- can be used for sensitivity analysis, for example to see how sensitive our portfolio selection decision is to changes in the data
- the model fails to take into account any statistical uncertainly (risk) in the data, it is a completely deterministic model, for example project j might have a (known or estimated) statistical distribution for its profit P_j and so we might need a model that takes this distribution into account

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Integer programming example 1994 MBA exam

A food is manufactured by refining raw oils and blending them together. The raw oils come in two categories:

- Vegetable oil:
  - VEG1
  - VEG2
- Non-vegetable oil:
  - OIL1
  - OIL2
  - OIL3

The prices for buying each oil are given below (in £/tonne)

```
VEG1  VEG2  OIL1  OIL2  OIL3
115   128   132   109   114
```

The final product sells at £180 per tonne. Vegetable oils and non-vegetable oils require different production lines for refining. It is not possible to refine more than 210 tonnes of vegetable oils and more than 260 tonnes of non-vegetable oils. There is no loss of weight in the refining process and the cost of refining may be ignored.

There is a technical restriction relating to the hardness of the final product. In the units in which hardness is measured this must lie between 3.5 and 6.2. It is assumed that hardness blends linearly and that the hardness of the raw oils is:

```
VEG1  VEG2  OIL1  OIL2  OIL3
8.8   6.2   1.9   4.3   5.1
```

It is required to determine what to buy and how to blend the raw oils so that the company maximises its profit.

- *Formulate* the above problem as a *linear program*. (Do not actually solve it).
- What assumptions do you make in solving this problem by linear programming?

The following extra conditions are imposed on the food manufacture problem stated above as a result of the production process involved:

- the food may never be made up of more than 3 raw oils
- if an oil (vegetable or non-vegetable) is used, at least 30 tonnes of that oil must be used
- if either of VEG1 or VEG2 are used then OIL2 must also be used

Introducing 0-1 integer variables extend the linear programming model you have developed to encompass these new extra conditions.

Solution

Variables

We need to decide how much of each oil to use so let x_i be the number of tonnes of oil of type i used (i=1,...,5) where i=1 corresponds to VEG1, i=2 corresponds to VEG2, i=3 corresponds to OIL1, i=4 corresponds to OIL2 and i=5 corresponds to OIL3 and where $x_i >=0$ i=1,...,5

Constraints

- cannot refine more than a certain amount of oil

$x_1 + x_2 <= 210$
$x_3 + x_4 + x_5 <= 260$

- hardness of the final product must lie between 3.5 and 6.2

$(8.8x_1 + 6.2x_2 + 1.9x_3 + 4.3x_4 + 5.1x_5)/(x_1 + x_2 + x_3 + x_4 + x_5) >= 3.5$
$(8.8x_1 + 6.2x_2 + 1.9x_3 + 4.3x_4 + 5.1x_5)/(x_1 + x_2 + x_3 + x_4 + x_5) <= 6.2$

Objective

The objective is to maximise total profit, i.e.

maximise $180(x_1 + x_2 + x_3 + x_4 + x_5) - 115x_1 - 128x_2 - 132x_3 - 109x_4 - 114x_5$

The assumptions we make in solving this problem by linear programming are:

- all data/numbers are accurate
- hardness does indeed blend linearly
- no loss of weight in refining
- can sell all we produce

Integer program

Variables

In order to deal with the extra conditions we need to decide whether to use an oil or not so let $y_i = 1$ if we use any of oil i (i=1,...,5), 0 otherwise

Constraints

- must relate the amount used (x variables) to the integer variables (y) that specify whether any is used or not

$x_1 <= 210y_1$
$x_2 <= 210y_2$
$x_3 <= 260y_3$
$x_4 <= 260y_4$
$x_5 <= 260y_5$

- the food may never be made up of more than 3 raw oils

$y_1 + y_2 + y_3 + y_4 + y_5 <= 3$

- if an oil (vegetable or non-vegetable) is used, at least 30 tonnes of that oil must be used

$x_i >= 30y_i \quad i=1,...,5$

- if either of VEG1 or VEG2 are used then OIL2 must also be used

$y_4 >= y_1$
$y_4 >= y_2$

Objective

The objective is unchanged by the addition of these extra constraints and variables.

Integer programming example 1985 UG exam

A factory works a 24 hour day, 7 day week in producing four products. Since only one product can be produced at a time the factory operates a system where, throughout one day, the same product is produced (and then the next day either the same product is produced or the factory produces a different product). The rate of production is:

Product	1	2	3	4
No. of units produced per hour worked	100	250	190	150

The only complication is that in changing from producing product 1 one day to producing product 2 the next day five working hours are lost (from the 24 hours available to produce product 2 that day) due to the necessity of cleaning certain oil tanks.

To assist in planning the production for the next week the following data is available:

Product	Current stock (units)	Demand (units) for each day of the week 1	2	3	4	5	6	7
1	5000	1500	1700	1900	1000	2000	500	500
2	7000	4000	500	1000	3000	500	1000	2000
3	9000	2000	2000	3000	2000	2000	2000	500
4	8000	3000	2000	2000	1000	1000	500	500

Product 3 was produced on day 0. The factory is not allowed to be idle (i.e. one of the four products must be produced each day). Stockouts are not allowed. At the end of day 7 there must be (for each product) at least 1750 units in stock.

If the cost of holding stock is £1.50 per unit for products 1 and 2 but £2.50 per unit for products 3 and 4 (based on the stock held at the end of each day) formulate the problem of planning the production for the next week as an integer program in which all the constraints are linear.

Solution

Variables

The decisions that have to be made relate to the type of product to produce each day. Hence let: