

# AN ANALYTICAL APPROACH FOR PERFORMANCE EVALUATION OF A CLASS OF RECEIVERS WITH JOINT EQUALIZATION AND CARRIER FREQUENCY SYNCHRONIZATION

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## ABSTRACT

In this paper we consider an SC-FDE (Single Carrier Frequency-Domain Equalization) block transmission with residual frequency errors. We employ a frequency-domain iterative DFE (Decision Feedback Equalization) denoted by Iterative Block-Decision Frequency Equalization (IB-DFE), which is combined with a Decision-directed (DD) Carrier Frequency Offset (CFO) estimator. This estimator produces a CFO estimate plus compensation within each iteration of the equalizer. We develop a model to predict the performance of IB-DFE receivers with joint detection and carrier frequency synchronization. Likewise, we propose a graphical tool allowing to predict the number of iterations needed to reach a given Bit Error Rate (BER) and the value of the BER for each iteration.

## 1. INTRODUCTION

Due to an increased demand for wireless services, future systems are required to support high quality of service at high data rates. For such high data rates, the time-dispersion effects associated to the multipath propagation can be severe. In this case, conventional time-domain equalization schemes are not practical. Alternative techniques employing block transmission with appropriate cyclic extensions and employing FDE (Frequency-Domain Equalization) techniques have been shown to be suitable for high data rate transmission over severely time-dispersive channels without requiring complex receivers. The most popular modulations based on this latter concept are the OFDM (Orthogonal Frequency Division Multiplexing) modulations [1]. An alternative approach based on the same principle are block transmission SC modulations (Single Carrier) combined with FDE (also denoted SC-FDE) [2] and [3]. Both schemes have similar implementation complexities; nevertheless SC-FDE schemes require simpler receivers. In this paper we consider the SC-FDE approach.

A promising IFDE (Iterative FDE) technique for SC-FDE, denoted IB-DFE (Iterative Block-Decision Feedback Equalizer), was proposed in [4]. This technique was later extended to diversity scenarios and layered space-time schemes. These IFDE receivers can be regarded as iterative DFE (Decision Feedback equalizer) receivers with the feedforward and the feedback operations implemented in the frequency domain (see [3] and references therein). An IB-DFE receiver with joint post-equalization carrier frequency synchronization was proposed in [5]. This receiver can be regarded as a modified turbo equalization scheme where, for each iteration, we perform DD (Decision-directed) CFO (Carrier Frequency Offset) estimation.

To maintain high power and spectral efficiencies, the cyclic extension, which has to be longer than the overall channel impulse response length, must be a small fraction of the block duration. As a consequence, we usually need large blocks for severely time-dispersive channels, with hundreds or even thousands of symbols and, typically the frequency errors cannot exceed a small fraction of the inverse of the block duration. This means that we have higher sensitivity to frequency errors for larger blocks, making accurate carrier synchronization mandatory. Frequency errors usually originate from the frequency mismatch between the oscillators at the transmitter and receiver. Another possible source of frequency errors is the Doppler frequency shift caused by relative motion between the transmitter and the receiver.

While discussing carrier synchronization, two synchronization levels have to be distinguished, namely, carrier phase and carrier frequency synchronization. Furthermore, depending on the magnitude of the carrier frequency offset two types of synchronization can be attained, namely, a coarse synchronization and a fine synchronization. Usually, algorithms for fine synchronization require some type of previous coarse frequency estimation. Typically, fine carrier frequency synchronization systems are designed to deal with frequency offsets less than 10% of the symbol duration. Also, based on the degree of knowledge that the receiver has on the transmitted signal, these systems can be separated into three categories:

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data-aided (DA); non-data aided (NDA); and decision-directed (DD) (see [6] and references therein).

A DD method for the estimation of the carrier frequency offset was presented in [7]. There, a CFO estimator that relies on data decisions to produce an estimate of the CFO is analyzed. If the receiver had perfect knowledge on the transmitted data, the CFO estimate would be unbiased. However, since we rely on estimates of the data symbols, which contain errors, the CFO estimates are biased. The statistics of the bias were determined and a model for the bias, as a function of the true CFO and of the signal-to-noise ratio (SNR), was established.

In this paper we consider a SC-FDE block transmission in the presence of residual frequency errors. In particular, we consider an IB-DFE receiver with joint carrier frequency synchronization. Moreover, the output of the frequency-domain equalizer is approximated by a Gaussian distribution [8], [9], [10] permitting to conveniently describe the output of the equalizer by an equivalent AWGN channel. This approximation allows the post-equalization carrier frequency synchronization step to be described by the model proposed in [7]. We propose to model a receiver structure with joint equalization and carrier synchronization and eventually predict the performance of the real system. Additionally, a graphical tool is developed which allows a prediction of the minimum BER achievable, the required number of iterations to reach that minimum and the BER for each iteration.

This paper is organized as follows. In Sec. 2 we describe the models for an iterative FDE receiver, for an iterative estimator of the carrier frequency offset and for a receiver with joint equalization with carrier frequency synchronization. Sec. 3 presents the performance results and Sec. 4 the conclusions.

## 2. SYSTEM MODEL

### 2.1. Iterative FDE Receiver

The time-domain output of the  $i$ th iteration of an FDE receiver can be described by a Gaussian distributed random variable [8], [9], [10]

$$\tilde{s}_n^{(i)} = s_n + \eta_n^{(i)}, \quad (1)$$

where  $\{s_n; n = 0, 1, \dots, N-1\}$  are the transmitted symbols and  $\{\eta_n^{(i)}; n = 0, 1, \dots, N-1\}$  are samples of channel noise plus interference. Samples  $\{\eta_n^{(i)}; n = 0, 1, \dots, N-1\}$  have the corresponding frequency-domain samples given by the Discrete Fourier Transform (DFT) operation,  $\{\eta_k; k = 0, 1, \dots, N-1\} = \text{DFT}\{\eta_n; n = 0, 1, \dots, N-1\}$ , with

overall power given by [9],

$$\begin{aligned} E[|\eta_k^{(i)}|^2] = & \\ & E[|F_k^{(i)} H_k - 1 - (\rho^{(i-1)})^2 B_k^{(i)}|] E[|S_k|^2] + \\ & + E[|B_k^{(i)}|^2] (\rho^{(i-1)})^4 (1 - (\rho^{(i-1)})^2) E[|S_k|^2] + \\ & + E[|F_k^{(i)}|^2] E[|N_k|^2], \end{aligned} \quad (2)$$

where  $\{H_k; k = 0, 1, \dots, N-1\}$  is the Channel Frequency Response (CFR),  $\{S_k; k = 0, 1, \dots, N-1\} = \text{DFT}\{s_n; n = 0, 1, \dots, N-1\}$  and  $\{N_k; k = 0, 1, \dots, N-1\} = \text{DFT}\{\nu_n; n = 0, 1, \dots, N-1\}$  where  $\{\nu_n; n = 0, 1, \dots, N-1\}$  is a complex white Gaussian sequence with variance  $2\sigma_\nu^2$ .

In (2)  $B_k$  and  $F_k$  are the optimum feedback and feedforward equalizer coefficients, respectively, and are given by [9],  $B_k^{(i)} = F_k^{(i)} H_k - 1$  and  $F_k^{(i)} = \check{F}_k^{(i)} / \gamma^{(i)}$ , where  $\check{F}_k^{(i)}$  is given by  $\check{F}_k^{(i)} = H_k^* / [\alpha^{-1} + (1 - (\rho^{(i-1)})^2) |H_k|^2]$ , with  $\alpha = E[|S_k|^2] / E[|N_k|^2]$  and  $\gamma^{(i)} = \frac{1}{N} \sum_{k=0}^{N-1} \check{F}_k^{(i)} H_k$ .

Using (2) may define the ‘‘Signal-to-Noise plus Interference Ratio’’ as,

$$SNIR^{(i)} = \frac{E[|S_k|^2]}{E[|\eta_k^{(i)}|^2]}. \quad (3)$$

Moreover, at the output of the FDE the signal resembles the output of an ideal AWGN channel. Therefore (3) can be regarded as the signal-to-noise ratio  $SNR$  of an *equivalent* AWGN channel and thus, for QPSK signaling with Gray mapping, the BER at the output of the FDE can be approximated by

$$P_b^{(i)} = \mathcal{Q}\left(\sqrt{SNR^{(i)}}\right), \quad (4)$$

where  $SNR^{(i)} = SNIR^{(i)}$  and  $\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy$ .

The correlation factor  $\rho^{(i-1)}$  in (2) relates with the BER (4) through the expression [9]

$$\rho^{(i-1)} = 1 - 2P_b^{(i-1)}. \quad (5)$$

For a given  $\alpha$ , assuming perfect knowledge of the CFR,  $H_k$ , equations (2)-(5) provide the means for predicting the BER performance of the FDE receiver for successive iteration orders [11]: 1) set  $\rho^{(0)} = 0$  and let  $i = 1$ ; 2) compute  $SNIR^{(i)}$  replacing  $E[|\eta_k|^2]$  in (3) by (2), note that (2) is computed using the CFR  $H_k$ ,  $\rho^{(i-1)}$  and the channel SNR  $\alpha$ ; 3) resorting to the Gaussian approximation to make  $SNR^{(i)} = SNIR^{(i)}$ , compute  $P_b^{(i)}$  using (4); 4) compute  $\rho^{(i)}$  using (5); 5) increment  $i$  and go to Step 2.

### 2.2. Iterative Estimator of the Carrier Frequency Offset

Let us now assume that there is a residual carrier frequency offset between the transmitter and the receiver local oscillators. In this case the transmitted time-domain data symbols  $\{s_n; n = 0, 1, \dots, N-1\}$  undergo a phase rotation,

$s'_n = s_n \exp(j2\pi\Delta f^{(1)}nT/N)$ . For the sake of simplicity, it is assumed that the residual frequency offset results exclusively from the transmitter; we also assume that the phase rotation is zero for  $n = 0$ . Since the  $n$ th data symbol is rotated by  $n\theta^{(i)}$ , with  $\theta = 2\pi\Delta f^{(i)}T/N$ , the corresponding BER for a QPSK constellation and an ideal Gaussian channel is  $P_{b,n}^{(i)} = \frac{1}{2} \left( p_{1,n}^{(i)} + p_{2,n}^{(i)} \right)$ , where

$$p_{1,n}^{(i)} = \mathcal{Q} \left( \left( \cos(n\theta^{(i)}) - \sin(n\theta^{(i)}) \right) \sqrt{SNR} \right), \quad (6)$$

$$p_{2,n}^{(i)} = \mathcal{Q} \left( \left( \cos(n\theta^{(i)}) + \sin(n\theta^{(i)}) \right) \sqrt{SNR} \right), \quad (7)$$

and the overall BER is the average of  $P_{b,n}^{(i)}$  over  $n$

$$P_b^{(i)} = \mathbb{E}[P_{b,n}^{(i)}] = \frac{1}{2N} \sum_{n=0}^{N-1} (p_{1,n}^{(i)} + p_{2,n}^{(i)}). \quad (8)$$

In [7] a model of the DD CFO estimator as a function of the true CFO and of the SNR, was presented. The estimates of the CFO were described by the sum of the CFO  $\Delta f$  and a bias component,  $\Delta f_{\text{bias}}$ , i.e.,  $\widehat{\Delta f} = \Delta f + \Delta f_{\text{bias}}$ , where the bias is approximately

$$\Delta f_{\text{bias}} \approx \frac{N}{2\pi MT} \arg\{\Upsilon\}, \quad (9)$$

with  $\Upsilon = (N - M)|s_n|^2 + \sum_{n=0}^{N-M-1} \bar{\zeta}_n$ . The contribution of the data detection errors to (9) is  $\bar{\zeta}_n = \mathbb{E}[\zeta_n] = \bar{\varepsilon}_{n+M}^* s_n + \bar{\varepsilon}_n s_{n+M}^* + \bar{\varepsilon}_{n+M}^* \bar{\varepsilon}_n$ , where, assuming  $D$  to be the minimum Euclidean distance, the mean value of the data decision error is  $\bar{\varepsilon}_n = \mathbb{E}[\varepsilon_n] = -Dp_{1,n} - jDp_{2,n}$ , with  $p_{1,n}$  and  $p_{2,n}$  given by (6) and (7), respectively.

Assuming an known initial CFO  $\Delta f^{(1)}$ , an algorithm for the prediction of the BER performance of a iterative estimator of the carrier frequency offset can be summarized by the following steps [12]: 1) set  $\Delta f_{\text{bias}}^{(0)} = -\Delta f^{(1)}$  and  $i = 1$ ; 2) compute  $p_{1,n}^{(i)}$  and  $p_{2,n}^{(i)}$  using (6) and (7), respectively, with  $\theta^{(i)} = -2\pi\Delta f_{\text{bias}}^{(i-1)}T/N$ ; 3) compute  $P_b^{(i)}$  using (8) and  $\Delta f_{\text{bias}}^{(i)}$  using (9); 4) increment  $i$  and go to Step 2.

### 2.3. Joint Equalization with Carrier Frequency Synchronization

The following algorithm predicts the performance of a receiver with joint equalization and carrier frequency synchronization as a function of the number of iterations used and the initial CFO,  $\Delta f^{(1)}$ : 1) set  $\rho^{(0)} = 0$ ,  $\Delta f_{\text{bias}}^{(0)} = -\Delta f^{(1)}$  and let  $i = 1$ ; 2) compute  $SNIR^{(i)}$  replacing  $E[|\eta_k|^2]$  in (3) by (2), note that (2) is computed using the CFR  $H_k$ , the correlation  $\rho^{(i-1)}$ , and the channel SNR  $\alpha$ ; 3) compute  $p_{1,n}^{(i)}$  and  $p_{2,n}^{(i)}$  using (6) and (7), respectively, with  $\theta^{(i)} = -2\pi\Delta f_{\text{bias}}^{(i-1)}T/N$  and  $SNR = SNIR^{(i)}$ ; 4) compute  $P_b^{(i)}$  using (8) and  $\Delta f_{\text{bias}}^{(i)}$  using (9); 5) compute  $\rho^{(i)}$  using (5); 6) increment  $i$  and go to Step 2.

## 3. PERFORMANCE RESULTS

In this section we present a set of experimental results where we consider SC-FDE modulations with blocks of  $N = 1024$  "useful" modulation symbols (corresponding to a duration of  $4\mu s$ ), plus an appropriate cyclic prefix. The modulation symbols belong to a QPSK constellation and are selected from the transmitted data according to a Gray mapping rule. We consider linear power amplification and perfect channel estimation. The propagation channel is characterized by the power delay profile type C for HIPERLAN/2 (High Performance Local Area Network) [13], with uncorrelated Rayleigh fading on different paths (similar results could be obtained for other severely time dispersive channel models with rich multipath propagation).

Let us define function  $G$  as a mapping of the  $SNIR$  at the output of the equalizer as a function of the correlation factor,  $\rho^{(i-1)}$ ,

$$SNIR^{(i)} = G(\alpha, \rho^{(i-1)}). \quad (10)$$

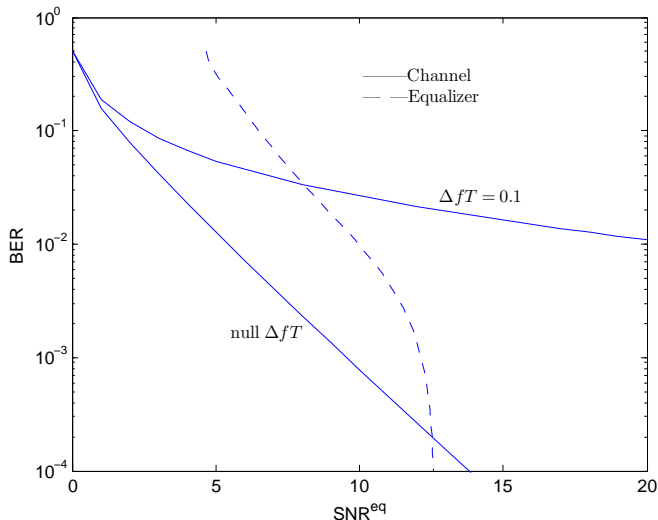
In (10)  $SNIR^{(i)}$  is given by (3) with  $E[|\eta_k|^2]$  replaced by (2). Yet, if we define  $J$  as being a function with inverse,  $J^{-1}$ , given by (5), results that (10) can be rewritten as

$$SNIR^{(i)} = G(\alpha, J^{-1}(P_b^{(i-1)})). \quad (11)$$

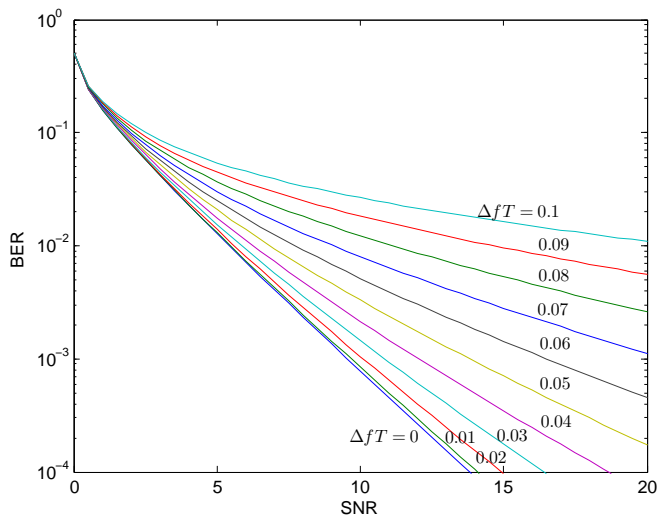
Thus (11) is the mapping of the  $SNIR$  at the output of the equalizer as a function of the BER at the input of that same equalizer. We will call (11) the "equalizer curve". In Fig. 1 we can see the "equalizer curve" for  $E_b/N_0 = 8\text{dB}$ .

On the other side we have the *equivalent* "channel curve" which is a function of the SNR and the CFO given by the BER (8). If a transmission with no CFO is considered, (8) is transformed into (4). These functions map the BER at the output of the FDE from the  $SNIR$  of the *equivalent* channel of the output of the FDE. In Fig. 1, adding to the "equalizer curve", we plotted the "channel curve" for an ideal AWGN channel with no CFO using (4) and the "channel curve" with a CFO  $\Delta fT = 0.1$  using (8). Fig. 2 "channel curves" for other values of the CFO.

In Fig. 3 we use a graphical tool for the prediction of the performance of a receiver with joint equalization with carrier frequency offset. On the  $x$ -axis we have the  $SNIR$  of the AWGN channel *equivalent* to the output of the FDE. On the  $y$ -axis we have the BER at the output of the receiver. Each iteration corresponds to a trajectory between the "channel curve" and the "equalizer curve" and back to the "channel curve". The horizontal straight lines are associated to the equalization process representing improvements in the  $SNIR$  at the output of the FDE, whereas the straight vertical lines represent the trajectory between the BER of the previous iteration and that of the present iteration. So, an iteration consists of a straight horizontal line followed by a straight vertical line. The points where the straight lines meet the "channel curve" are the predicted values for the BER corresponding to that iteration.



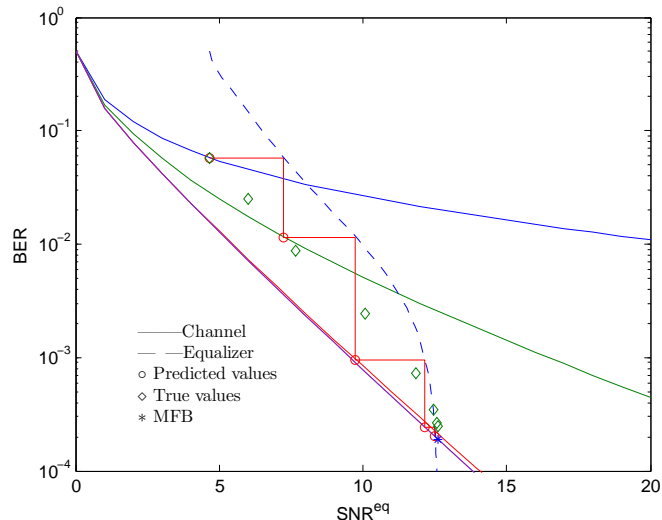
**Fig. 1.** “equalizer curve” for  $E_b/\mathcal{N}_0 = 8\text{dB}$  and two “channel curves”, one for an ideal AWGN channel (null  $\Delta f$ ) and another for an AWGN channel with CFO ( $\Delta fT = 0.1$ ).



**Fig. 2.** “Channel curves” for  $\Delta fT = 0, 0.01, \dots, 0.1$ .

Note that, graphically, the “channel curve” which has the largest CFO corresponds to the uppermost curve. In the particular case of Fig. 3,  $\Delta fT^{(1)} = 0.1$ . Due to the synchronization process for each iteration we have that the CFO changes into a lower value and since the “channel curve” is a function of the CFO it results that for each iteration we have a new “channel curve” (see Fig. 2). Eventually, as the number of iterations increases we will have a “channel curve” close to that of an ideal AWGN channel.

Depicted in Fig. 4 is the evolution of the BER for successive iterations of the IB-DFE receiver with joint carrier synchronization. In the figure we compare the simulation results,

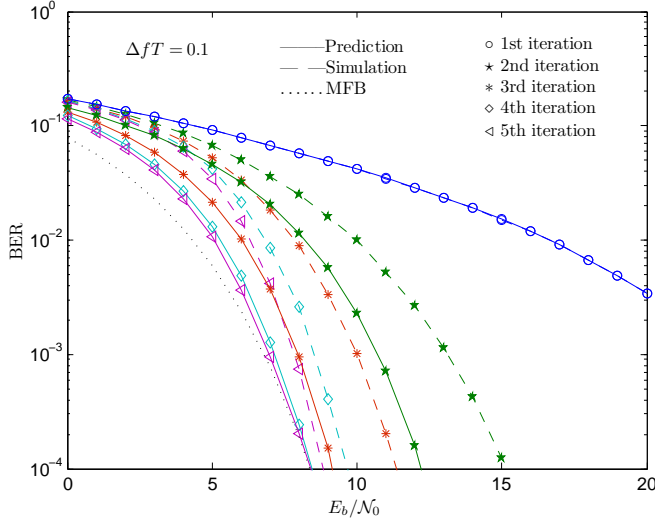


**Fig. 3.** IB-DFE performance with joint carrier synchronization for  $E_b/\mathcal{N}_0 = 8\text{dB}$  and initial CFO  $\Delta f^{(1)}T = 0.1$ . Along each “channel curve” “o” identifies the theoretically predicted decreasing BER achieved with  $i = 1, 2, \dots$  iterations. The associated experimentally obtained values for the bit error rate are identified by “ $\diamond$ ”. Additionally, the MFB is identified by “\*” (see Appendix).

obtained through Monte Carlo trials, with those obtained using the algorithm presented in Sec. 2.3. We can see that the predicted values for the BER have some differences in relation with the simulated ones, with differences around 2.5dB for the 2nd iteration. Therefore, the prediction is considered optimistic. These differences in the BER curves can be justified due to the differences found between the true value of the bias of the CFO estimates and the analytical value found using (9). Additionally, as the iteration order increases, the difference between the theoretical and the true value of the bias of the CFO estimates also increases, this being due to the propagation of the difference between the true bias and the analytical one. On the other side, as the iteration order increases, the estimates of the CFO became less biased.

#### 4. CONCLUSION

We considered an IB-DFE with joint detection and carrier frequency synchronization for SC-FDE block transmission in the presence of residual frequency errors. We described the output of the frequency-domain equalizer by a Gaussian distribution and defined an equivalent AWGN channel which allowed us to propose a model for the carrier frequency synchronization procedure. By using the Gaussian approximation for the equalizer and by modeling the bias as a function of both the CFO and the SNR we built a model for the IB-DFE with joint carrier frequency synchronization. This model can be used for predicting the behavior of the true system and was



**Fig. 4.** BER performance for the IB-DFE receiver with  $\Delta f^{(1)}T = 0.1$ . Additionally, we also included the MFB as a benchmark (see Appendix).

extended for the use as a graphical tool.

## 5. APPENDIX

Assuming QPSK signalling and Gray mapping, the Matched Filter Bound (MFB) is given by [14]

$$P_{b,\text{MFB}} = \mathcal{Q} \left( \sqrt{\frac{2E_b}{\mathcal{N}_0} \frac{1}{N} \sum_{k=0}^{N-1} |H_k|^2} \right), \quad (12)$$

which, for frequency selective channels, is the best performance that we can have if no ISI exists.

## 6. REFERENCES

- [1] L. Cimini Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. 33, pp. 400–411, July 1985.
- [2] H. Sari, G. Karam, and I. Jeanclaude, "An analysis of orthogonal frequency-division multiplexing for mobile radio applications," in *Proc. IEEE Vehicular Technology Conference, (VTC'94)*, Stockholm, June 1994, pp. 1635–1639.
- [3] N. Benvenuto, R. Dinis, D. Falconer, and S. Tomasin, "Single carrier modulation with nonlinear frequency domain equalization: An idea whose time has come—again," *Proceedings of the IEEE*, vol. 98, no. 1, pp. 69–96, Jan. 2010.
- [4] N. Benvenuto and S. Tomasin, "Block iterative DFE for single carrier modulation," *IEEE Electronic Letters*, vol. 39, no. 19, pp. 1144–1145, Sep. 2002.
- [5] T. Araújo and R. Dinis, "Iterative equalization and carrier synchronization for single-carrier transmission over severe time-dispersive channels," in *Proc. IEEE Global Communications Conference (GLOBECOM'04)*, Dallas, Texas, USA, 29 Nov. - 3 Dec. 2004, vol. 5, pp. 3103–3107.
- [6] C. Georghiades and E. Serpedin, *The handbook of communications*, chapter 19—Synchronization, CRC Press, 2002.
- [7] P. Pedrosa, R. Dinis, and F. Nunes, "Performance evaluation of decision-directed carrier synchronization for SC-FDE schemes," in *Proc. IEEE International Symposium on Wireless Communication Systems (ISWCS'08)*, Oct. 2008, pp. 113–117.
- [8] M. Tüchler, R. Koetter, and A. Singer, "Turbo equalization: principles and new results," *IEEE Trans. Commun.*, vol. 50, pp. 754–767, May 2002.
- [9] R. Dinis, P. Silva, and A. Gusmão, "IB-DFE receivers with space diversity for CP-assisted DS-CDMA and MC-CDMA systems," *Eur. Trans. Telecomm.*, vol. 18, no. 7, pp. 791–802, 2007.
- [10] M. Sabbaghian and D. Falconer, "An analytical approach for finite block length performance analysis of turbo frequency-domain equalization," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 3, pp. 1292–1301, Mar. 2009.
- [11] S. Beheshti and G.W. Wornell, "Iterative interference cancellation and decoding for spread-signature cdma systems," in *Proc. IEEE 47th Vehicular Technology Conference (VTC'97)*, May 1997, pp. 26–30.
- [12] P. Pedrosa, R. Dinis, and F. Nunes, "A model of an iterative estimator of the carrier frequency offset," in *Proc. IASTED Signal Processing, Pattern Recognition and Applications (SPPRA'10)*, Innsbruck, Austria, Feb. 2010.
- [13] ETSI, "Channel models for HIPERLAN/2 in different indoor scenarios," *ETSI EP BRAN 3ER1085B*, pp. 1–8, Mar. 1998.
- [14] R. Dinis, A. Gusmão, and N. Esteves, "On broadband block transmission over strongly frequency-selective fading channels," in *IEEE Wireless'03*, Calgary, Canada, July 2003.