$$\boxed{s^2 \overset{\text{def}}{=} \frac{1}{n-1}\sum_i \left(x_i - \bar{x}\right)^2}$$

$$(n-1)s^2 = \sum_i \left(x_i - \bar{x}\right)^2$$

$$(n-1)s^2 = \sum_i \left(x_i - \bar{x}\right)^2 = \sum_i \left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right) =$$
$$= \sum_i x_i^2 - 2\bar{x}\sum_i x_i + n\bar{x}^2$$

$$(n-1)s^2 = \sum_i x_i^2 - 2\left(\frac{1}{n}\sum_i x_i\right)\sum_i x_i + n\left(\frac{1}{n}\sum_i x_i\right)^2$$

$$(n-1)s^2 = S\left(x^2\right) - \frac{2}{n}S^2(x) + \frac{1}{n}S^2(x)$$

$$(n-1)s^2 = S\left(x^2\right) - \frac{1}{n}S^2(x)$$

$$\boxed{s^2 \overset{\text{calc}}{=} \frac{1}{n-1}\left[S\left(x^2\right) - \frac{S^2(x)}{n}\right]}$$

❖