

# Mathematical Programming Glossary Supplement: More for Less Paradox

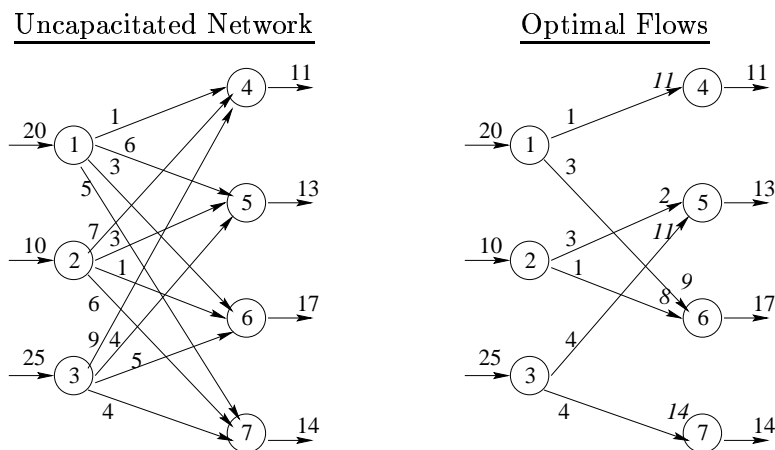
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This illustrates the *More-for-less paradox* [1]:

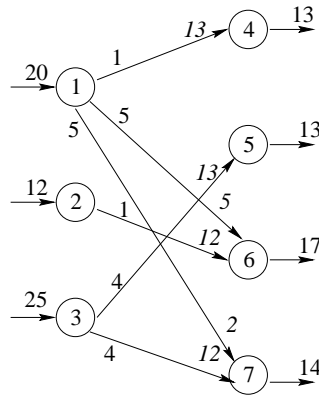
*It is possible to send more flow from supply to demand nodes at lower cost, even if all arc costs are positive.*

To illustrate, consider the following transportation network, where the arc numbers are unit costs.



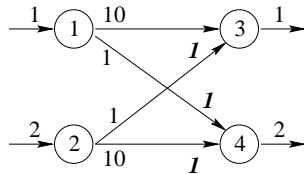
The total flow across the network is 55 units, and a minimum cost flow is shown in italics next to the demand nodes. (Missing arcs have zero flow.) What is the cost of the optimal flow? Verify that it is optimal.

Now increase the supply at node 2 and the demand at node 4, each by 2 units, so the flow increases to 57 units. However, the minimum cost flow for this new transportation is one of lower cost (determine what that total cost is):

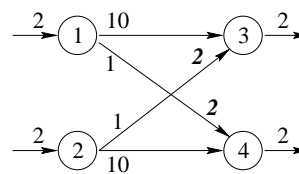


Here is a smaller example by Michael J. Hennebery (verbal communication):

Flow = 3; Min cost = \$12



Flow = 4; Min cost = \$4



The original network on the left must use the arc (2,4), which costs \$10. Upon increasing the supply at node 1 and demand at node 3 (network on right), the cross arcs can be used to reduce the total cost while sending more flow across the network.

## References

- [1] A. Charnes and D. Klingman. The “more for less” paradox in the distribution model. *Cahiers du Centre D’Etudes de Recherche Operationnelle*, 13:11–22, 1971.