1º Exame Electromagnetismo e Óptica (14-01-2008)

LEAN e MEAer

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Resolução Suscinta

(I) **(a)** $qE = mg \quad \left(\vec{F}_{EL} + \vec{P} = \vec{0}\right)$

$E = \frac{mg}{q} = \frac{1.67\times10^{-26}\times9.81}{1.6\times10^{-18}} = 1.02 \times 10^{-7}\ V/m$

(b) $\vec{E} = -\nabla\phi \Rightarrow \int_A^B d\phi = -\int_A^B \vec{E} \cdot d\,\vec{r}$

Campo eléctrico é uniforme: $\int_A^B d\phi = -\left|\vec{E}\right| \Delta x$

$\Delta\phi = -\left|\vec{E}\right| \Delta x$ logo $|\Delta\phi| = 1.02 \times 10^{-7} \times 5 \times 10^{-2} = 5.1 \times 10^{-9}\ V$

(II) $a = 30 \times 10^{-2} m$, $b = 50 \times 10^{-2} m$ e $R = 5\Omega$;

(a) $\Phi = \int\int_S \vec{B}.\vec{n}\ dS = \left|\vec{B}\right| A = \left|\vec{B}\right| ab\ (\vec{B} \ //\ \vec{n})$;

(b) (Lei de Faraday-Lenz) $\varepsilon = -\frac{\partial\Phi}{\partial t} = -\dot{B}A \Rightarrow Ri_{IND} = \dot{B}A$

$\dot{B} = \alpha t \Rightarrow \alpha = \frac{Ri_{IND}}{tA} = \frac{5\times300\times10^{-6}}{10\times1500\times10^{-4}} = \frac{1}{2} \times 2 \times \frac{10^{-6}}{10^{-3}} = 10^{-3}\ Ts^{-2}$.

(c) Sentido horário.

(d) $B(t_2) = \frac{1}{2}\alpha t^2 - B(t_1)$ e $B(t_1) = \frac{1}{2} \times 10^{-3} \times 10^2 = 5 \times 10^{-2}\ T$

$B(t_2) = \frac{1}{2} \times 10^{-3} \times 400 - 5 \times 10^{-2} = 0.2 - 0.05 = 0.15\ T$

$i_{IND} = \frac{\alpha t A}{R} = \frac{10^{-3}\times20\times0.15}{5} = 6 \times 10^{-4}\ A$

(III) **(a)** $\vec{B}_{EXT} = \vec{0} \Rightarrow \vec{E}_{EXT} = \vec{0}$

(b) $B_{INT} = \mu_0 In = \mu_0 In$

$\varepsilon = -\frac{\partial\Phi}{\partial t}$ com $\Phi = B_{INT}\pi r^2$;

$\varepsilon = \int_\Gamma \vec{E}_{INT} \cdot d\,\vec{r} = 2\pi r \left|\vec{E}_{INT}\right|$

$\dot{\Phi} = \dot{B}_{INT}\pi r^2 = \mu_0 n I_0 \omega \sin(\omega t)\pi r^2 \Rightarrow \vec{E}_{INT} = \frac{1}{2}\mu_0 n I_0 \omega \sin(\omega t) r\ \vec{e}_\varphi$

(IV) $\vec{E} = 20\cos(6 \times 10^8 t - 2z + 30°)\ \vec{e}_x + 20\cos(6 \times 10^8 t - 2z + 120°)\ \vec{e}_y$ (V/m);

$\Delta\phi = 90°$, $\omega = 6 \times 10^8 rads^{-1}$, $k_z = 2m^{-1}$, $\vec{k} = 2\ \vec{e}_z \Rightarrow k = 2m^{-1}$

$\vec{n} = \frac{\vec{k}}{k} = \vec{e}_z$.

(a) Polarização:

$\cos(a+b) = \cos a \cos b \mp \sin a \sin b$

$\cos(a + \pi/2) = -\sin a$

$\vec{E} = E_{x_0}\cos(\omega t - k_z + 30°)\ \vec{e}_x - E_{y_0}\sin(\omega t - k_z + 30°)\ \vec{e}_z$

$t = 0$, $z = 0$: $E_x = E_{x_0}\cos 30°$ e $E_y = E_{y_0}\sin 30°$;

$t = 0^+$: $E_x < E_{x_0}\cos 30°$ e $E_y < E_{y_0}\sin 30°$;

$\Rightarrow$ Polarização no sentido horário (direita).

(b) $k = \frac{\omega}{v}$, $k = 2m^{-1}$, $v = \frac{\omega}{k} = \frac{6\times10^8}{2} = 3 \times 10^8\ m/s = c$

(c) $\vec{B} = \sqrt{\varepsilon_0\mu_0}(\vec{n} \times \vec{E})$

$= \frac{1}{c}(\vec{n} \times \vec{E})$

$\vec{B} = \frac{1}{c}\left(-E_y\ \vec{e}_x + E_x\ \vec{e}_y\right)$

$\vec{B} = \left[-6.67\cos\left(6 \times 10^8 t - 2z + 120^\circ\right)\ \vec{e}_x + 6.67\cos\left(6 \times 10^8 t - 2z + 30^\circ\right)\ \vec{e}_y\right] \times 10^{-8}\ (T)$