Bayesian Approach for the Estimation of Phase Noise in SC-FDE schemes

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***Abstract*—A solution for the problem of estimating the PN (Phase Noise) from the observation of the channel output in burst communications is to establish a state-model for the PN and determine the *a posteriori* probability density function (pdf) of the state conditioned on all measurement data, thus providing the means to compute an optimal estimate with respect to any criterion, *e.g.*, Minimum Mean-Squared Error (MMSE). However, except in the Gaussian case, it is extremely difficult to determine and propagate this density function. As a result, and since the observation model of the PN is non-linear, the *a posteriori* pdf becomes non-Gaussian, and a sub-optimal solution for the problem must be found. In this paper we approximate the non-Gaussian pdf by a weighted sum of Gaussians. Additionally, we compare the performance results obtained when modeling the density function by a weighted sum of Gaussians with those obtained with a single Gaussian using a SC-FDE (Single Carrier-Frequency Domain Equalizer) scheme.**

***Index Terms*—Bayesian filtering; Kalman filtering; Gaussian sum filter; phase noise; carrier frequency offset; single carrier-frequency domain equalization;**

I. INTRODUCTION

Due to an increased demand for wireless services, future systems are required to support high quality of service at high data rates. Single Carrier (SC) with Cyclic Prefix (CP) [1] is seen as an alternative to Orthogonal Frequency Domain Multiplexing (OFDM) [2] since it can efficiently cope with the time-dispersion effects associated to the multipath propagation using Frequency Domain Equalization (FDE). Moreover, SC-FDE has a simpler transmitter structure which makes it the natural choice for the uplink, *i.e.*, from the Mobile Terminal (MT) to the Base Station (BS), and experiences a lower Peak-to-Average Power Ratio (PAPR), enabling the use of non-linear power amplifiers [3], [4].

A study on the impact of phase noise on OFDM and SC-FDE as been conducted in [5]. Zamorano *et al.* compared the impact of the Phase Noise (PN) on both schemes. They concluded that PN causes the same Common Phase Error (CPE) on both schemes, as well as leading to Inter-Carrier Interference (ICI) in OFDM and Inter-Symbol Interference (ISI) in SC-FDE. It was also shown that CPE is the dominant impairment if the PN cut-off frequency is smaller than the subcarrier spacing. In [6], Sabbagian and Falconer proposed a synchronization method to compensate for the effect of Carrier Frequency Offset (CFO) and PN for a Turbo Frequency Domain Equalizer (TFDE). It is a joint turbo equalization and synchronization method in which the feedback information, generated iteratively by the decoder, is used for synchronization. The synchronization method is based on Maximum a Posterior (MAP) criterion which can be considered as a decision directed phase lock loop. In [7], Stark and Raphaeli investigated the combination of Decision Feedback Equalizers (DFE) and Digital Phase Lock Loop (DPLL) for the symbol detection in channels impaired by ISI and PN. The analysis of the estimation schemes was conducted in terms of residual phase jitter, optimal loop gain, and complexity of implementation. In [8], Petrovic *et al.* introduced an algorithm for pilot based CPE estimation using an Extended Kalman Filter (EKF) for OFDM.

The solution for the problem of estimating the PN (Phase Noise) from the observation of the channel output in bursts communications is to define a state model from the PN and determine the *a posteriori* probability density function (pdf) of the state conditioned on all measurement data, thus providing the means to compute an optimal estimate with respect to any criterion, *e.g.*, Minimum Mean-Squared Error (MMSE). However, except in the Gaussian case, it is extremely difficult to determine this density function. As a result, and since the observation of the PN is non-linear the resulting *a posteriori* pdf is a non-Gaussian function and a sub-optimal solution for the problem must be found. In this paper we approximate the *a posteriori* non-Gaussian pdf by a weighted sum of Gaussians. Additionally, we compare the performance results obtained when modeling the pdf by a weighted sum of Gaussians with those obtained using a single Gaussian. We assume perfect channel estimation and symbol synchronization. Moreover, we embed our PN estimator in a iterative FDE equalizer to produce a joint equalizer and synchronization receiver.

The paper is organized in the following manner: Sec. II establishes the models of the channel output and of the PN and also presents the Bayesian equations. Sec. III defines the Gaussian sum filter, with subsections dedicated to the Gaussian sum approximation, the Gaussian fitting of the likelihood, and the solution of the Bayesian equations. Sec. IV concerns

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implementation aspects of the algorithm. Sec. V is dedicated to the presentation of the Iterative FDE receiver, while Sec. VI presents the performance results and Sec. VII concludes this paper.

II. System Model

Consider the transmission of the Binary Phase Shift-Keying (BPSK)[1] symbol $\{s_n = e^{j\phi_n}; n = 0, \ldots, N-1\}$, where $\phi_n \in \{0, \pi\}$, over an Additive White Gaussian (AWGN) channel in the presence of PN,

$$y_n = e^{j(\phi_n + \theta_n)} + v_n; \qquad n = 0, \ldots, N-1. \tag{1}$$

In (1), $\{v_n; n = 0, \ldots, N-1\}$ is a sequence of complex zero-mean white Gaussian noise with variance of the real or imaginary components $\sigma_v^2 = \mathcal{N}_0/T_b$, where $\mathcal{N}_0/2$ is the AWGN power spectral density and T_b is the bit duration, and $\{\theta_n; n = 0, \ldots, N-1\}$ is a Wiener process described by the linear stochastic difference equation

$$\theta_n = \theta_{n-1} + w_n; \qquad n = 0, \ldots, N-1, \tag{2}$$

modeling the PN. In (2), $\{w_n; n = 0, \ldots, N-1\}$ is a sequence of real zero-mean white Gaussian noise with variance σ_w^2.

The noise sequences $\{v_n; n = 0, \ldots, N-1\}$ and $\{w_n; n = 0, \ldots, N-1\}$ are assumed to be mutually independent and independent of the initial state θ_{-1}. The pdfs of $\{v_n; n = 0, \ldots, N-1\}$ and $\{w_n; n = 0, \ldots, N-1\}$ are denoted by p_v and p_w, respectively.

A. Bayesian Filtering

The conditional density function, $p(\theta_n|Y_n = y_0, y_1, \ldots, y_n)$, contains all required statistics to derive the MMSE estimate of the PN $\hat{\theta} = \mathrm{E}[\theta_n|Y_n]$. Although, in general, $p(\theta_n|Y_n)$ is not easy to obtain, it can be determined recursively by the following equations [9]:

Prediction step

$$p(\theta_n|Y_{n-1}) = \int p(\theta_n|\theta_{n-1}) p(\theta_{n-1}|Y_{n-1}) d\theta_{n-1}; \tag{3}$$

Filtering step

$$p(\theta_n|Y_n) = \frac{p(y_n|\theta_n) p(\theta_n|Y_{n-1})}{\int p(y_n|\theta_n) p(\theta_n|Y_{n-1}) d\theta_n}. \tag{4}$$

Since the transition pdf is

$$p(\theta_n|\theta_{n-1}) = p_w(\theta_n - \theta_{n-1}) \tag{5}$$

the operation in (3) is a convolution of pdfs $p_w(\theta_{n-1})$ and $p(\theta_{n-1}|Y_{n-1})$.

The initial conditional density function $p(\theta_{-1}|Y_{-1})$ is given by

$$p(\theta_{-1}|Y_{-1}) = p(\theta_{-1}) \tag{6}$$

where we assume $p(\theta_{-1})$ to be Gaussian. The integral in (4) is just a normalizing factor and, in certain cases, may not be evaluated.

[1] Our derivations are presented using the BPSK constellation due to its simplicity. This is done with no loss of generalization to other M-PSK constellations, for which the extension of the proposed solution is straightforward.

The function $p(y_n|\theta_n)$ is non-Gaussian, rendering the integration indicated in (4) impossible to accomplish in closed form. Since the computation of the exact solution is too demanding, an adequate approximation of $p(\theta_n|Y_n)$ must be found. The choice of what approximation to use to propagate the *a posterior* pdf depends on the problem in hand, existing no general solution.

There are two main approaches for the non-linear problem: one consists of linearizing the observation and the dynamics equations about the best estimate of the state vector by retaining the linear term of the Taylor series expansion; the other, consists of finding a function that models conveniently the pdf $p(\theta_n|Y_n)$, but with parameters easy enough to compute on each iteration. The former corresponds to the approach applied in the Extended Kalman Filter (EKF). The latter is the approach implemented in this work, since we approximate the non-Gaussian function $p(y_n|\theta_n)$ by a weighted sum of Gaussians such that (3)-(4) can be computed analytically by resorting to a Gaussian Sum Filter (GSF). This approach was considered, for instance, in [10], [11], [12].

III. Gaussian Sum Filter

A. Gaussian Sum Approximation

Following the Gaussian sum approximation, any pdf p can be approximated as closely as desired by a density function p_A of the form [10]

$$p_A(x) = \sum_{i=1}^{m} \alpha^{(i)} \mathcal{N}(x - \mu^{(i)}, \sigma^{2(i)}), \tag{7}$$

for some sufficiently large integer m, where

$$\mathcal{N}(x - \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right) \tag{8}$$

is the normal density function with mean μ and variance σ^2. Weights $\alpha^{(i)}$ are non-negative and sum to one. Representation (7) can be used to approximate the non-Gaussian multi-modal densities in a problem.

B. Gaussian Fitting of the Likelihood

From (1) and the statistics of v_n we have [13]

$$\begin{aligned} p(y_n|\theta_n, \phi_n) &= p_v\left(y_n - e^{j(\phi_n+\theta_n)}\right) = \\ &= \frac{1}{2\pi\sigma_v^2} e^{-\frac{\left|y_n - e^{j(\theta_n+\phi_n)}\right|^2}{2\sigma_v^2}}, \end{aligned} \tag{9}$$

which is a non-Gaussian function of θ_n.

Expanding the squared modulus of (9) results in

$$\begin{aligned} &\left|y_n - e^{j(\theta_n+\phi_n)}\right|^2 = \\ &= 1 + |y_n|^2 - 2|y_n|\cos(\theta_n + \phi_n - \eta_n), \end{aligned} \tag{10}$$

where

$$\eta_n = \arg\{y_n\}, \tag{11}$$

with $\arg\{\cdot\}$ being the complex argument.

Considering (10) we can write (9) as

$$p(y_n|\theta_n,\phi_n)=\frac{1}{2\pi\sigma_v^2}e^{-\gamma_n}e^{\beta_n\cos(\theta_n+\phi_n-\eta_n)}, \quad (12)$$

where

$$\gamma_n=\frac{1+|y_n|^2}{2\sigma_v^2} \quad (13)$$

and

$$\beta_n=\frac{|y_n|}{2\sigma_v^2}. \quad (14)$$

Marginalizing $p(y_n|\theta_n,\phi_n)$ with respect to ϕ_n we are able to write $p(y_n|\theta_n)$ in (4) as a sum of exponentials,

$$\begin{aligned}&p(y_n|\theta_n)=\\&=\frac{1}{2}\int_{-\infty}^{+\infty}p(y_n|\theta_n,\phi_n)[\delta(\phi_n)+\delta(\phi_n-\pi)]d\phi_n=\\&=\frac{1}{4\pi\sigma_v^2}e^{-\gamma_n}\left(e^{\beta_n\cos(\eta_n-\theta_n)}+e^{-\beta_n\cos(\eta_n-\theta_n)}\right),\end{aligned} \quad (15)$$

where the transmitted symbols are assumed to be equiprobable, *i.e.*, $\mathrm{Prob}(\phi=0)=\mathrm{Prob}(\phi=\pi)=1/2$.

From the observation of (15) it is clear that the observation mechanism $p(y_n|\theta_n)$ is π-periodic. Resorting to the Gaussian sum approximation, (7), we can write (15) as

$$p(y_n|\theta_n)\approx\sum_{j=1}^{p}\kappa\mathcal{N}\left(\theta_n-(\eta_n'+a^{(j)}),R_n\right) \quad (16)$$

where $a^{(j)}\in\{\ldots,-2\pi,-\pi,0,\pi,2\pi,\ldots\}$, $\kappa=1/p$, and

$$\eta_n'=\eta_n-g(\hat{\theta}_{n-1})\pi. \quad (17)$$

In (17), we get rid of the digital modulation by resorting to the decision-directed hard-decision criterion

$$g(\hat{\theta}_{n-1})=\begin{cases}1, & |\eta_n-\hat{\theta}_{n-1}|>\frac{\pi}{2}\\ 0, & |\eta_n-\hat{\theta}_{n-1}|\le\frac{\pi}{2}\end{cases}, \quad n=1,\ldots,N. \quad (18)$$

With this decision criterion we implement a very simple bit detection scheme. The rationale behind the choice of $g(\cdot)$ consists of assuming that $\eta_n-\theta_n\approx\eta_n-\hat{\theta}_{n-1}\approx\phi_n$, according to (11). This assumption is supported by our experimental results.

C. Solution of the Bayesian Equations

The idea of the GSF is that both the prediction density function (3) and the filtering density function (4) are weighted sums of Gaussians, *i.e.*,

$$p(\theta_n|Y_{n-1})=\sum_{i=1}^{m}\alpha_n^{(i)}\mathcal{N}\left(\theta_n-\hat{\theta}_{n|n-1}^{(i)},P_{n|n-1}\right). \quad (19)$$

Replacing (16) and (19) in (4) we obtain the filtering density

$$p(\theta_n|Y_n)=\sum_{i=1}^{m}\sum_{j=1}^{p}\gamma_n^{(i,j)}\mathcal{N}\left(\theta_n-\hat{\theta}_{n|n}^{(i,j)},P_{n|n}\right). \quad (20)$$

Taking into account that

$$\mathcal{N}(x-a,\sigma_a^2)\cdot\mathcal{N}(x-b,\sigma_b^2)=\mathcal{N}(x-\mu,\sigma^2)\cdot\mathcal{N}(a-b,\sigma_a^2+\sigma_b^2) \quad (21)$$

with

$$\mu=\frac{a\sigma_b^2+b\sigma_a^2}{\sigma_a^2+\sigma_b^2} \quad (22)$$

$$\sigma^2=\frac{\sigma_a^2\sigma_b^2}{\sigma_a^2+\sigma_b^2} \quad (23)$$

and replacing (16) and (19) in (4) we obtain the filtering density

$$p(\theta_n|Y_n)=\sum_{i=1}^{m}\sum_{j=1}^{p}\gamma^{(i,j)}\mathcal{N}(\theta_n-\hat{\theta}_{n|n-1}^{(i)},P_{n|n}) \quad (24)$$

where

$$P_{n|n}=(R_n^{-1}+P_{n|n-1}^{-1})^{-1}; \quad (25)$$

$$\hat{\theta}_{n|n}^{(i,j)}=\frac{1}{R_n+P_{n|n-1}}[(\eta_n'-a^{(j)})P_{n|n-1}+\hat{\theta}_{n|n-1}^{(i)}R_n]; \quad (26)$$

$$\gamma_n^{(i,j)}=\alpha^{(i)}\mathcal{N}(\eta_n'-a^{(j)}-\hat{\theta}_{n|n-1}^{(i)},R_n+P_{n|n-1}). \quad (27)$$

If the filtering density $p(\theta_{n-1}|Y_{n-1})$ is a Gaussian sum then the prediction density $p(\theta_n|Y_{n-1})$ resulting from the convolution of Gaussian functions, is also a Gaussian sum with means

$$\hat{\theta}_{n|n-1}^{(l)}=\hat{\theta}_{n-1|n-1}^{(l)} \quad (28)$$

and variance

$$P_{n|n-1}=P_{n-1|n-1}+Q_{n-1} \quad (29)$$

where $Q_n=\sigma_w^2$.

The Gaussian sum approximation method is used to approximate $p(y_n|\theta_n)$ in order that the convolution and products in (3)-(4) are accomplished in a simple manner, also allowing the MMSE estimate to be computed analytically and efficiently as

$$\begin{aligned}\hat{\theta}_n&=\int_{-\infty}^{+\infty}\theta_n p(\theta_n|Y_n)d\theta_n=\\&=\int_{-\infty}^{+\infty}\theta_n\sum_{i=1}^{m}\gamma_n^{(i)}\mathcal{N}(\theta_n-\hat{\theta}_{n|n}^{(i)},P_{n|n})d\theta_n=\\&=\frac{\sum_{i=1}^{m}\gamma_n^{(i)}\hat{\theta}_{n|n}^{(i)}}{\sum_{i=1}^{m}\gamma_n^{(i)}}\end{aligned} \quad (30)$$

IV. Implementation Aspects

A. Variance of the Observations

The observation factor (9) is not a density function. So, before we can use approximation (16) we must proceed to the normalization of (9), *i.e.*,

$$q(y_n|\theta_n)=\frac{p(y_n|\theta_n)}{\int p(y_n|\theta_n)d\theta_n}. \quad (31)$$

According to (15) we may write (31) as

$$q(y_n|\theta_n)=\frac{e^{\beta_n\cos(\eta_n-\theta_n)}+e^{\beta_n\cos(\eta_n+\pi-\theta_n)}}{\int e^{\beta_n\cos(\eta_n-\theta_n)}+e^{\beta_n\cos(\eta_n+\pi-\theta_n)}d\theta_n}. \quad (32)$$

which is a function of β_n. Consequently, the variance of (16) is also a function of β_n. Since $\beta_n=\frac{2|y_n|}{\sigma_v^2}$, results that to a large variance of the observation noise σ_v^2 corresponds a small value of β_n.

In order to lighten our algorithm, we compute off-line the variance of the observation, given by

$$\sigma_y^2 = \int_{\theta_n - \frac{\pi}{2}}^{\theta_n + \frac{\pi}{2}} (\eta_n - \theta_n)^2 q(y_n|\theta_n) d\theta_n, \qquad (33)$$

for several values of β_n and then interpolate the variance of the observation for any given β_n.

B. Growing Memory Problem

The operation of multiplication in (4) entails the growth of the number of Gaussian modes, as shown in (25)-(27). In order to keep the GSF efficient one must keep the number of modes as small as possible without losing significant information. A number of techniques have been developed to restrain the number of terms in a Gaussian mixture to a finite number or below a maximum number of terms (see [14], [15], [11]).

For its simplicity, our preferred component reduction procedure is to preserve m elemental distributions with the largest weights and discard the remaining (pruning procedure). Thus, if we initialize the algorithm with $m = 3$ Gaussian modes and assuming $p = 3$, the filtering density will have always $mp = 9$ Gaussian modes which are reduced again to $m = 3$ modes by the pruning procedure.

V. Iterative FDE Receiver

Based on a previously developed receiver structure, for the joint equalization and carrier synchronization [16], we present a joint equalization and PN compensation for SC-FDE schemes. The estimator presented in section III can be used in this structure (see Fig. 1) since we assume the time-domain the equalizer output, $\{\tilde{s}_n^{(i)};\ n = 0, \ldots, N-1\}\,(i = 1, 2, \ldots)$, to have a Gaussian distribution despite the presence of residual ISI [17].

Let us assume that our transmission is impaired with PN, (2). In this case, the received time-domain block is $\{y'_n; n = 0, 1, \ldots, N-1\}$ and the corresponding frequency-domain block is $\{Y'_k; k = 0, 1, \ldots, N-1\}$, with

$$Y'_k = S'_k H_k + N_k \qquad (34)$$

where the block of frequency-domain symbols $\{S'_k; k = 0, 1, \ldots, N-1\}$ is the DFT of the effectively transmitted block of time-domain data symbols, $\{s'_n; n = 0, 1, \ldots, N-1\}$, with

$$s'_n = s_n \exp(j\theta_n), \qquad (35)$$

With $\{s_n; n = 0, 1, \ldots, N-1\}$ denoting the nth data symbol to be transmitted, selected from a given constellation (*e.g.*, a QAM or a M-PSK constellation). And where $\{H_k; k = 0, 1, \ldots, N-1\}$ and $\{N_k; k = 0, 1, \ldots, N-1\}$ denote the channel transfer function and the channel noise, respectively, for the kth subchannel.

For the sake of simplicity, it is assumed that the PN and the CFO results exclusively from the transmitter (*i.e.* the Mobile Terminal (MT), since we are considering the uplink); we also assume that the phase rotation is 0 for $n = 0$.

For a given iteration i, the frequency-domain samples at the output of the FDE are given by

$$\tilde{S}_k'^{(i)} = F_k^{(i)} Y'_k - B_k^{(i)} \hat{S}_k'^{(i-1)} \qquad (36)$$

where $\{F_k^{(i)}; k = 0, 1, \ldots, N-1\}$ are the feedforward coefficients and $\{B_k^{(i)}; k = 0, 1, \ldots, N-1\}$ are the feedback coefficients. $\{\hat{S}_k'^{(i-1)}; k = 0, 1, \ldots, N-1\}$ denotes the DFT of the block of time-domain data estimates associated to the previous iteration, $\{\hat{s}_n'^{(i-1)}; n = 0, 1, \ldots, N-1\}$, *i.e.*, the phase rotated version of the hard-decisions associated to the time-domain block at the output of the FDE, $\{\tilde{s}_n^{(i)}; n = 0, 1, \ldots, N-1\}$ = IDFT $\{\tilde{S}_k^{(i)}; k = 0, 1, \ldots, N-1\}$.

It can be shown that the optimum feedback coefficients are

$$B_k^{(i)} = F_k^{(i)} H_k^{(l)} - 1 \qquad (37)$$

and the feedforward coefficients given by

$$F_k^{(i)} = \frac{\breve{F}_k^{(i)}}{\gamma^{(i)}}, \qquad (38)$$

with

$$\breve{F}_k^{(i)} = \frac{H_k^*}{\alpha + (1 - (\rho^{(i-1)})^2)|H_k|^2}, \qquad (39)$$

where $\alpha = E[|N_k|^2]/E[|S_k|^2]$,

$$\gamma^{(i)} = \frac{1}{N} \sum_{k=0}^{N-1} \breve{F}_k^{(i)} H_k \qquad (40)$$

and the correlation factor $\rho^{(i-1)}$ is defined as

$$\rho^{(i-1)} = \frac{E[\hat{s}_n^{(i-1)} s_n^*]}{E[|s_n|^2]}. \qquad (41)$$

VI. Performance Results

In this section we display some experimental results. The "useful" block size is $N = 512$ symbols, corresponding to a duration of 4μs, plus an appropriate cyclic prefix. The channel is a highly frequency-selective channel. More specifically, the propagation channel is characterized by the power delay profile type C for HIPERLAN/2 (HIgh PERformance Local Area Networks) [18], with uncorrelated Rayleigh fading on different paths (similar results could be obtained for other severely time-dispersive channel models with rich multipath propagation).

In Fig. 2 we plot the BER versus the standard deviation of the noise (Wiener process), σ_w, for $E_b/\mathcal{N}_0 = 10$ and BPSK constellation when using joint equalization and carrier synchronization. Additionally, we include the curves for the case for which no compensation is performed. It is evident from the figure that the phase estimation and compensation process produces outstanding results, achieving a significant reduction on the BER for the lower values of σ_w. Additionally, as expected, our filter performs better with $p = 3$ Gaussian modes than with $p = 1$, particularly for values of $\sigma_w > 0.1$. For values of $\sigma_w < 0.1$ one should consider using $p = 1$, since the algorithm is computationally lighter. Also notable is that increasing the iteration order corresponds to an improvement in the quality of the data reception.

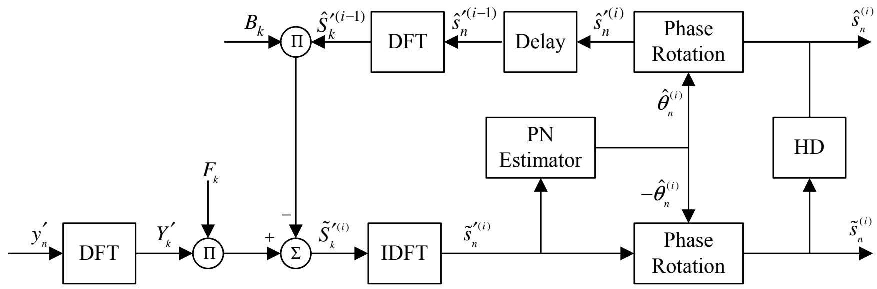

Fig. 1. IB-DFE with PN compensation.

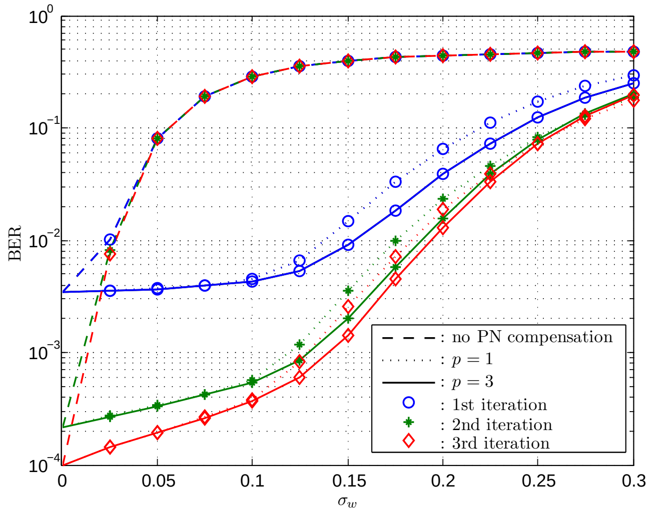

Fig. 2. BER versus the σ_w for $E_b/\mathcal{N}_0 = 10$dB for iteration orders $i = 1, 2, 3$.

VII. Conclusions

We proposed a solution for the problem of estimating the PN by approximating the non-Gaussian probability density function by a weighted sum of Gaussians. Additionally, we compare the performance results obtained when modeling the density function by a weighted sum of Gaussians with those obtained using a single Gaussian. We observed that for practical applications of the algorithm, the use of a single Gaussian suffices. Moreover, through experimental results we observed that the algorithm introduced presents outstanding results for the case in which we have perfect channel estimation and symbol synchronization.

References

[1] H. Sari, G. Karam, and I. Jeanclaude, "An analysis of orthogonal frequency-division multiplexing for mobile radio applications," in *Proc. IEEE Vehicular Technology Conference, (VTC'94)*, Stockholm, Jun. 1994, pp. 1635–1639.

[2] L. Cimini Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Transactions on Communications*, vol. 33, pp. 400–411, Jul. 1985.

[3] A. Gusmão, R. Dinis, J. Conceição, and N. Esteves, "Comparison of two modulation choices for broadband wireless communications," in *IEEE Vehic. Tech. Conf., VTC'2000 (Spring)*, vol. 2, Tokyo, Japan, May 2000, pp. 1300–1305.

[4] D. Falconer, S. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Comm. Mag.*, vol. 4, no. 4, pp. 58–66, Apr. 2002.

[5] J. Zamorano, J. Nsenga, W. Van Thillo, A. Bourdoux, and F. Horlin, "Impact of phase noise on OFDM and SC-CP," in *Global Telecommunications Conference, 2007. GLOBECOM '07. IEEE*, 26–30 2007, pp. 3822–3825.

[6] M. Sabbaghian and D. Falconer, "Joint turbo frequency domain equalization and carrier synchronization," *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 204–212, Jan. 2008.

[7] A. Stark and D. Raphaeli, "Combining decision-feedback equalization and carrier recovery for two-dimensional signal constellations," *Communications, IEEE Transactions on*, vol. 55, no. 10, pp. 2012–2021, oct. 2007.

[8] D. Petrovic, W. Rave, and G. Fettweis, "Phase noise suppression in OFDM using a Kalman filter," in *in Proc. WPMC*, 2003, p. 03.

[9] A. Jazwinski, *Stochastic Processes and Filtering Theory*. New York: Academic, 1970.

[10] D. Alspach and H. Sorenson, "Nonlinear bayesian estimation using gaussian sum approximations," *IEEE Trans. Automatic Control*, vol. 17, no. 4, pp. 339–448, Aug. 1972.

[11] J. Leitão and J. Moura, "Acquisition in phase demodulation: application to ranging in radar/sonar systems," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 31, no. 2, pp. 581–599, Apr. 1995.

[12] W. Tam, K. Plataniotis, and D. Hatzinakos, "An adaptive gaussian sum algorithm for radar tracking," *Signal Processing*, vol. 77, no. 1, pp. 85–104, Aug. 1999.

[13] A. Papoulis, *Probabilities, Random Variables, and Stochastic Processes*, 3rd ed. New York: McGraw-Hill, 1991.

[14] D. Lainiotis, "Partitioning: A unifying framework for adaptative systems I: Estimation," in *Proc. IEEE 64*, 1976, pp. 1126–1143.

[15] Y. Bar-Shalom and X. Li, *Estimation and Tracking: Principles, Techniques and Software*. New York: Artech House, 1986.

[16] R. Dinis, T. Araújo, P. Pedrosa, and F. Nunes, "Joint turbo equalization and carrier synchronization for SC-FDE schemes," *Euro. Trans. Telecomm.*, vol. 21, no. 2, pp. 131–141, Feb.

[17] M. Tüchler, R. Koetter, and A. Singer, "Turbo equalization: principles and new results," *IEEE Trans. Commun.*, vol. 50, pp. 754–767, May 2002.

[18] ETSI, "Channel models for HIPERLAN/2 in different indoor scenarios," *ETSI EP BRAN 3ERI085B*, pp. 1–8, Mar. 1998.