = 0$.

Since the nth data symbol is rotated by $n\theta$, with $\theta = 2\pi\Delta fT/N$, the corresponding BER for a QPSK constellation and an ideal Gaussian channel is $P_{b,n} = \frac{1}{2}(p_{1,n} + p_{2,n})$, where

$$p_{1,n} = \mathcal{Q}\left(\big(\cos(n\theta) - \sin(n\theta)\big)\sqrt{2E_b/\mathcal{N}_0}\right), \tag{6}$$

$$p_{2,n} = \mathcal{Q}\left(\big(\cos(n\theta) + \sin(n\theta)\big)\sqrt{2E_b/\mathcal{N}_0}\right), \tag{7}$$

$2E_b/\mathcal{N}_0 = \mathrm{SNR}$, and the overall BER is the average of $P_{b,n}$ over n

$$P_b = \mathrm{E}[P_{b,n}] = \frac{1}{2N}\sum_{n=0}^{N-1}(p_{1,n}+p_{2,n}). \tag{8}$$

B. Model of the Carrier Frequency Offset Estimator

In [12] a model of an iterative estimator of the carrier frequency offset as a function of the true CFO and of the SNR is introduced. The estimates of the CFO are described by the sum of the CFO Δf and a bias component, Δf_{bias}, i.e., $\widehat{\Delta f} = \Delta f + \Delta f_{\mathrm{bias}}$, where the bias is approximately

$$\Delta f_{\mathrm{bias}} \approx \frac{N}{2\pi MT}\arg\{\Upsilon\}, \tag{9}$$

where $\Upsilon = (N-M)|s_n|^2 + \sum_{n=0}^{N-M-1}\bar{\zeta}_n$. Defining the data decision error as $\varepsilon_n = \hat{s}_n - s_n$ and assuming D to be the minimum Euclidean distance, the mean value of the data decision error is $\bar{\varepsilon}_n = \mathrm{E}[\varepsilon_n] = -Dp_{1,n} - jDp_{2,n}$, with $p_{1,n}$ and $p_{2,n}$ given by (6) and (7), respectively. In addition, the contribution of the data detection errors to (9) is $\bar{\zeta}_n = \mathrm{E}[\zeta_n] = \bar{\varepsilon}^*_{n+M}s_n + \bar{\varepsilon}_n s^*_n + \bar{\varepsilon}^*_{n+M}\bar{\varepsilon}_n$.

C. Model for the joint equalization with carrier frequency synchronization

In this section we combine the analytical results obtained for the IB-DFE receiver with those obtained for the synchronization of the carrier frequency offset, expecting to produce a model for the IB-DFE with joint carrier frequency synchronization. The model for the carrier synchronization block presumes the use of an AWGN channel, whereas the output of the IB-DFE receiver is modeled as an equivalent Gaussian channel. Since the synchronization procedure is conducted after the equalization procedure, we are in conditions to produce a model for the joint equalization and carrier frequency synchronization. The model for the iterative algorithm can be described by the following steps. Firstly, we do $i = 1$ and assume to have an initial CFO $\Delta f^{(1)}$ and $\rho^{(0)} = 0$. Secondly, $\rho^{(i-1)}$ is used in (3), and (3) in (2) to determine the SNR of the equivalent Gaussian channel. Thirdly, after having the $SNR^{(i)}$ and the CFO $\Delta f^{(i)}$ we compute the BER using (8). Also with $SNR^{(i)}$ and $\Delta f^{(i)}$, we compute (9) which predicts the CFO present in the signal in the next iteration, *i.e.*, $\Delta f^{(i+1)} = -\Delta f^{(i)}_{\mathrm{bias}}$. Finally, we update the algorithm by using the BER $P_b^{(i)}$ in (5) to compute the $\rho^{(i)}$ and by incrementing the iteration order. These steps are graphically summarized in Fig. 2.

IV. PERFORMANCE RESULTS

In this section we present a set of experimental results, where we compare the performance of the proposed system with that predicted by our model. We consider SC-FDE modulations with blocks of $N = 1024$ "useful" modulation symbols (corresponding to a duration of $4\mu s$), plus an appropriate cyclic prefix. The modulation symbols belong to a QPSK constellation and are selected from the transmitted data according to a Gray mapping rule. We consider linear power amplification and perfect channel estimation. The propagation channel is characterized by the power delay profile type C for HIPERLAN/2 (HIgh PERformance Local Area Network) [13], with uncorrelated Rayleigh fading on different paths (similar results could be obtained for other severely time dispersive channel models with rich multipath propagation).

Depicted in Figs. 3 and 4 is the evolution of the BER and the correlation factor, respectively, for successive iterations of the IB-DFE receiver. In both figures we compare the simulation results, obtained through Monte Carlo trials, with those obtained using the Gaussian approximation, with the BER given by (4) and the correlation factor by (5). In Fig. 3 we can see that predicted values for the BER are very close to the simulated ones, with differences around 0.5dB for the 2nd iteration. Additionally, can say that the analytical values are slightly optimistic. From Fig. 4, where the correlation factor is depicted, we can see that there is a close match between the predicted and the simulated curves.

In Figs. 5, 7 and 9 we consider a CFO normalized to the symbol duration $\Delta fT = 0.05$, whereas for Figs. 6, 8 and 10 we consider a CFO normalized to the symbol duration $\Delta fT = 0.1$.

Depicted in Figs. 5 and 6 is the true bias of the CFO estimates, obtained through Monte Carlo trials, and the predicted bias, obtained using (9). We see that for the 2nd iteration, the one for which a first estimate of the CFO is produced, the difference between the prediction and its true value is around or below 1.5dB, and around or below 2dB for $\Delta fT = 0.05$ and $\Delta fT = 0.1$, respectively. However, as the iteration order increases this difference also increases. This growth in the error results from using for a given iteration the bias predicted in the previous iteration as the CFO, which, as we have seen from the 2nd iteration, differs from the true value. Additionally, it can be referred that the model for the bias, (9), is optimistic.

In Figs. 7 and 8 we compare the evolution of the BER with that predicted by the proposed model. The results depicted refer to successive iteration orders. For the first iteration, the analytical curve and the simulation curve are very close. This results from using just the Gaussian approximation for the model, since no estimate of the CFO is produced. For the remaining iterations, which apply (9), the analytical values can be considered optimistic with differences around or below 1dB, when $\Delta fT = 0.05$, and around or below 2.5dB, when $\Delta fT = 0.1$. This for the 2nd iteration. We also include the MFB (Matched Filter Bound) performance, defined as [14]

$$P_{b,\mathrm{MFB}} = E\left[\mathcal{Q}\left(\sqrt{\frac{2E_b}{\mathcal{N}_0}\frac{1}{N}\sum_{k=0}^{N-1}|H_k|^2}\right)\right], \tag{10}$$

as a benchmark. Additionally, it can be referred that the model for the joint iterative equalization with carrier frequency synchronization is optimistic.

Depicted in Figs. 9 and 10 is the evolution of the correlation factor, ρ, together with the predicted values given by (5), with P_b given by the proposed model (see Figs. 7 and 8). Thus,

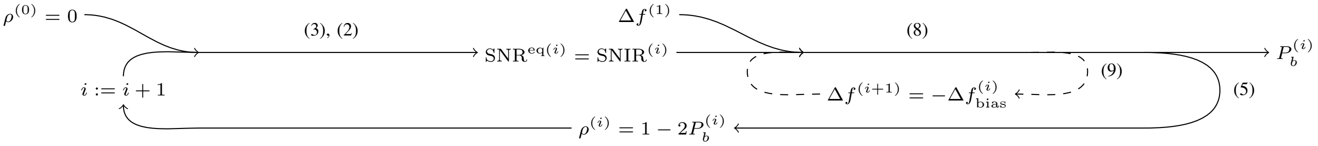

Fig. 2. Algorithm modeling the joint equalization with carrier frequency synchronization.

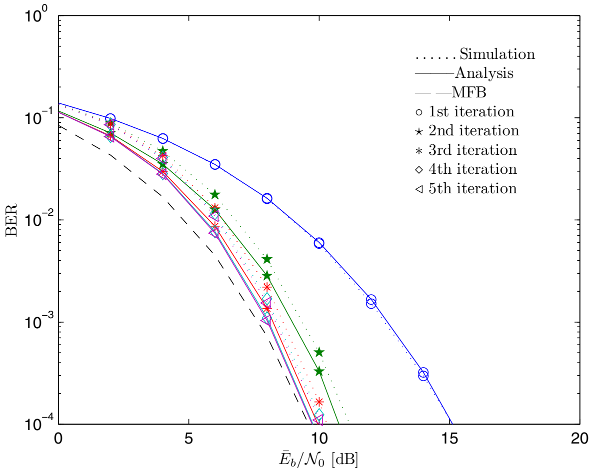

Fig. 3. BER performance for a IB-DFE receiver with joint carrier synchronization.

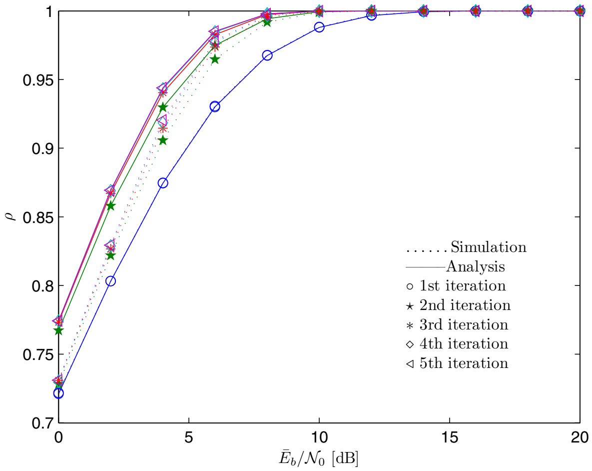

Fig. 4. Correlation factor ρ versus $\bar{E}_b/\mathcal{N}_0$.

the quality of the prediction of $\rho^{(i)}$ is directly linked to the quality of the prediction of the BER. From Figs. 9 and 10 we can see that the analytical values, with exception for the 1st iteration, tend to be overestimated.

V. CONCLUSIONS

We considered an IB-DFE with joint detection and carrier frequency synchronization for SC-FDE block transmission in the presence of residual frequency errors. We described the output of the frequency-domain equalizer by a Gaussian distribution and defined an equivalent AWGN channel which allowed us to propose a model for the carrier frequency synchronization procedure. By using the Gaussian approximation for the equalizer and by modeling the bias as a function of both the CFO and the SNR we built a model for the IB-DFE with joint carrier frequency synchronization. This model can be used for predicting the behavior of the true system.

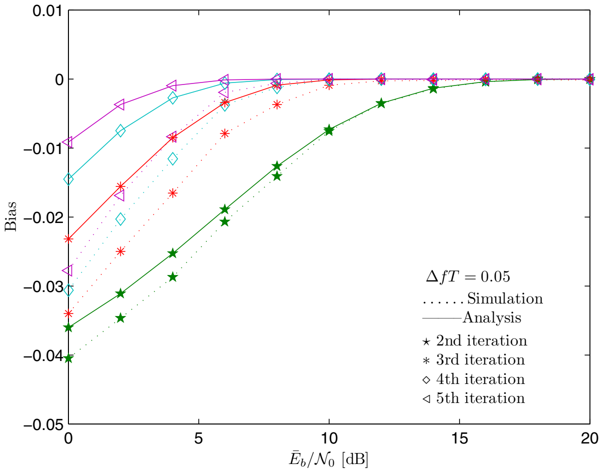

Fig. 5. Bias of the CFO estimates per symbol.

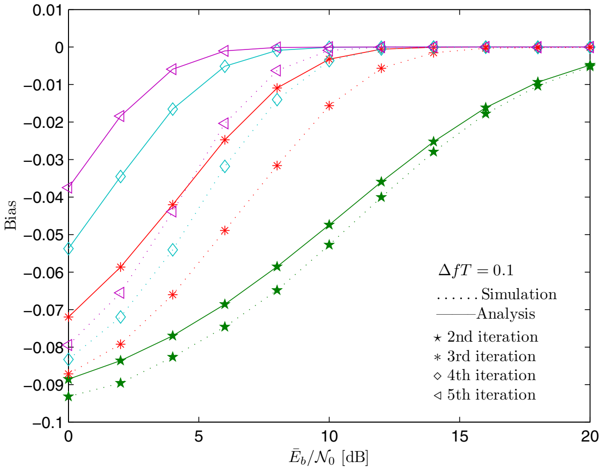

Fig. 6. Bias of the CFO estimates per symbol.

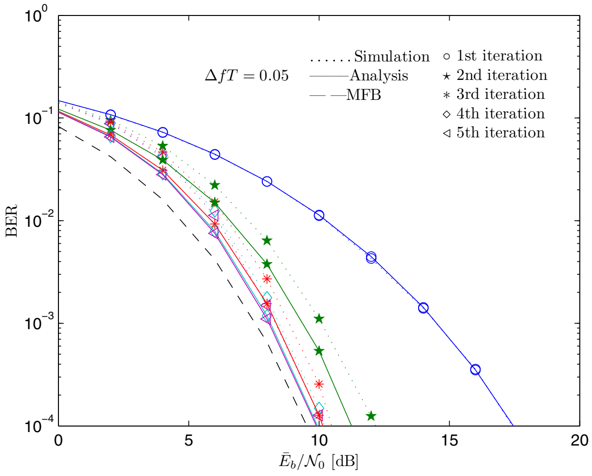

Fig. 7. BER performance for a IB-DFE receiver with joint carrier synchronization.

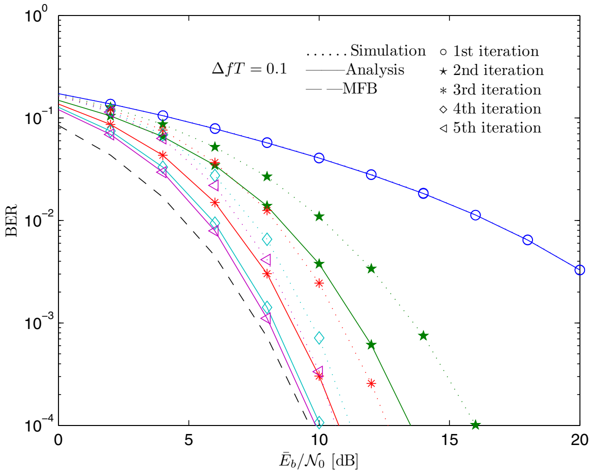

Fig. 8. BER performance for a IB-DFE receiver with joint carrier synchronization.

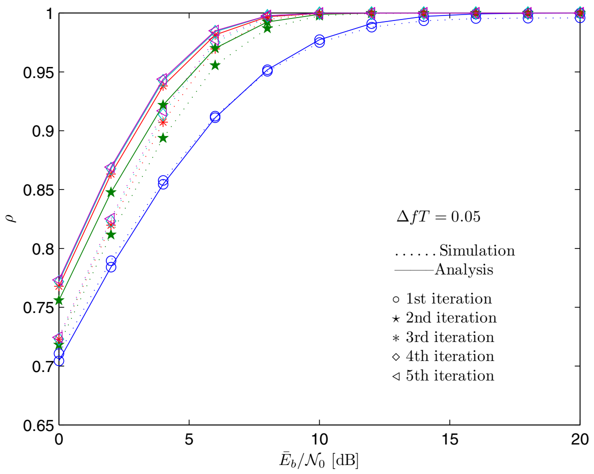

Fig. 9. Correlation factor ρ versus $\bar{E}_b/\mathcal{N}_0$.

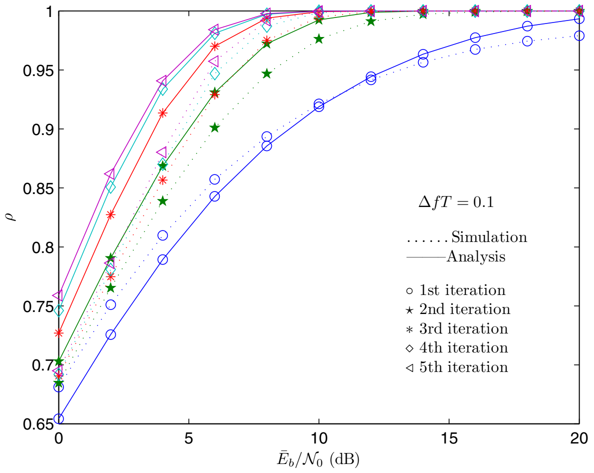

Fig. 10. Correlation factor ρ versus $\bar{E}_b/\mathcal{N}_0$.

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