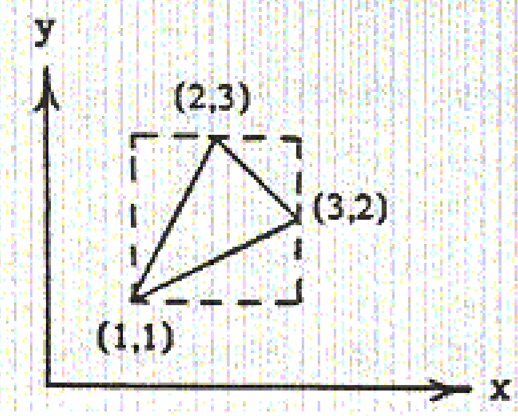
246. Triangle Area

1.5. Enclose the triangle in a square:



248. Rope Around the Earth

One day you start walking while trailing a long rope behind you. You don't stop until you get back to where you started, so the rope makes a circle around the Earth's center with a diameter of 7913 miles. Then you let out 3 more feet of rope and make another circle around the Earth. How high off the ground is the rope?

249. Rope Around the Earth (*continued*)

Now suppose you take the rope in Problem 248, with the 3 feet added, and pull it straight up away from the Earth's center until there is no slack. How high off the ground is the high point of the rope? Find the answer to the nearest foot.

The required answer is 27.4, or 5 hours.

248. Rope Around the Earth

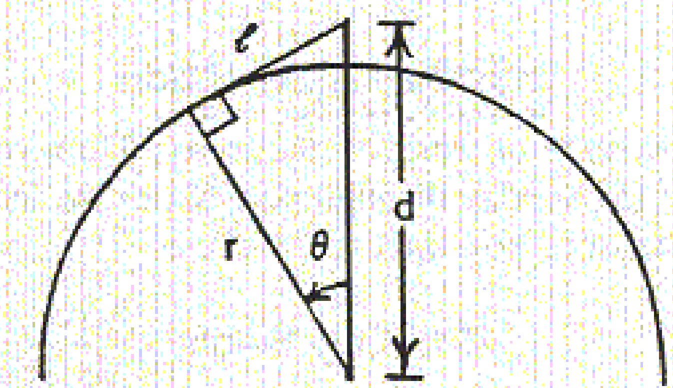
.5 feet. If r_1 is the radius of the earth, r_2 is the radius of the rope-circle, then the rope is $r_2 - r_1$ off the ground. The three feet added to the rope is the difference in the circumferences of the two circles, so we have

$$3 = 2\pi r_2 - 2\pi r_1 = 2\pi(r_2 - r_1) \Rightarrow r_2 - r_1 = \frac{3}{2\pi} = .5$$

Note that we didn't have to use the Earth's diameter of 7913 miles. In fact, the answer to the problem is the same .5 feet for a basketball or any other sphere.

249. Rope Around the Earth (*continued*)

375 feet. In the following figure, the radius of the Earth in feet is $r = .5(7913)(5280)$, and ℓ is the length of the rope from the point of tangency to the highest point:



The required height is $d - r$, and since $\cos\theta = r/d$, we have $d - r = (r/\cos\theta) - r$. The hard part is to find θ. If $x = 3$ feet is the length added the rope, then ℓ must be the arc length $r\theta$ plus $.5x$. This leads to

$$\tan\theta = \frac{\ell}{r} = \frac{r\theta + .5x}{r} = \theta + \frac{.5x}{r}$$

Unfortunately, there appears to be no way to find θ other than systematic trial and error. This can be done on APL or any other computing system that gives $\tan\theta$. With the method of bisection for finding the root of an equation, we let

$$y = \tan\theta - \theta - \frac{.5x}{r} = 0$$

Then we find a θ_1 such that $y_1 < 0$, and a θ_2 such that $y_2 > 0$. $\theta_1 = 0$ and $\theta_2 = \pi/4$ do the job. The desired θ, which gives $y = 0$, must lie between θ_1 and θ_2. We close in on the value by *bisecting* etc.

d – r = 375 ft

You will need a ladder.