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Acceptance sampling by variables for double specification limits: save space, and compute on the Web

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***Abstract*— In this paper, we discuss computations in Acceptance Sampling "by variables", the classical complement to Statistical Process Control. We focus on the case of double (lower and upper) specification limits (Gaussian variable), and relate the cases of single and double specification limits. After briefly reviewing the topic, we expose a detail in its core mathematical basis, relating the quality index to the estimated percent defective, in contrast to the verbosity in literature, and provide a web page for the direct computation. There, the interested reader, needing no additional software or tables, can verify our computations in a clear, trustworthy manner, rare in the literature. This methodology also points to an easier collaboration between Academia and Industry.**

Keywords – Quality Control; acceptance sampling by variables; Web computing; "Form 2"; international standards

I. 1 Background and Purpose

Quality, if we couple it to Statistics, has been around already since one century and is everywhere, namely in manufacture and in business in general. A sketch of the history of Quality is in Fig. 1, noticing the leap in the "Early 20.th century", from the American Society for Quality (ASQ) website[2], where pioneers, such as Shewhart, Deming, Juran, Ishikawa, are naturally mentioned.

In this paper, in a simplified manner, we separate what occurs inside an organization and what happens across its frontiers. In terms familiar to us, Quality partisans, these would be Statistical Process Control (SPC) and Acceptance Sampling (AS), respectively. We address the latter, to (*i*) recollect some of its statistical basis, (*ii*) bring it to the open light of execution on a constructed web page, shortening the scientific path, and (*iii*) criticizing simultaneous verbosity and gaps in essential support documents.

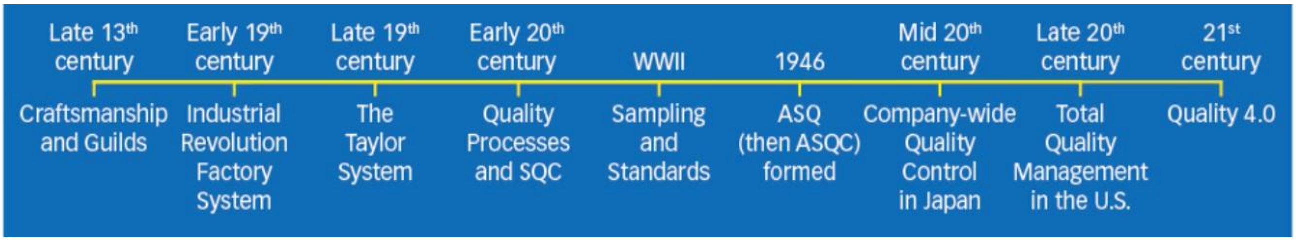

Fig. 1. ASQ: "The History of Quality".

Acceptance sampling by variables to inspect a lot of similar items (detailed in III.A) is, briefly:

- From the lot, draw a sample and calculate its average, $\bar{X}$, and standard deviation, *s*.
- Is $\bar{X}$ "safely" inside (*L*, *U*), i.e., at a good distance of them, in terms of *s* ?
- If Yes, Accept the lot; if No, Reject the lot.

We also review basic aspects of Quality, often diluted in management concepts, as once remarked by (Gunther 1998). The fact is that the international standards underlying our subject keep being updated and maintained (e.g., (ISO/TC 69/SC 5 2022b)). In the areas of Engineering, these authors' field of interest, such as Chemical or Computer Science Engineering, Acceptance Sampling becomes more purposeful when, simultaneously, the continuous variable inspected is subject to the two specification limits, lower and upper[3]. In the Quality terminology, this is "AS by variables" under "Form 2" (ISO/TC 69/SC 5 2022a), whereas Form 1 is meant for single specification limits. This becomes computationally accessible, as will be seen, and is advantageous, namely, to non-specialized decision makers.

In the following parts: Section II, "Approach to bibliographical reproach", comments some unsatisfactory literature features; Section III, "Inspection procedure and mathematical transformation", describes the sampling procedure according

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[2] URL: https://asq.org/quality-resources/history-of-quality
[3] Not to be confused with the lower and upper control limits, LCL and UCL, in SPC.

to the applicable standards and the said transformation; Section IV, "Results and discussion", shows and explains the web-based functionality, and settles our results; and Section v, "Conclusions", sums up the usefulness of the study.

II. Approach to bibliographical reproach

As a first thought, we note that, despite the presence of Informatics[4] in most domains (Engineering, Medicine, Linguistics, etc.), we (these non-Informatics Engineering authors) regret the often-weak connection from the informatic engineers, leading to poor results in the said domains. Second thought, in the scientific literature, is the lack of links to web pages for the verification of results by the reader, a fact about which we are not neutral. We always provide them, but found no literature examples to cite, apart from our own, such as (Casquilho 2013), (Carolino et al. 2016), (Barros and Casquilho 2019), (Casquilho and Buescu 2022), (Casquilho et al. 2025), (Casquilho and Pacheco 2023), (Casquilho 2025), as verifiable, e.g., in (Kuntsche et al. 2011)[5]. One reason for this absence may be the editors' worry (reasonably, as seen in next paragraph) that the targets disappear, which leads to the broader issue of the permanence of the publications themselves, nowadays as 'pdf' documents on the Web, no longer in paper.

A remarkable case of web computing (not typical articles) is Ponce's (Ponce 1999)[6] website (Hydraulics) with hundreds of dynamic problems. Another example of web computing, also academic, is Arsham's (Arsham 1994) website, a large repository on Operations Research and Statistics. As concrete article, see, e.g., (Kuntsche et al. 2011), on Chemical Engineering, reporting a website for process simulation for which one of the two links is "not found" (well, since 2011) and showing no clickable illustration. (We resort to open the 'pdf's with the free version of (PDF Xchange 2026), usually able to detect links.)

These authors will be thankful for any examples of (open access or not) technological literature articles that include links for the verification by the interested readers. We could not find them even with the help from several AI (artificial intelligence) engines.

III. Inspection procedure and mathematical transformation

We now detail the inspection procedure for practical application, as prescribed in the standard, and show the mathematical transformation of the acceptance constant, k, into the maximum estimated percent defective, M.

A. Inspection procedure

The use of AS by variables, as defined in the applicable international standards, assumes that the material to be inspected is constituted by a series, the "lot", of individual, measurable, similar parts (such as screws, resistors, bags of cereal, bottles of chemicals with prescribed composition, etc.). The inspection of incoherent material, the so-called "bulk sampling" (ISO/TC 69/SC 5 2022a), such as powders, slurry, liquids, is not in our scope.

The inspection procedure observes the following steps, beginning with the establishment of the size of the random sample to be drawn from the whole material, i.e., the lot, of, say, size N = 100 (to match the example used further below):

The inspection procedure begins with the establishment of the size of the random sample to be drawn from the whole material, i.e., the lot, of, say, size N = 100 (to match the example used further below):

a) Identify the size of the lot, N , to be "sentenced" (accept or reject).

b) Stipulate the two specification limits (the "specs"), lower spec, L, and upper spec, U.

c) Stipulate the AQL (Acceptable Quality Limit) for each spec, AQL_L and AQL_U (percentage).

d) Find the size, n, of the sample to be randomly drawn from the lot.

e) Follow the decision procedure.

In the above list, step 'd)' (finding the adequate sample size) is implemented by the sampling plan, given in a table in the standard as follows:

a) Supply the lot size, N.

b) Select the "sampling level", which we admit Normal, in the present context.

c) Obtain the sample size letter from 'Table A-2', in Fig. 2 (assuming the neutral "inspection level" IV). The letter (F) will become n = 10 in 'Table B-3', in Fig. 3.

We illustrate the procedure with the example in (Schilling and Neubauer 2009), 2.[nd] ed. 288.[th] page), which reappears in (Schilling and Neubauer 2017), 3.[rd] ed. (p. 318), itself taken from the standard. For a lot of N = 100 (from the standard, 'Table B-3', in Fig. 3), it is n = 10 items, with values such as the following:

$$\mathbf{x} = 643\ 651\ 619\ 627\ 668\ 670\ 673\ 642\ 638\ 650$$

These values are supposed to be a sample of measured electrical resistance, in ohm, with stipulated specification limits of (650 ± 30) Ω and AQL's

$$L = 620,\ \text{AQL}_L = 1\%,\ U = 680,\ \text{AQL}_U = 2.5\%\ .$$

[4] Or "Information Science", "Computer Science" (in the USA), etc..
[5] No visible link: https://mosaic-modeling.de/?page_id=5490 , accessed 2026-Feb-01.
[6] V. M. Ponce, OnlineCalc, San Diego State University, https://ponce.sdsu.edu/online_calc.php, accessed 2026-Feb-01.

The numerical procedure to take a decision ("sentence" the lot) is detailed in Table 1. A lot is accepted if (only if) the three criteria are Ac ("accept"); and a lot is rejected if any criterion is Re ("reject"). So, in this example, the lot would be rejected.

Contrary to Form 1, which just concludes Ac or Re, Form 2 yields a real, more intuitive notion of how far (above or below, respectively) the inferred quality is from the limiting percent defective. (The words percent and defective are, respectively, an adjective and a noun, i.e., a substantive or nominalized adjective.) So, Form 2 is generally recommended for communication with the deciders.

In this study, one of our main points is that 'Table B-3', in Fig. 3, is a mathematical transformation (and a simple one) from 'Table B-1', in Fig. 4, as will be seen.

MIL-STD-414 Table A-2—Sample Size Code Letters[a]

	Inspection Levels				
Lot Size	**I**	**II**	**III**	**IV**	**V**
3–8	B	B	B	B	C
9–15	B	B	B	B	D
16–25	B	B	B	C	E
26–40	B	B	B	D	F
41–65	B	B	C	E	G
66–110	B	B	D	F	H
111–180	B	C	E	G	I
181–300	B	D	F	H	J
301–500	C	E	G	I	K
501–800	D	F	H	J	L
801–1,300	E	G	I	K	L
1,301–3,200	F	H	J	L	M
3,201–8,000	G	I	L	M	N
8,001–22,000	H	J	M	N	O
22,001–110,000	I	K	N	O	P
110,001–550,000	I	K	O	P	Q
550,001 and over	I	K	P	Q	Q

Source: United States Department of Defense, *Military Standard, Sampling Procedures and Tables for Inspection by Variables for Percent Defective*, MIL-STD-414, U.S. Government Printing Office, Washington, DC, 1957, p. 4.

[a] Sample size code letters given in body of table are applicable when the indicated inspection levels are to be used.

Fig. 2. Table A-2 (MIL-STD-414), giving the sample size, *n*, from the lot size, *N*.

B. Transformation of k into M

Let us return to the example problem in Table 1 and take AQL_U = 2.5%. In this case, for the said n = 10, Table B-1' (in Fig. 4) gives k = 1.41. (In this article, we are not analyzing this table, also called "the k s-table", so its underlying consumer and producer's risks are assumed.)

We now cite the books by (Sarkadi and Vincze 1974) and by (DeVor et al. 2007). The first book (415 pp), after 229 pages of mathematical theory, presents its Table 27 (pp 326–329), "Table for estimating the lot fraction defective" (from Table C-5 of MIL-STD-414)". Thus, surprisingly: a heavily theoretical book reproduces a table from a standard where that table was calculated from theory; and the discourse suddenly switches to the "range method", the alternative measure of variability. (The expected table would be Table B-5, not C-5.) No mention was found there to the incomplete beta distribution.

The second book (809 pp) just reads (p 392) that "the objective of acceptance sampling is to arrive at a decision of either accepting or rejecting a fixed and generally large lot of products in the basis of a small sample of observations of that lot", so no mention to the sought underlying theory.

Our last attempt was the fundamental book by (Duncan 1986), 1145 pp, certainly upstream from the two previously cited. Notice that Duncan co-authored another book in 1944 and probably made the original edition in 1952, according to the vast Japanese international repository CiNii [7]. After describing Form 2 (which he also calls Procedure 2), the conversion of Q into $\hat{p}$ (our p) is passed to a "chart" (a nomogram), a crafty way of saving many pages in a table but mentioning nothing of the theory. We resort to our own longtime personal knowledge (Alves 1970), theoretically corroborated now by AI.

[7] CiNii Research, NII (National Institute of Informatics, Tokyo, Japan), Acheson Duncan (search "1952"), https://cir.nii.ac.jp/books?q=Acheson+Duncan, accessed 2026-02-28.

$$M = \mathrm{B}\left(x;a,b\right) = \\ = \mathrm{B}\left(\frac{1}{2}-\frac{1}{2}k\frac{\sqrt{n}}{n-1};\frac{n}{2}-1;\frac{n}{2}-1\right) \quad (1)$$

where: **B**(*x*; *a*. *b*) is the cumulative incomplete beta function (cumulative beta probability function) of equal parameters *a* and *b*; *n* is the sample size; and *k* is the acceptance constant for which *M* is the correspondent maximum allowable fraction defective.

If we use Excel to calculate (1), we get

M = BETADIST(x, α, β, cumulative) = BETADIST(0.2522882, 4, 4, True) = 7.269% (2)

which does reproduce the value of 7.27% in 'Table B-3' (Fig. 3) for the pair (*n*, AQL) = (10, 2.5%).

If we use Python (Python.scipy.special.betainc, n.d.),

```
import numpy as np
import scipy.special as sc
n = 10; k = 1.41
x = 0.5 - 0.5 * k * np.sqrt(n) / (n - 1); a = 0.5 * n - 1
print ('x, a,', x, a)
print ('M,', sc.betainc(a, a, x))
```

we also get *M*, 0.072687.

The above segment derogates the fact that a fundamental work, such as (Schilling and Neubauer 2009) (2.[nd] ed.) and (Schilling and Neubauer 2017) (idem, 3.[rd] ed.), spends, respectively, 12 pages (626.[th]–637.[th] [8]) and 14 pages (pp 735–748) giving the "estimation of the lot percent defective" from values of Q_L or Q_U and sample sizes. These pages are necessarily limited to a range of quality indices, *Q*, and several sample sizes, *n*, whereas the simple formulas above might replace or complement those long tables (Lieberman and Resnikoff 1957).

MIL-STD-414 Table B-3—Master Table for Normal and Tightened Inspection for Plans Based on Variability Unknown: Standard Deviation Method (Double Specification Limit and Form 2, Single-Specification Limit)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (Normal Inspection)													
		.040	.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00
		M	M	M	M	M	M	M	M	M	M	M	M	M	M
B	3	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	7.59	18.86	26.94	33.69	40.47
C	4	⇩	⇩	⇩	⇩	⇩	⇩	⇩	1.53	5.50	10.92	16.45	22.86	29.45	36.90
D	5	⇩	⇩	⇩	⇩	⇩	⇩	1.33	3.32	5.83	9.80	14.39	20.19	26.56	33.99
E	7	⇩	⇩	⇩	⇩	0.422	1.06	2.14	3.55	5.35	8.40	12.20	17.35	23.29	30.50
F	10	⇩	⇩	⇩	0.349	0.716	1.30	2.17	3.26	4.77	7.29	10.54	15.17	20.74	27.57
G	15	0.099	0.186	0.312	0.503	0.818	1.31	2.11	3.05	4.31	6.56	9.46	13.71	18.94	25.61
H	20	0.135	0.228	0.365	0.544	0.846	1.29	2.05	2.95	4.09	6.17	8.92	12.99	18.03	24.53
I	25	0.155	0.250	0.380	0.551	0.877	1.29	2.00	2.86	3.97	5.97	8.63	12.57	17.51	23.97
J	30	0.179	0.280	0.413	0.581	0.879	1.29	1.98	2.83	3.91	5.86	8.47	12.36	17.24	23.58
K	35	0.170	0.264	0.388	0.535	0.847	1.23	1.87	2.68	3.70	5.57	8.10	11.87	16.65	22.91
L	40	0.179	0.275	0.401	0.566	0.873	1.26	1.88	2.71	3.72	5.58	8.09	11.85	16.61	22.86
M	50	0.163	0.250	0.363	0.503	0.789	1.17	1.71	2.49	3.45	5.20	7.61	11.23	15.87	22.00
N	75	0.147	0.228	0.330	0.467	0.720	1.07	1.60	2.29	3.20	4.87	7.15	10.63	15.13	21.11
O	100	0.145	0.220	0.317	0.447	0.689	1.02	1.53	2.20	3.07	4.69	6.91	10.32	14.75	20.66
P	150	0.134	0.203	0.293	0.413	0.638	0.949	1.43	2.05	2.89	4.43	6.57	9.88	14.20	20.02
Q	200	0.135	0.204	0.294	0.414	0.637	0.945	1.42	2.04	2.87	4.40	6.53	9.81	14.12	19.92
		.065	**.10**	**.15**	**.25**	**.40**	**.65**	**1.00**	**1.50**	**2.50**	**4.00**	**6.50**	**10.00**	**15.00**	
		Acceptability Quality Levels (tightened inspection)													

Source: United States Department of Defense, *Military Standard, Sampling Procedures and Tables for Inspection by Variables for Percent Defective*, MIL-STD-414, U.S. Government Printing Office, Washington, DC, 1957, p. 45.
All AQL and table values are in percent defective.
⇩ Use first sampling plan below arrow, that is, both sample size as well as *M* value. When sample size equals or exceeds lot size, every item in the lot must be inspected.

Fig. 3. Table B-3, giving *M* , maximum allowable percent defective..

[8] This numbering is from a 'pdf' reader, since in this case our copy (pdf) has no page numbers.

Table 1: Decision procedure to accept (Ac) or reject (Re)

Item	Value	Source
$\bar{X}$; s	647.0, 17.2	—
$Q_L = \frac{\bar{X}-L}{s}$	1.57	—
$Q_U = \frac{U-\bar{X}}{s}$	1.92	—
$Q_L \Rightarrow p_L$	4.92%	(From Table B-5)
$Q_U \Rightarrow p_U$	1.62%	(From Table B-5)
M_L	3.26%	(From Table B-3)
M_U	7.29%	(From Table B-3)
$p_L < M_L$?	No (Re)	—
$p_U < M_U$?	Yes (Ac)	—
$p_L + p_U$	6.54%	—
$p_L + p_U < \max(M_L, M_U)$?	Yes	(See below)

MID-STD-414 Table B-1—Master Table for Normal and Tightened Inspection for Plans Based on Variability Unknown: Standard Deviation Method (Single-Specification Limit, Form 1)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (Normal Inspection)													
		.04	.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00
		k	k	k	k	k	k	k	k	k	k	k	k	k	k
B	3	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	⇩	1.12	.958	.765	.566	.341
C	4	⇩	⇩	⇩	⇩	⇩	⇩	⇩	1.45	1.34	1.17	1.01	.814	.617	.393
D	5	⇩	⇩	⇩	⇩	⇩	⇩	1.65	1.53	1.40	1.24	1.07	.874	.675	.455
E	7	⇩	⇩	⇩	⇩	2.00	1.88	1.75	1.62	1.50	1.33	1.15	.955	.755	.536
F	10	⇩	⇩	⇩	2.24	2.11	1.98	1.84	1.72	1.58	1.41	1.23	1.03	.828	.611
G	15	2.64	2.53	2.42	2.32	2.20	2.06	1.91	1.79	1.65	1.47	1.30	1.09	.886	.664
H	20	2.69	2.58	2.47	2.36	2.24	2.11	1.96	1.82	1.69	1.51	1.33	1.12	.917	.695
I	25	2.72	2.61	2.50	2.40	2.26	2.14	1.98	1.85	1.72	1.53	1.35	1.14	.936	.712
J	30	2.73	2.61	2.51	2.41	2.28	2.15	2.00	1.86	1.73	1.55	1.36	1.15	.946	.723
K	35	2.77	2.65	2.54	2.45	2.31	2.18	2.03	1.89	1.76	1.57	1.39	1.18	.969	.745
L	40	2.77	2.66	2.55	2.44	2.31	2.18	2.03	1.89	1.76	1.58	1.39	1.18	.971	.746
M	50	2.83	2.71	2.60	2.50	2.35	2.22	2.08	1.93	1.80	1.61	1.42	1.21	1.00	.774
N	75	2.90	2.77	2.66	2.55	2.41	2.27	2.12	1.98	1.84	1.65	1.46	1.24	1.03	.804
O	100	2.92	2.80	2.69	2.58	2.43	2.29	2.14	2.00	1.86	1.67	1.48	1.26	1.05	.819
P	150	2.96	2.84	2.73	2.61	2.47	2.33	2.18	2.03	1.89	1.70	1.51	1.29	1.07	.841
Q	200	2.97	2.85	2.73	2.62	2.47	2.33	2.18	2.04	1.89	1.70	1.51	1.29	1.07	.845
		.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00	
		Acceptability Quality Levels (tightened inspection)													

Source: United States Department of Defense, *Military Standard, Sampling Procedures and Tables for Inspection by Variables for Percent Defective*, MIL-STD-414, U.S. Government Printing Office, Washington, DC, 1957, p. 39.

All AQL values are in percent defective.

⇩ Use first sampling plan below arrow, that is, both sample size as well as k value. When sample size equals or exceeds lot size, every item in the lot must be inspected.

Fig. 4. Table B-1, giving k, acceptance constant.

IV. Results and Discussion

We now present the web page where a user can test a sample, by automatically following the decision procedure defined in the standard, and verify some values of 'Table B-3' (as in Fig. 3) in order to discuss whether it is rendered superfluous, and more so the extensive, verbose 'Table B-5' (as in Fig. 5).

TABLE T12.7: MIL-STD-414 Table B-5—Table for estimating the lot percent defective using standard deviation method.[a]

Q_U or Q_L	Sample Size															
	3	4	5	7	10	15	20	25	30	35	40	50	75	100	150	200
0	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
.10	47.24	46.67	46.44	46.26	46.16	46.10	46.08	46.06	46.05	46.05	46.04	46.04	46.03	46.03	46.02	46.02
.20	44.46	43.33	42.90	42.54	42.35	42.24	42.19	42.16	42.15	42.13	42.13	42.11	42.10	42.09	42.08	42.08
.30	41.63	40.00	39.37	38.87	38.60	38.44	38.37	38.33	38.31	38.29	38.28	38.27	38.25	38.24	38.22	38.22
.31	41.35	39.67	39.02	38.50	38.23	38.06	37.99	37.95	37.93	37.91	37.90	37.89	37.87	37.86	37.84	37.84
.32	41.06	39.33	38.67	38.14	37.86	37.69	37.62	37.58	37.55	37.54	37.52	37.51	37.49	37.48	37.46	37.46
.33	40.77	39.00	38.32	37.78	37.49	37.31	37.24	37.20	37.18	37.16	37.15	37.13	37.11	37.10	37.09	37.08
.34	40.49	38.67	37.97	37.42	37.12	36.94	36.87	36.83	36.80	36.78	36.77	36.75	36.73	36.72	36.71	36.71

(a) Table B-5 (title).

1.40	0.00	3.33	5.88	6.98	7.44	7.69	7.80	7.86	7.90	7.92	7.94	7.97	8.01	8.02	8.04	8.05
1.41	0.00	3.00	5.66	6.80	7.27	7.53	7.64	7.70	7.74	7.77	7.79	7.82	7.86	7.87	7.89	7.90
1.42	0.00	2.67	5.44	6.62	7.10	7.37	7.49	7.55	7.59	7.62	7.64	7.67	7.71	7.73	7.74	7.75
1.43	0.00	2.33	5.23	6.45	6.94	7.22	7.34	7.40	7.44	7.47	7.50	7.52	7.56	7.58	7.60	7.61
1.44	0.00	2.00	5.01	6.27	6.78	7.07	7.19	7.26	7.30	7.33	7.35	7.38	7.42	7.44	7.46	7.47
1.45	0.00	1.67	4.81	6.10	6.63	6.92	7.04	7.11	7.15	7.18	7.21	7.24	7.28	7.30	7.31	7.33
1.46	0.00	1.33	4.60	5.93	6.47	6.77	6.90	6.97	7.01	7.04	7.07	7.10	7.14	7.16	7.18	7.19
1.47	0.00	1.00	4.39	5.77	6.32	6.63	6.75	6.83	6.87	6.90	6.93	6.96	7.00	7.02	7.04	7.05
1.48	0.00	.67	4.19	5.60	6.17	6.48	6.61	6.69	6.73	6.77	6.79	6.82	6.86	6.88	6.90	6.91
1.49	0.00	.33	3.99	5.44	6.02	6.34	6.48	6.55	6.60	6.63	6.65	6.69	6.73	6.75	6.77	6.78

(b) Table B.5 (continued), sector for $1.40 \leqslant Q \leqslant 1.49$.

1.70	0.00	0.00	0.66	2.62	3.41	3.84	4.02	4.12	4.18	4.22	4.25	4.30	4.35	4.38	4.41	4.42
1.71	0.00	0.00	0.55	2.51	3.31	3.75	3.93	4.02	4.09	4.13	4.16	4.20	4.26	4.29	4.31	4.32
1.72	0.00	0.00	0.45	2.41	3.21	3.65	3.83	3.93	3.99	4.04	4.07	4.11	4.17	4.19	4.22	4.23
1.73	0.00	0.00	0.36	2.30	3.11	3.56	3.74	3.84	3.90	3.94	3.98	4.02	4.08	4.10	4.13	4.14
1.74	0.00	0.00	0.27	2.20	3.02	3.46	3.65	3.75	3.81	3.85	3.89	3.93	3.99	4.01	4.04	4.05
1.75	0.00	0.00	0.19	2.11	2.93	3.37	3.56	3.66	3.72	3.77	3.80	3.84	3.90	3.93	3.95	3.97
1.76	0.00	0.00	0.12	2.01	2.83	3.28	3.47	3.57	3.63	3.68	3.71	3.76	3.81	3.84	3.87	3.88
1.77	0.00	0.00	0.06	1.92	2.74	3.20	3.38	3.48	3.55	3.59	3.63	3.67	3.73	3.76	3.78	3.80
1.78	0.00	0.00	0.02	1.83	2.66	3.11	3.30	3.40	3.47	3.51	3.54	3.59	3.64	3.67	3.70	3.71
1.79	0.00	0.00	0.00	1.74	2.57	3.03	3.21	3.32	3.38	3.43	3.46	3.51	3.56	3.59	3.63	3.63

(c) Table B-5 (continued), sector for $1.70 \leqslant Q \leqslant 1.79$.

Fig. 5. Table B-5 (10 pages, 628 – 637, parts), with $0 \leqslant Q \leqslant 3.90$.

A. Test your sample

The adopted example, in III.A (Table 1), or others supplied by the interested reader can be run at the web page "Test your sample" in List 1, where web pages related to the subject are mentioned. Web page 'b)' applies directly to this subsection,

List 1. Web pages for computations

a) "Transforming Q into ω in AS by variables"

→ http://web.tecnico.ulisboa.pt/ mcasquilho/compute/qc/Fx-ASvarQtoLotFD.php

b) "Test your sample"

→ http://web.tecnico.ulisboa.pt/mcasquilho/compute/icqem2026/accByVars/P.testsample.php

The example in Table 1 embodies the default data in the web page, as depicted in Fig. 6. With these default values, the user just clicks the 'Execute' button to obtain the results in a new, temporary web page that reproduces all the computed results (equal to those already shown, omitted here for shortness) and shows a plot of the sample data, as shown in Fig. 7. In this particular case, it happens that one point, $x_3 = 619$, lies below the lower spec, $L = 620$. This would be a defective if our AS were by attributes, but, strangely to the diffident layman, that criterion is extraneous to inspection by variables.

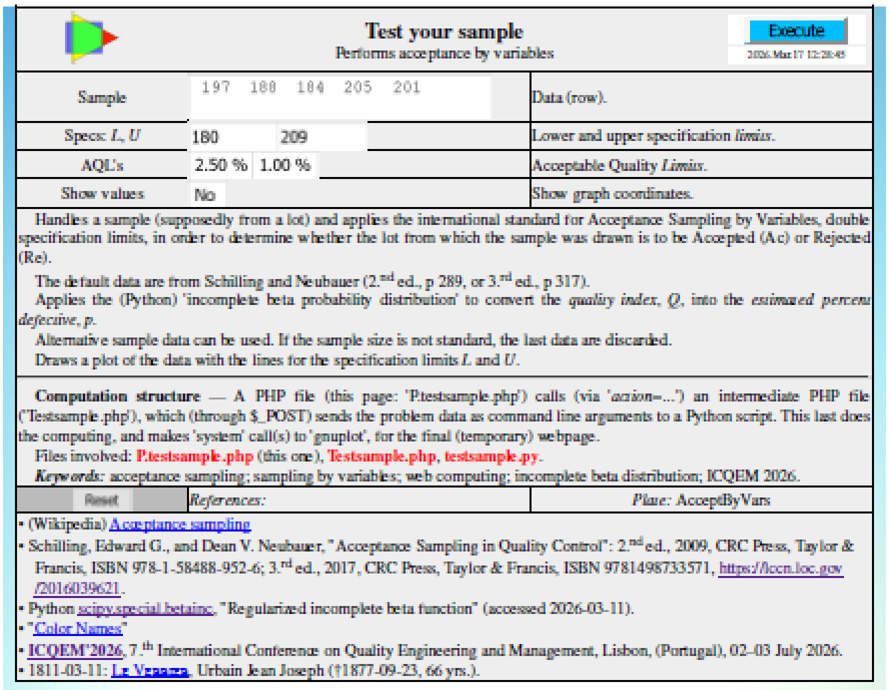

Fig. 6. Web page to test a sample in AS by variables (Form 2).

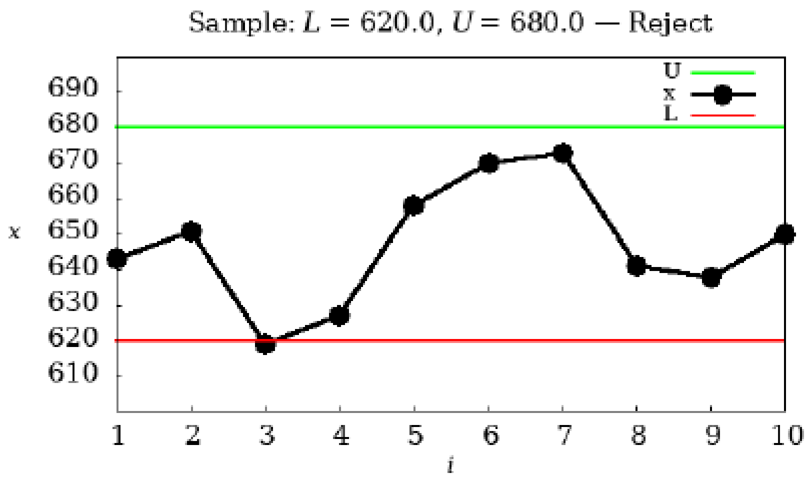

Fig. 7. Plot of the example sample, with given specification limits, *L* and *U*.

B. *Quasi-superfluous tables*

In the expression "quasi-superfluous tables". we intermix two cases: tables that make sense for the easy use of a standard; and extensive tables that simple mathematical expressions can replace or, at least, accompany.

To illustrate the idea above, we recover the example given in Table 1. To confirm our objective about the nonnecessity of certain tables, either superfluous or quasi-superfluous, we will use 'a)' and 'b)' to obtain 'c)':

a) *k* in 'Table B-1' (Fig. 4)
b) Eq. (1) or, equivalently, (2) or the 'scipy.special.betainc' in the code in III.B

c) *M* in 'Table B-3' (Fig. 3)

Thus, from the example, let us take sample size letter F (n = 10) and AQL = 1%. It is k = 1.72 (if rounded from 1.715) from 'Table B-1' (Fig. 4). Using, e.g., Excel, as in (2), we get M = 3.261%, which rounds to 3.26%, reproducing the value in Table 3. For the other k = 1.41 (if rounded from 1.409), we get M = 7.285%, rounding to 7.29%, reproducing again the value in the table. Now, by using the conversion in the verbose 'Table B-5' (Fig. 5), we read, respectively, 3.21% and 7.27%, which (before rounding) coincide with the above.

In sum, we did the verification as above for a region of the tables of the k values (Table B-1', Fig. 4), and of the M values (Table B-3', Fig. 3). The result is on Fig. 8. With one exception, n = 5, AQL = 1%, where the k corresponding to M does not round correctly (1.52 to replace 1.53), all the values of M are, indeed, the direct transformation we mentioned in (1) or (2), according to our affirmations in the article.

	k					M				
n \ AQL	0,65%	1%	1,50%	2,50%	4%	0,65%	1%	1,50%	2,50%	4%
5	1,65	1,53	1,40	1,24	1,07					
	1,64630	1,52494	1,40230	1,23660	1,06800	1,33%	3,33%	5,83%	9,80%	14,39%
7	1,75	1,62	1,50	1,33	1,15					
	1,7470	1,6210	1,4956	1,3258	1,1510	2,14%	3,55%	5,35%	8,40%	12,25%
10	1,84	1,72	1,58	1,41	1,23					
	1,8400	1,7150	1,5810	1,4090	1,2335	2,17%	3,26%	4,77%	7,29%	10,54%
15	1,91	1,79	1,65	1,47	1,3					
	1,9148	1,7870	1,6550	1,4745	1,2965	2,11%	3,05%	4,31%	6,56%	9,46%
20	1,96	1,82	1,69	1,51	1,33					
	1,9570	1,8230	1,6930	1,5125	1,3315	2,05%	2,95%	4,09%	6,17%	8,92%
25	1,98	1,85	1,72	1,53	1,35					
	1,9840	1,8495	1,7162	1,5344	1,3515	2,00%	2,86%	3,97%	5,97%	8,64%
30	2	1,86	1,73	1,55	1,36					
	2,0000	1,8630	1,7292	1,5467	1,3640	1,98%	2,83%	3,91%	5,86%	8,47%
35	2,03	1,89	1,76	1,57	1,39					
	2,0300	1,8905	1,7580	1,5740	1,3885	1,87%	2,68%	3,70%	5,57%	8,10%
40	2,03	1,89	1,76	1,57	1,39					
	2,03480	1,89100	1,75950	1,57497	1,39050	1,88%	2,71%	3,72%	5,58%	8,09%
50	2,08	1,93	1,80	1,61	1,42					
	2,0800	1,9330	1,7970	1,6117	1,4240	1,71%	2,49%	3,45%	5,20%	7,61%

Fig. 8. Verification of values of M from (original) k.

V. Conclusions

In this study in Quality, we addressed Acceptance Sampling (AS), the complement to Statistical Process Control, focusing on its component of AS "by variables" (vs. "by attributes") in the case of double specification limits (lower and upper). We started by the basic operations in AS by variables as stated in the applicable international standards and related the cases of single and double specification limits, the so-called Form 1 and Form 2, respectively, this latter being clearer and more intuitive, namely in the context of organizations' management.

We discussed the conversion of the *acceptance constants*, "*k*", into the *maximum allowable percent defective*, "*M*", which are associated to an extensive, verbose table (around 15 pages) in the applicable standard. We showed that at least this latter table is superfluous and its computation is direct and easy, a simple use of the incomplete beta distribution. Yet, surprisingly, the information about this function is silently absent in fundamental sources, while overloading these documents with many pages reducible to a mere mathematical expression.

In the study, besides its content, its form is to be emphasized, both through the available web page and by the concomitant insertion of hyperlinks for the verification of our computations. In the technological publications, such links, again surprisingly not found, would permit the interested reader to verify and try with other data. This methodology makes the information more useful and scientifically trustworthy, and promotes mutual advantages for Academia and Industry.

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