2º Exame Electromagnetismo e Óptica (30-01-2008)

LEAN e MEAer

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Resolução Sucinta

(I) $\tan\theta = \frac{F_C}{mg}$ e $d = 2l\sin\theta$

$\left|\vec{F_C}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2} = 0.056\ N$, $m = \frac{F_C}{g\tan\theta} = 9.7\times 10^{-3}\ kg.$

(II) $l = 10m$, $V \equiv \Delta\phi = 1V$, $q = |e| = 1.6\times 10^{-19}C$, $I = \int_S \vec{J}\cdot\vec{n}\ dS = \left|\vec{J}\right| A = NqvA$

$V = RI = RNqvA$ and $R = \rho\frac{l}{A} \Rightarrow V = \rho l N q v \Rightarrow v = \frac{V}{\rho l N q}$

$\rho = \rho_0[1 + \alpha(T - T_0)] = 2.93\times 10^{-8}\ \Omega m$

$26.93\times 10^{-3}\ kg \rightarrow 6.023\times 10^{-8}$ electrões

$2.7\times 10^{3}\ kg/m^3 \rightarrow N$

$N = 6.027\times 10^{28}$ electrões$/m^3 \Rightarrow v = 3.54\times 10^{-4} m/s.$

(III) $n = 50\ espiras/m$, $i = 5\ A$, $\mu_R = 5000$, $\mu = \mu_R\mu_0$, $\vec{B} = \mu\ \vec{H}$

$\vec{B}_{INT} = \mu n i\ \vec{e}_z$, $\vec{B}_{EXT} = \vec{0}$

$\vec{H}_{INT} = n i\ \vec{e}_z$, $\vec{H}_{EXT} = \frac{\vec{B}_{EXT}}{\mu_0} = \vec{0}$

$\left|\vec{B}_{INT}\right| = \frac{\pi}{2}\ T$

$\left|\vec{H}_{INT}\right| = 2.5\times 10^{2}\ A/m$

(IV) $\vec{B}(\vec{r}, t) = 3\times 10^{-7}\left(B_0\ \vec{e}_x + 2\ \vec{e}_y\right)\cos\left[12\pi\times 10^3 t - 2\pi\alpha\times 10^{-5}(\sqrt{2}x - y - z)\right]$ (T);

(a) $\omega = 12\pi\times 10^3\ rads^{-1}$, $\omega = 2\pi\nu \Rightarrow \nu = 6\times 10^3\ Hz.$

$\vec{k} = 2\pi\alpha\times 10^{-5}(\sqrt{2}\ \vec{e}_x) - \vec{e}_y - \vec{e}_z \Rightarrow \left|\vec{k}\right| = 2\pi\alpha\times 10^{-5}\sqrt{2+2} = 4\pi\alpha\times 10^{-5}\ m^{-1}$

$k = \frac{\omega}{c} \Rightarrow 4\pi\alpha\times 10^{-5} = \frac{12\pi\times 10^3}{3\times 10^8} \Rightarrow \alpha = 1$

$k = 4\pi\times 10^{-5}\ m^{-1}$ $\lambda = \frac{2\pi}{k} = 5\times 10^4\ m$

$\vec{n} = \frac{\vec{k}}{\left|\vec{k}\right|} = \frac{\sqrt{2}}{2}\ \vec{e}_x - \frac{1}{2}\ \vec{e}_y - \frac{1}{2}\ \vec{e}_z.$

(b) $\vec{B}\cdot\vec{k} = 0 \Rightarrow B_0\sqrt{2} - 2 = B_0 = \sqrt{2}.$

(c) $\vec{E} = z\left(\vec{H}\times\vec{n}\right)$, $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$, $\frac{Z_0}{\mu_0} = c$,

$\vec{H} = \frac{\vec{B}}{\mu_0}$

$\vec{E} = c\times 3\times 10^{-7}(\sqrt{2}\ \vec{e}_x + 2\ \vec{e}_y)\cos\left[12\pi\times 10^3 t - 2\pi\times 10^{-5}(\sqrt{2}x - y - z)\right]\times\left(\frac{\sqrt{2}}{2}\ \vec{e}_x - \frac{1}{2}\ \vec{e}_y - \frac{1}{2}\right.$

$\vec{E} = 90\cos\left[12\pi\times 10^3 t - 2\pi\times 10^{-5}(\sqrt{2}x - y - z)\right]\ \vec{Y}$ (V/m)

$\vec{Y} = \left(\sqrt{2}\ \vec{e}_x + 2\ \vec{e}_y\right)\times\left(\frac{\sqrt{2}}{2}\ \vec{e}_x - \frac{1}{2}\ \vec{e}_y - \frac{1}{2}\ \vec{e}_z\right) = -\ \vec{e}_x + \frac{\sqrt{2}}{2}\ \vec{e}_y - 3\frac{\sqrt{2}}{2}\ \vec{e}_z.$