DANTZIG, George B., and Mukund N. THAPA, 1997, "Linear Programming: 2, theory and extensions", Springer (448 pages) — pp 13–14

Definition of *simplex*

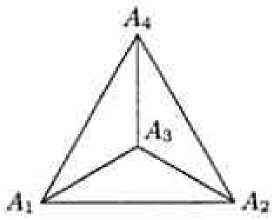

Figure 1-8: A Three-Dimensional Simplex

1.2 SIMPLEX DEFINED

There is a close connection between the Simplex Method and the the simplest higher-dimensional polyhedral set, the simplex.

> *Definition (m-Dimensional Simplex)*: In higher dimensions, say m, the convex hull of $m+1$ points in general position (see definition below) is called an *m-dimensional simplex*.

Thus

- a zero-dimensional simplex is a point;
- a one-dimensional simplex is a line segment;
- a two-dimensional simplex is a triangle and its interior;
- a three-dimensional simplex is a tetrahedron and its interior. (See Figure 1-8).

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Definition (General Position): Let $A_j = (a_{1j}, a_{2j}, \dots, a_{mj})$ be the coordinates of a point A_j in m-dimensional space. Algebraically a set of $m+1$ points $[A_1, A_2, \dots, A_{m+1}]$ of points in m dimensions is said to be in *general position* if the determinant of their coordinates and a row of ones, as in (1.34), is nonvanishing,

$$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ a_{11} & a_{12} & \cdots & a_{1,m+1} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{m,m+1} \end{vmatrix} \neq 0. \tag{1.34}$$

Definition (Algebraic Definition of an m-Dimensional Simplex): The set of all points,

$$x = \lambda_1 A_1 + \lambda_2 A_2 + \cdots + \lambda_{m+1} A_{m+1}, \tag{1.35}$$

generated by all choices of λ such that $\sum_{j=1}^{m+1} \lambda_j = 1$, $\lambda_j \geq 0$ is defined to be an m-dimensional simplex if the determinant (1.34) is nonvanishing.

Definition (Vertices of a Simplex): The points $x = A_j$ in (1.35) are called *vertices* or *extreme points* of the simplex.

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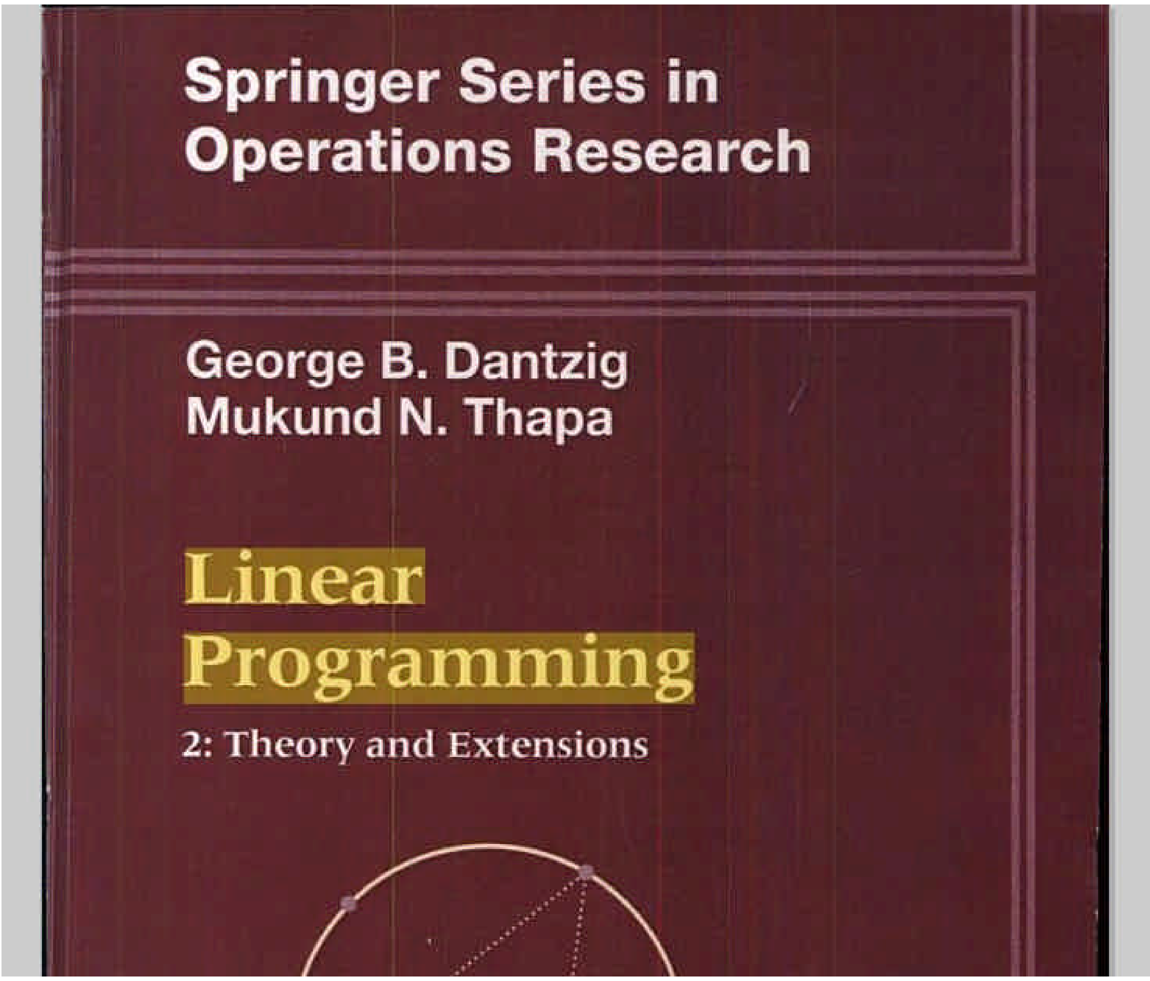
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