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Article

The Web as a Laboratory for Integral Calculus: Solving Area Problems Between Curves

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Abstract: The use of new methodologies in teaching and learning mathematics is very important, as it allows the creation of new possibilities for learning mathematical objects, favoring a more active student's interaction with knowledge. Web resources can be used as a teaching methodology to support mathematics learning for engineering students. With the main objective of evaluating the contribution of using a Web resource in approaching the thematic domain of Mathematical Analysis, more specifically, in calculating areas through the definite integral, a case study was implemented. In this study, through a web page, the aim was to assess the level of development of technological competencies, specific and transversal mathematical competencies, knowledge, and skills. It took place in a normal academic environment, with students from the 1.st year of the Degree in Electrical and Computer Engineering at the Institute of Engineering of Coimbra. The analysis to which the data was subjected, collected through various information collection techniques and instruments, allowed us to conclude that the resources used to support the implemented didactic approach had positive repercussions in terms of technological and mathematical development related to the topics mentioned.

Keywords: New technologies; ICT; Web-based application; mathematics learning; Engineering education

1. Introduction

Engineering is a very diverse and stimulating science that is based on the evolution of technologies. With these, engineers discover new ways to solve problems in more efficient ways. As time passes and new challenges are faced, the essential skills required of engineers remain the same. A useful knowledge of mathematics and physics will always be desired along with a capacity for innovation. In engineering education, technologies must also be present. Mathematics must be taught with the support of real simulations, Web applications, or other technologies [1].

The development of learning objects through the construction and provision of simulators and support content in a Web environment is an added value for teaching, especially for engineering teaching [2]. Many applications based on real systems or experimental systems have received particular attention in the context of distance learning, as a complementary means to traditional teaching or in their integrated use in networks of partners sharing the distinct laboratory capabilities of different institutions [3]. Visualizing and manipulating the content of a subject integrated into educational planning can enhance teaching and facilitate the learning process [4]. Virtual simulation has the advantages of high security, controllability and repeatability, strong sense of experience and remote operation, and has a wide range of application spaces [5]. In mathematics, virtual simulation has also been used to teach engineering, placing students in real situations [6]–[9]. Web applications for teaching and learning mathematics in engineering are, however, scarce, or practically non-existent. Casquilho [10][11] has developed some web pages with direct engineering applications available for use by any student.

The integration of information and communication technologies (ICT, or Informatics) in education needs to be done in a creative and critical way, allowing students to develop autonomy and reflection, so that they are not just recipients of information [12]. Several studies have demonstrated that the appropriate use of ICT modifies the teaching and learning process, enabling the diversification and expansion of knowledge, providing a more motivating environment of discovery and reflection for students [13][14] and the development a learning with of more conducive activities, individualized teaching and student autonomy [15][16].

In this study, with the aim of enhancing learning in the mathematical concept of calculate areas through the definite integral, a didactic sequence was proposed and implemented to be carried out for engineering students in the classroom where some ICT tools were used, such as the web page [17].

This article is organized as follows. This first section introduces the study. Section 2 describes the methodology applied in the classroom to (re)solve integral calculus applications to find areas between polynomial curves on the Web. Section 3 presents the results of 19 answers from a questionnaire, and its analysis with some comments, and finally the conclusions appear in Section 4.

2. Methodology

The applications of definite integrals in calculating the area of limited flat regions are one of the Mathematical Analysis syllabuses widely used in engineering and which requires great graphical and analytical knowledge on the part of students. The great difficulty that students have in visualizing regions limited by curves and their description through integrals led the authors to implement teaching techniques and methodologies that could complement and motivate students in their study of these syllabuses. Thus, during 3 practical hours of the Mathematical Analysis curricular unit, 25 students from the 1.st year of the Degree in Electrical and Computer Engineering at the Coimbra Institute of Engineering, carried out an activity with a set of problems using the web page [17] developed by Casquilho and represented in Figure 1.

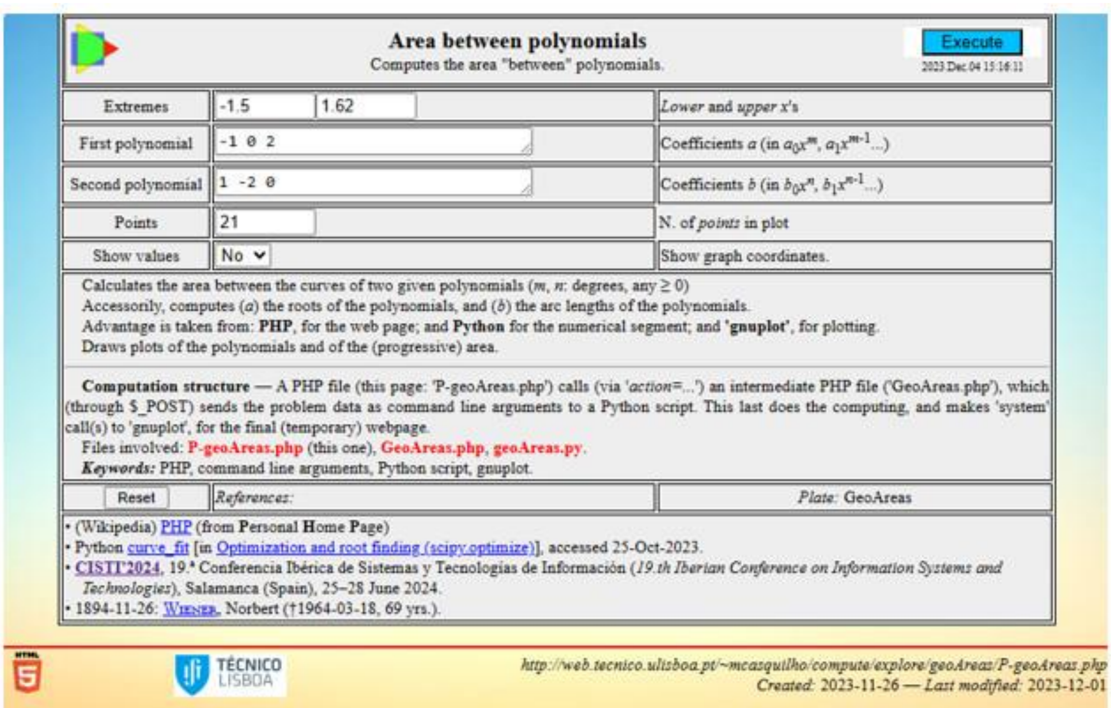

Figure 1.“Area between polynomials” web page by Casquilho [17].

The activity focused on learning the applications of definite integrals to the calculation of areas, in this case, their use in defining regions limited by two polynomial curves. Polynomials must be introduced in the web page, by their coefficients from the highest degree to the lowest degree, that is, the introduction of the coefficients -1 0 2 corresponds to the polynomial $-x^2 + 0x + 2$. It is also necessary to pay attention to which polynomial limits the region superiorly, inferiorly, and what the points of intersection are. In the first application, presented in Fig. 2, on the left, the shaded region must be calculated. To do this, students will have to identify the polynomials that limit it $f(x) = x^2 - 2x$ and $g(x) = -x^2 + 2$ and their points of intersection A (-0.62, 1.62) and B (1.62, -0.62).

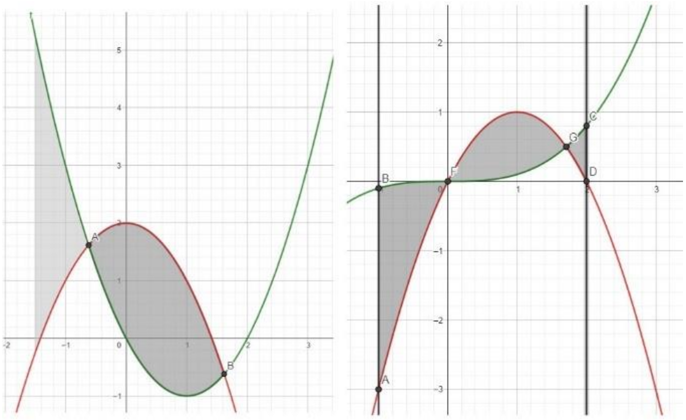

Figure 2. Applications of calculating areas between polynomial curves: 2 regions defined by different colors (left) and 3 regions defined by a single color (right).

To calculate the limited area between the functions $f(x)$ and $g(x)$ from $x = a$ to $x = b$, Eq. (1) was used.

$$\boldsymbol{A = \int_a^b [f(x) - g(x)]\ dx} \tag{1}$$

In the case of Figure 2 (left), the area is divided into two subareas, defined by two different colors to facilitate student understanding. Fig. 3 shows the calculations obtained for the first application (Figure 2, left) using the web page [17].

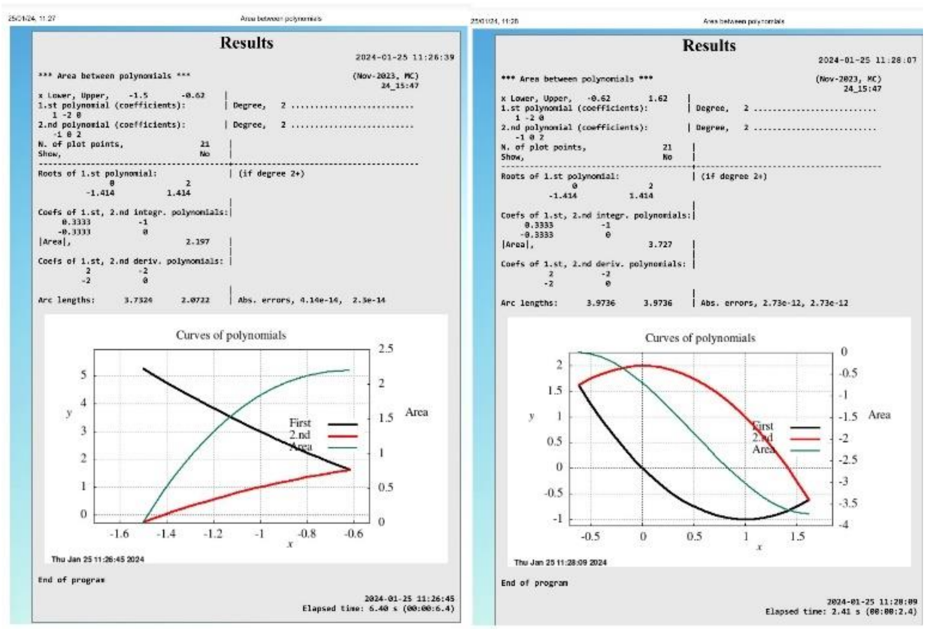

Figure 3. The first application (Figure 2, left) using the web page [17].

Some questions arise here because the area of the darkest region is negative. It's possible? In the calculations that the students made manually, the area was negative, and, on the web page, it is positive! In this case, students will have to explain what happened and change the order of the polynomials in the integral function. Therefore, students must enter the following data on the web page: *x* Lower -0.62, *x* Upper 1.62, 1.st polynomial (coefficients) -1 0 2 and 2.nd polynomial (coefficients) 1 -2 0, as represented in Fig. 4 (left) to obtain the area with a positive value whose representation is in Fig. 4 (right).

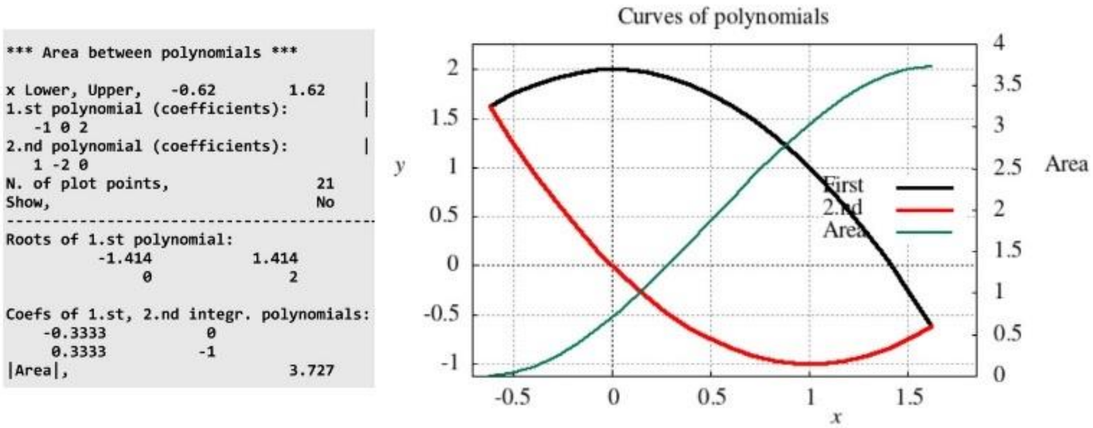

Figure 4. Correct calculation of area A_2 on the web page: values entered (left) and visualization (right).

Thus, the total area (A_t) will be given by the sum of the brighter area A_1 between $x = -1.5$ and $x = -0.62$ limited by $f_1(x) = x^2 - 2x$ and $g_1(x) = -x^2 + 2$, and the darker area A_2 between $x = -0.62$ and $x = 1.62$, limited by $g_1(x)$ and $f_1(x)$ as represented in Eq. (2).

$$\boldsymbol{A_t} = A_b + A_g = \int_{-1.5}^{-0.62} [\boldsymbol{f_1(x)} - \boldsymbol{g_1(x)}]\ \boldsymbol{dx} + \int_{-0.62}^{1.62} [\boldsymbol{g_1(x)} - \boldsymbol{f_1(x)}]\ \boldsymbol{dx} \qquad (2)$$

In the case of the application, presented in Figure 2 on the right, the region is represented in a single color and defined by the functions $f_2(x) = \frac{x^3}{10}$ and $g_2(x) = -x^2 + 2x$, between $x = -1$ and $x = 2$. Here, it is also necessary to divide the region into three subareas, which increases the student's difficulty in deciding which regions to define, which curves to choose and which lateral limits to designate. After calculating the intersection points $F(0,0)$ and $G(1.71{,}0.5)$, the total area will be defined by Eq. (3),

$$A_t = \int_{-1}^{0} [f_2(x) - g_2(x)]\ dx + \int_{0}^{1.71} [g_2(x) - f_2(x)]\ dx + \int_{1.71}^{2} [f_2(x) - g_2(x)]\ dx \quad (3)$$

Other experiments were carried out, varying the degrees of the polynomials and the complexity of the regions. A new application was presented to the students with the need for them to initially construct two polynomials that fit a banana, as shown in Figure 5.

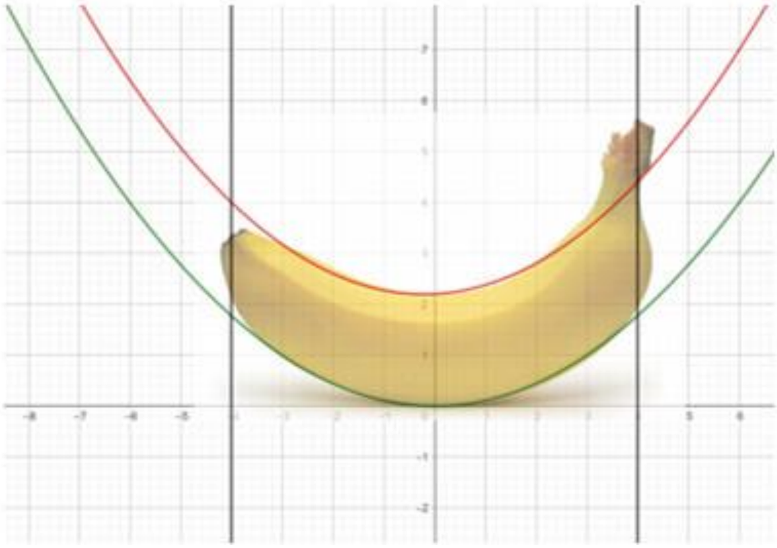

Figure 5. Definition of the region that limits a banana with two second degree polynomials.

Here it is first necessary to adjust one parabola (second degree polynomial) to the upper limits and another to the lower limits of the banana. After defining the region limited superiorly and inferiorly by parabolas, the extremes of integration were defined by vertical lines. The student now must define the area through integration and perform the calculations. In the case presented in Fig. 5, the area of the banana is just one and given by Eq. 4.

$$A_{banana} = \int_{-4}^{4} \left[\left(\frac{(x+0.2)^2}{8} - 2.2 \right) - \left(\frac{x^2}{9} \right) \right]\ dx \approx 23.04 \quad (4)$$

A final application was presented to the students introducing the symmetry of the regions in relation to coordinate axes. Thus, the calculation of the mask area (A_{mask}) in Fig. 6, can be presented by twice the area of the region that lies on the positive part of the x-axis (since the region is symmetrical in relation to the y-axis), designated by A_2. On the other hand, this region A_2 will have to be subdivided into two parts: A_{21} between point $G(0,-2)$ and point $D(2,0)$ and A_{22} between $D(2,0)$ and point $E(4,2)$, since the polynomial that limits the region inferiorly is changed at point D, moving from $g_3(x) = \left(\frac{x}{\sqrt{2}}\right)^2 - 2$ to $h_3(x) = -\left(\frac{x-4}{\sqrt{2}}\right)^2 + 2$, maintaining the polynomial that limits the region superiorly defined by $f_3(x) = -\left(\frac{x}{2}\right)^2 + 6$.

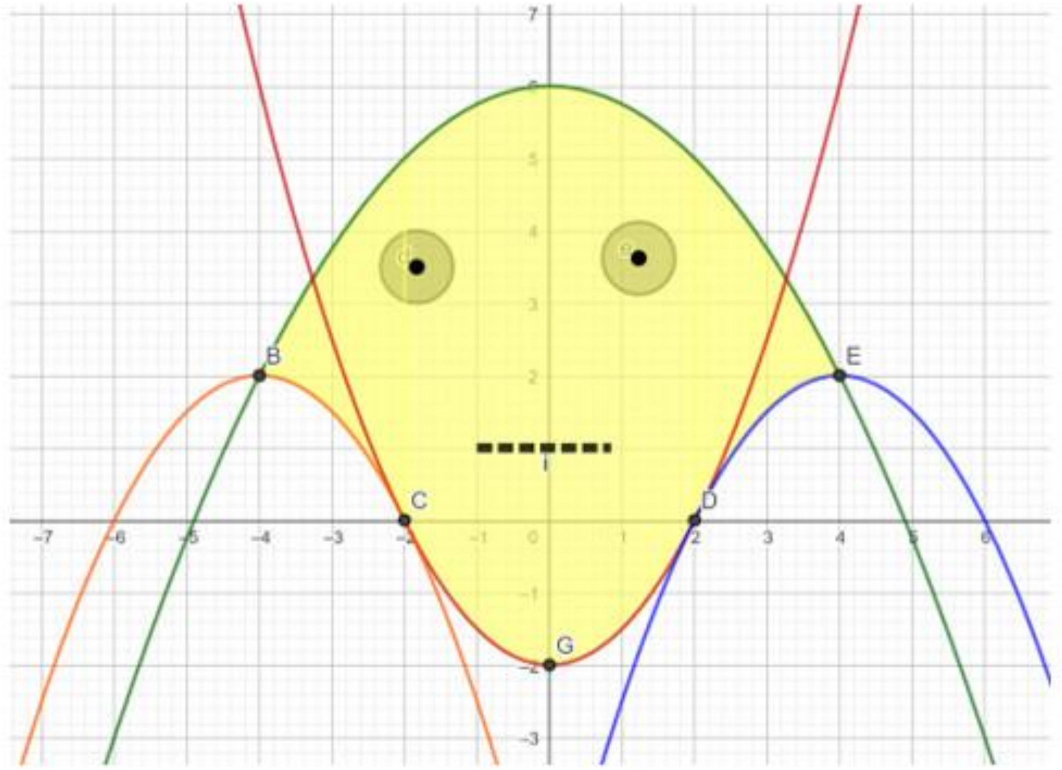

Figure 6. Calculation of the mask area using symmetry in relation to the *y*-axis.

Thus, the calculations performed are represented in Eq. (5) and justified in Fig. 7.

$$\begin{aligned} \boldsymbol{A_{mask}} &= \boldsymbol{2A_2} = \boldsymbol{2(A_{21} + A_{22})} \\ &= \boldsymbol{2\left[\int_0^2 [f_3(x) - g_3(x)]\ dx + \int_2^4 [f_3(x) - h_3(x)]\ dx\right]} \\ &= \boldsymbol{2(14 + 4.67) \approx 37.33} \end{aligned} \tag{5}$$

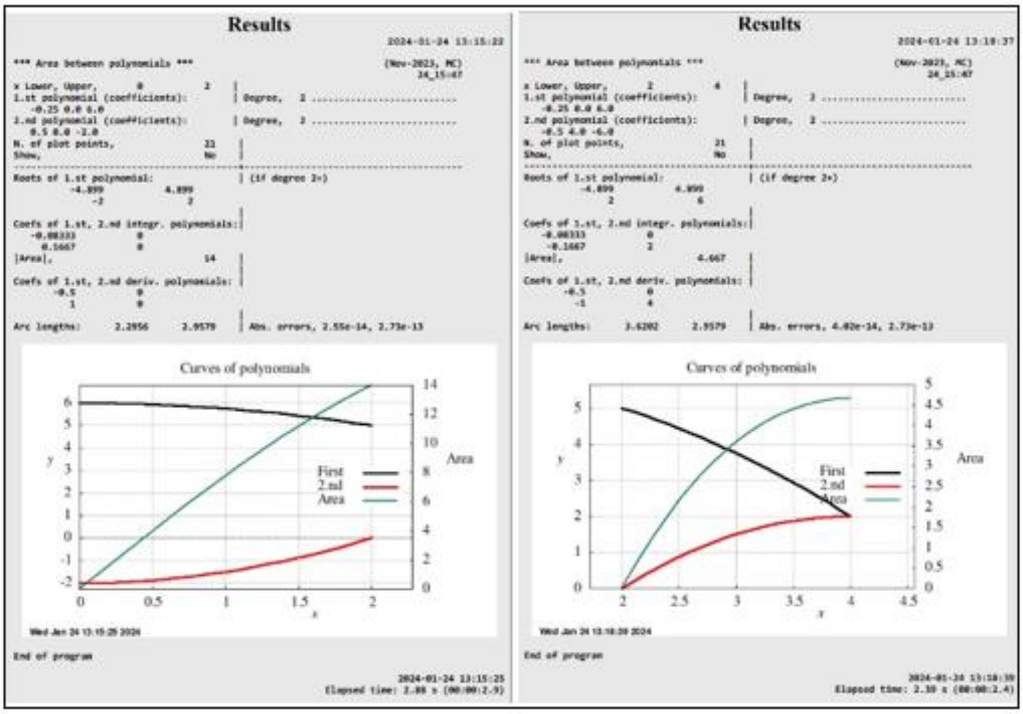

Figure 7. The mask application (Figure 6) using the web page [17].

3. Results and Discussions

At the end of the activity, students were asked to answer an online questionnaire. In it, 19 of the 25 students answered the 5 questions in the questionnaire. The first 3 questions have a closed Yes/No answer and the last two have an open answer. In Table 1 the students answer to the first 3 questions are show.   In the first question, 100% of students responded that they enjoyed using the web page to support their learning. This confirms that the use of active methodologies and ICTs in engineering learning allows students to feel connected to the content taught and tempted to explore it in the way that most motivates them [18][19].

Table 1. Students’ answer to the first 3 yes/no questions.

Questions	Yes # (%)	No # (%)
Did you enjoy using the web page to confirm your calculations ?	19 (100%)	
Was it easy to use the web page ?	16 (84.2%)	3 (15.8%)
Do you think it is important to use these types of technology in mathematics classes ?	18 (95%)	1 (5%)

In the question about the ease of use of the web page, 16 (84.2%) students answered Yes and 3 (15.8%) No. It should be noted that the web page used is built to allow access to a set of classic mathematics problems applied to engineering, as support for teaching-learning, not as a specific teaching tool. Only 3 students did not find it easy to use. In the third question, 18 (95%) students think it is important to use this type of technology in mathematics classes and only 1 (5%) does not think it is important. Students in general, as well as teachers, believe that technologies allow greater motivation for students in the teaching-learning process, because it develops and stimulates cognitive skills, such as critical thinking and student curiosity [20].

Regarding the open question, about what students like most about carrying out the activity, it was found that support in confirming results (45.8% of students' answers) and ease of obtain results (25% of students' answers) were the answers most given by students. Visual representation (12.5% of students' answers), support when solving problems (8.3% of students' answers) and the interest in having applications of this type to help students in their learning (8.3% of students' answers) were also identified, as represented in Figure 8.

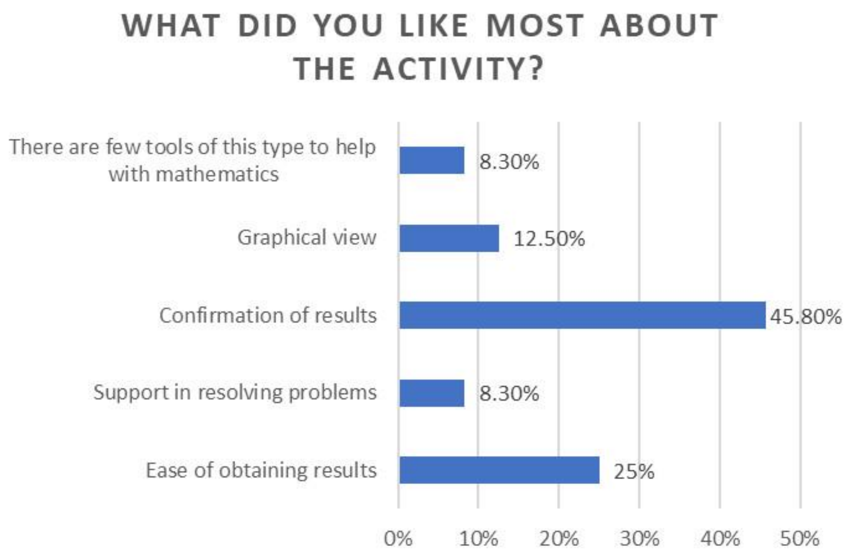

Figure 8. Answers to the question: "what did you like most about the activity ?"

In relation to what the students liked the least during the activity, as can be seen in Figure 9, it was the initial difficulty in interacting with the web page (40% of students' answers), which is natural when presenting a new application for the first time, the limitation that the web page only works with polynomial functions (35% of students' answers), since many other problems presented to students use trigonometric, logarithmic functions, etc., and time is limited to solve the activity (5% of students' answers). The remaining students (20% of students' answers) say they liked everything.

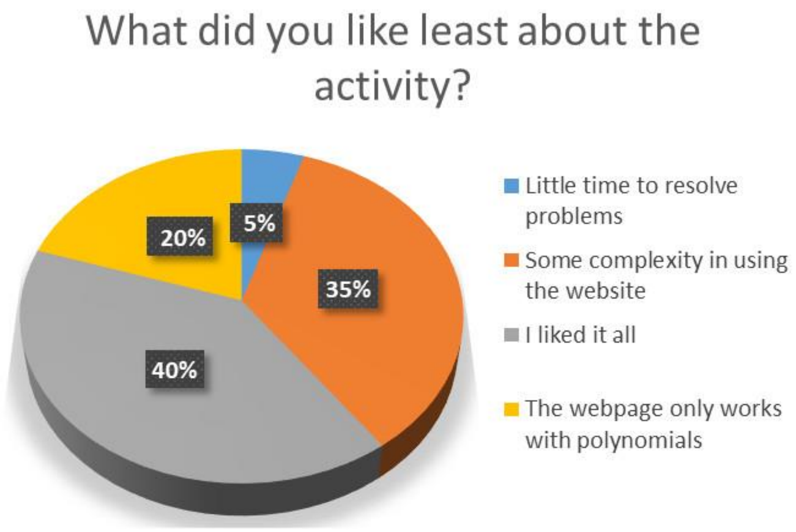

Figure 9. Answers to the question: "What did you like least about the activity ?".

3. Conclusions and future work

The main objective of this study, in general, is to understand the potential of using a web page as a teaching tool and verify its impact on the learning process and promoting student autonomy. In this way, the web page [17] was used as a learning tool, considering an active methodology support, based on learning through problem solving, developing skills, attitudes, and values in the mobilization of curricular knowledge. This tool places students at the center of the teaching and learning process and encourages active and collaborative intervention, in an environment of trust and satisfaction.

The objectives of this article focus on transforming the teaching and learning of mathematics into an innovative and adaptive methodology that responds to the specific challenges of practical engineering teaching in a real context. In the use of ICT, it improves the pedagogical experience for engineering students and focuses on essential practical skills and in-depth theoretical understanding.

This article explored the activity developed and the implementation of a web-focused pedagogical scenario within an engineering learning solution. The approach integrates theory, online simulation, experimentation, and reflection, providing students with a holistic learning experience [21] with technological resources available on the Web, more specifically on the web page mentioned.

The results obtained in the pedagogical scenario used showed strong positive indications, with students reinforcing their understanding of the case studies, improving their practical application skills, and expressing a high level of engagement. The questionnaires administered to the students provided valuable feedback that will guide the continuous improvement of the pedagogical approach to be followed in the next academic year.

Furthermore, it is important to note that Web simulation is becoming increasingly essential in today's educational context, although scarce in the technical literature. Unpredictable circumstances, such as lockdowns and restrictions, have highlighted the importance of developing flexible and adaptive teaching methods that allow students to continue their training without interruption.

In short, this pedagogical scenario illustrates the importance of innovation in higher education, adapting to the new needs of students using resources available on the Web to offer quality education. It paves the way for similar learning approaches in other domains, helping to mark a significant and necessary educational transformation in a context where practical teaching using technologies is becoming an obligation rather than a choice. Furthermore, it should be noted that the activity described here is just a case study of ICT in teaching mathematics to engineers and is a solution with great educational interest. Generalizing this approach to a wide range of mathematical content, offering students a comprehensive and adaptable learning experience to different areas of engineering, improving the skills and abilities of future engineers is fundamental, and will be implemented in the near future.

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