Optimal price subsidies under uncertainty

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Abstract

This paper analyzes the effects of three finite-lived subsidies (fixed price, fixed premium, and minimum price guarantee policies) on investment timing and social welfare. We show how these policies can eliminate the under-investment inefficiency when considering a generic demand function with an exogenous multiplicative shock. We highlight the importance of optimally setting subsidy levels depending on the exogenous shock and demand function parameters. We thus analyze these subsidies, and the main findings are threefold. First, the optimal premium subsidy is independent of the exogenous shock. Second, the optimal fixed price subsidy is affected only by uncertainty. Lastly, the optimal minimum price guarantee changes with the drift rate and volatility.

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1 Introduction

Subsidies are popular policy instruments for incentivizing investment decisions in critical areas for society, such as renewable energy (Bigerna et al., 2019), airports (Chow et al., 2021), and roads (Shi et al., 2020). Subsidies can come in many forms. According to Schwartz and Clements (1999), there are several examples of subsidies, such as cash payments, tax reductions, and purchase of goods/services at prices above the market. The authors also state that effective subsidies must target a specific group at a minimum cost. However, there are many examples of ineffective subsidies that turn into a burden for society and lead to lower levels of social welfare. Hence, policymakers must carefully design subsidies to make them as effective as possible.

A large body of literature analyzes subsidies and their impact on society. For example, Bian et al. (2020) compares two environmental subsidies and how they can affect the investment decision of green technologies and emission-reducing technologies. The results show that consumer subsidies lead to higher social welfare when compared with manufacturer subsidies. He et al. (2019) investigate how subsidies can impact manufacturers on the selection of channel structures within supply chains. The findings show that it is possible to influence manufacturers with appropriate subsidy levels but at the expense of the environment and subsidy expenditure. Shao et al. (2017) proposed a framework for policymakers that finds the optimal subsidies or optimal price discount rates to incentivize the adoption of electric vehicles.

Another exciting avenue of research focuses on models that analyze investment decisions under uncertainty and how subsidies can influence these decisions. Under market uncertainty, Bigerna et al. (2019) derive a monopolistic firm's optimal capacity and investment timing as a function of a subsidy level. The results show that increasing a subsidy level accelerates the investment decision at the expense of a lower capacity. To circumvent this capacity reduction, they derive an optimal subsidy level that induces a firm to invest at a given target capacity, which may occur after a given deadline. In this case, policymakers offer a conditional subsidy that causes the firm to invest before the deadline. Barbosa et al. (2020) analyze the optimal investment thresholds of four different finite-lived subsidies under market and policy uncertainty. They assume that firms operate in a price-taker scenario and model policy uncertainty as an event that might reduce the subsidy level. Hence, the results show that firms anticipate the investment to obtain a higher subsidy level (before the reduction event). Azevedo et al. (2021) derive a monopolistic firm's optimal capacity and investment timing considering fixed and variable subsidies to the investment cost. In particular, they assume that a firm optimally chooses the capacity and investment timing while the government adopts a zero-cost tax-subsidy policy. The findings show that welfare can be maximized for an optimal tax rate with a zero-incremental-cost package.

However, to the best of our knowledge, no work has derived optimal finite-lived price subsidies that induce a firm to invest at the optimal social welfare. Thus, our novel contribution is to derive and analyze three optimal finite-lived price subsidies under market uncertainty, focusing on social welfare. This paper starts by examining the difference between a monopolistic firm's optimal investment timing and the social planner's optimal threshold. In the absence of a subsidy, our analytical results show that the monopolistic firm under-invests because it invests later than the social planner. We also show that all three subsidy policies can eliminate the under-investment inefficiency, considering a generic demand function with an exogenous multiplicative shock. Lastly, we derive the optimal subsidy level for each policy that induces the firm to invest at the social-optimum threshold.

In particular, the subsidies that we study are the fixed price, fixed premium, and minimum price guarantee. The fixed-price subsidy has been used in public transport (Tirachini and Antoniou, 2020) and renewable energy projects (Couture and Gagnon, 2010; Boomsma et al., 2012), which is known as the feed-in tariff. The fixed-premium subsidy has been used in renewable energy projects (Bigerna et al., 2019; Boomsma et al., 2012) and is typically called a feed-in premium. Lastly, the minimum price guarantee subsidy has been employed in projects for agriculture (Alizamir et al., 2019), renewable energy (Barbosa et al., 2018), and infrastructure with PPP agreements (Marzouk and Ali, 2018).

Although our work is closely related to Barbosa et al. (2020), Bigerna et al. (2019), and Azevedo et al. (2021), we also make distinct contributions that set us apart. In particular, Barbosa et al. (2020) also analyze the fixed price, fixed premium, and minimum price guarantee. However, they compare a firm's optimal investment threshold across different subsidies but do not analyze the optimal subsidy level from a social planner's viewpoint. Additionally, they consider a price-taker scenario, while we consider a monopolistic firm facing a generic demand function.

Bigerna et al. (2019) analyze only the fixed-premium subsidy and aim to find the optimal subsidy level. Despite the similarity, they solve a different optimization problem by finding an optimal subsidy level to obtain a target capacity and using a conditional subsidy to guarantee that the firm will invest before a deadline. In contrast, our work finds an optimal subsidy level that induces a firm to invest at a time that maximizes social welfare. While they consider only a linear demand function with an additive exogenous shock, we use a generic demand function with a multiplicative exogenous shock. Additionally, we consider finite-lived subsidies, whereas they use a perpetual subsidy.

Azevedo et al. (2021) present a welfare analysis of a firm's investment decision when considering both taxes and subsidies to investment. The results show that a social planner can maximize social welfare by choosing an appropriate tax-subsidy package. However, our findings are significantly different due to two reasons. First, they consider a fixed and variable subsidy that depends on the investment size, while our work analyzes price subsidies. Second, we analytically derive the optimal subsidy level that maximizes the social welfare, while they only show the existence of this maximum point within a numerical analysis.

While analyzing how these optimal subsidies depend on the exogenous shock and demand function, we find the following results: i) the optimal fixed-price and fixed-premium subsidy levels do not depend on demand uncertainty, ii) the optimal minimum price guarantee increases with uncertainty, iii) the optimal fixed-premium subsidy level is independent of the expected growth of the demand shock; iv) the optimal fixed price and price floor decrease with the growth rate and v) for the linear demand case, a higher slope requires larger optimal subsidy levels. These results highlight the importance of optimally setting the different subsidies' price levels and how they depend on the exogenous shock and demand function parameters.

The remaining sections of this paper are organized as follows. Section 2 presents our models, deriving the optimal investment threshold for each case with and without subsidies. Section 3 studies how an investment subsidy policy can be used to maximize social welfare. In section 4, we perform an analytical and numerical comparative statics. Finally, section 5 concludes and suggests further research.

2 The value of investment

We assume that a firm is sufficiently large and has enough market power within an industry. For instance, some electricity markets present this property, such as the Italian electricity market (Bigerna et al., 2019). In these cases, it is inappropriate to model the market with perfect competition. Hence, we consider a framework with a monopolistic firm facing an investment decision. In addition, the same framework can be used for monopolistic firms with a concession contract that includes an expansion option (Marques et al., 2021), such as the option to expand a highway.

This section aims to present a monopolistic firm's investment thresholds for different subsidies, which builds on the work from Barbosa et al. (2020) and Dixit and Pindyck (1994) while using different profit functions. We also assume that a firm has a perpetual American option to invest with a finite-lived subsidy. In addition, our model assumes that a firm operates in a market with the following generic demand function:

$$P(X_t) = X_t D(Q) \tag{1}$$

where Q is the total market output, and $X = \{X_t, t \geqslant 0\}$ is a demand shock at time t. The multiplicative shock X is the solution of the following stochastic differential equation:

$$dX_t = \alpha X_t dt + \sigma X_t dB_t \tag{2}$$

where $B = \{B_t, t \geqslant 0\}$ is the Brownian motion, $\alpha < r$ is the risk-neutral drift, r is the risk-free interest rate, and σ is the volatility. The generic demand function (1) can be substituted, for example, by a linear demand function or by an isoelastic demand function.

We analyze the firm's investment decision for three finite-lived subsidy schemes and compare them with the plain (no-subsidy) case. We use the subscript S to denote these four cases, the subscript W for the case without subsidy, the subscript F for the fixed-price policy, the subscript P for the premium-price policy, and the subscript M for the minimum price guarantee policy. The firm's profit functions for each case are the following:

- No subsidy, for which $\Pi_W(X) = P(X)Q$.
- Fixed-price scheme, for which $\Pi_F(X) = \theta Q$.
- Fixed-premium scheme, for which $\Pi_P(X) = (P(X) + \theta)Q$.
- Minimum price guarantee, with $\Pi_M(X) = \max(P(X), \theta)Q$.

When the subsidy contract ends, we consider that the firm's profit depends only on the market price and a production/output quantity Q, which we assume to be fixed over time. We believe that this is a reasonable assumption in many projects. For instance, an infrastructure project with limited capacity (e.g., roads, railways, or airports). Another example is a renewable energy project where a firm receives a subsidy for a given capacity. In these projects, the government may optimally choose the capacity, and the monopolistic firm optimizes investment timing given the capacity and subsidies.

In addition, τ is the time at which investment takes place, T is the contract duration, and π_S denotes the profit function upon investment[1][2]:

$$\pi_S(X_t) = \begin{cases} \Pi_S(X_t) & \tau \leqslant t \leqslant \tau + T \\ P(X_t)Q & t > \tau + T \end{cases} \tag{3}$$

The firm receives the following amount when it invests at time τ:

$$V_S(X) = E\left[\int_{\tau}^{\tau+T} \Pi_S(X_t)e^{-rt}dt + \int_{\tau+T}^{+\infty} X_t D(Q)Qe^{-rt}dt | X_0 = X\right] \tag{4}$$

The firm's optimization problem, whereby the firm chooses an optimal time that maximizes $V_S(X)$ net of the investment sunk cost I, is the following:

$$F_S(X) = \sup_{\tau} E\left[\int_{\tau}^{\tau+T} \Pi_S(X_t)e^{-rt}dt + \int_{\tau+T}^{+\infty} X_t D(Q)Qe^{-rt}dt - Ie^{-r\tau} | X_0 = X\right] \tag{5}$$

When X is smaller than the investment threshold X_S^*, the investment is not yet optimal. Therefore, the value function in this region is the solution of the following differential equation:

$$\alpha\, X \frac{\partial F_S(X)}{\partial X} + 0.5\sigma^2\, X^2\, \frac{\partial^2 F_S(X)}{\partial X^2} - rF_S(X) = 0 \tag{6}$$

In general, the solution of (6) is the following (Dixit and Pindyck, 1994):

$$F_S(X) = AX^{\beta_1} \tag{7}$$

Therefore, the solution of the optimization problem in 5, known as the value of the investment opportunity, takes the following (general) form:

$$F_S(X) = \begin{cases} AX^{\beta_1} & X < X_S^* \\ \\ V_S(X) - I & X \geqslant X_S^* \end{cases} \tag{8}$$

with β_1 being the positive root of the fundamental quadratic equation:

$$Q(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta + r = 0, \tag{9}$$

[1]For the sake of simplicity we ignore operational costs, that would be easy to incorporate. The main results of our work would not change.

[2]For concession contracts, the second case in 3 has to be eliminated because the concessions ends after $\tau + T$. Notice, however, that from a social planner's perspective, the project would still exist after the concession, and the only change is the optimal investment timing decision by the firm. Optimal subsidies, naturally different, inducing the firm to invest at social planner's optimal timing would still exist.

i.e.:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left(\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2} \right)^{\frac{1}{2}} > 1 \tag{10}$$

and A and X_S^* are found solving $AX_S^{*\beta_1} = V_S(X_S^*) - I$ and $\beta_1 AX_S^{*\beta_1 - 1} = V_S'(X_S^*)$ (i.e., the value matching and smooth pasting conditions).

Next, we derive the value of the investment option and the optimal investment threshold for each scenario, namely the scenario without subsidy and the three subsidy policies.

Investment without subsidy

For the plain (no-subsidy) case, the value of the project is the expected present value of the revenue stream, as follows:

$$V_W(X) = E\left[\int_0^{+\infty} X_t D(Q) Q e^{-rt} dt | X_0 = X\right] = \frac{XD(Q)Q}{r-\alpha} \tag{11}$$

Proposition 1. *The value of the investment opportunity when the firm does not receive any subsidy is:*[3]

$$F_W(X) = \begin{cases} (V_W(X_W^*) - I)\left(\frac{X}{X_W^*}\right)^{\beta_1} & X < X_W^* \\ V_W(X) - I & X \geqslant X_W^* \end{cases} \tag{12}$$

where X_W^* is the firm's optimal investment threshold:

$$X_W^* = \frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{D(Q)Q} I \tag{13}$$

Subsidy policies

For the three subsidy policies studied in this paper, Barbosa et al. (2020) derive the value functions and the investment thresholds for the case of a price-taker firm. Following similar steps, but for the case of a monopolistic firm facing the generic demand function (1), it is possible to obtain similar functions and thresholds.

[3] All proofs can be found in Appendix A.

The present value of the revenue streams for the three cases is:

$$
\begin{aligned}
V_F(X) &= E\left[\int_0^T \theta Q e^{-rt}dt + \int_T^{+\infty} X_t D(Q) Q e^{-rt} dt | X_0 = X\right] \\
&= \frac{\theta Q}{r}\left(1-e^{-rT}\right) + \frac{XD(Q)Q}{r-\alpha}e^{-(r-\alpha)T}
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
V_P(X) &= E\left[\int_0^T (X_t D(Q)+\theta) Q e^{-rt}dt + \int_T^{+\infty} X_t D(Q) e^{-rt} dt | X_0 = X\right] \\
&= \frac{\theta Q}{r}\left(1-e^{-rT}\right) + \frac{X_t D(Q)Q}{r-\alpha}
\end{aligned} \tag{15}
$$

$$
V_M(X) = V_{MP}(X) - S(X) + \frac{XD(Q)Q}{r-\alpha}e^{-(r-\alpha)T} \tag{16}
$$

where

$$
V_{MP}(P) = \begin{cases} A_1 X^{\beta_1} + \dfrac{\theta Q}{r} & X < X_\theta \\ B_2 X^{\beta_2} + \dfrac{XD(Q)Q}{r-\alpha} & X \geqslant X_\theta \end{cases} \tag{17}
$$

with $X_\theta = \theta / D(Q)$, and

$$
\begin{aligned}
S(X) = {} & A_1 X^{\beta_1} N(-d_{\beta_1}) + \frac{\theta Q}{r} e^{-rT} N(-d_0) \\
& + B_2 X^{\beta_2} N(d_{\beta_2}) + \frac{XD(Q)Q}{r-\alpha} e^{-(r-\alpha)T} N(d_1),
\end{aligned} \tag{18}
$$

with:

$$
A_1 = \frac{\theta^{1-\beta_1}}{\beta_1 - \beta_2}\left(\frac{\beta_2}{r} - \frac{\beta_2 - 1}{r-\alpha}\right) Q D(Q)^{\beta_1} \tag{19}
$$

$$
B_2 = \frac{\theta^{1-\beta_2}}{\beta_1 - \beta_2}\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{r-\alpha}\right) Q D(Q)^{\beta_2} \tag{20}
$$

$$
d_\beta = \frac{\ln X - \ln \dfrac{\theta}{D(Q)} + \left(\alpha + \sigma^2\left(\beta - \dfrac{1}{2}\right)\right) T}{\sigma\sqrt{T}}, \qquad \beta \in \{0, 1, \beta_1, \beta_2\} \tag{21}
$$

and $N(.)$ denoting the standard normal cumulative distribution, and β_2 is the negative root of the fundamental quadratic in (9):

$$
\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \left(\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}\right)^{\frac{1}{2}} < 0 \tag{22}
$$

Notice that the value of the project with a finite-lived subsidy $V_M(X)$ is equal to the value of the perpetual minimum price guarantee $V_{MP}(X)$ minus the value of the project with a delayed perpetual minimum price guarantee $S(X)$ (i.e., a minimum price guarantee that starts in the future moment T) plus the expected value of the total profit after the end of the subsidized contract.

Proposition 2. *The value of the investment opportunity under a price subsidy is:*

$$F_S(X) = \begin{cases} (V_S(X_S^*) - I)\left(\dfrac{X}{X_S^*}\right)^{\beta_1} & X < X_S^* \\ \\ V_S(X) - I & X \geqslant X_S^* \end{cases} \tag{23}$$

where $X_S^*(S \in \{F, P, M\})$ is the optimal investment threshold, respectively:

$$X_F^* = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{e^{-(r-\alpha)T}D(Q)Q}\left(I - \frac{\theta Q}{r}\left(1 - e^{-rT}\right)\right), \tag{24}$$

$$X_P^* = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{D(Q)Q}\left(I - \frac{\theta Q}{r}\left(1 - e^{-rT}\right)\right), \tag{25}$$

and X_M^* is the numerical solution of one of the two following equations:

$$\begin{cases} -(\beta_1 - \beta_2)B_2 X_M^{*\beta_2} N(d_{\beta_2}) \\ \quad +(\beta_1 - 1)\dfrac{X_M^* D(Q)Q}{r-\alpha}e^{-(r-\alpha)T}(1 - N(d_1)) \\ \quad +\beta_1\left(\dfrac{\theta Q}{r}(1 - e^{-rT}(1 - N(d_0))) - I\right) = 0 & X_M^* < X_\theta \\ \\ (\beta_1 - \beta_2)B_2 X_M^{*\beta_2}(1 - N(d_{\beta_2})) \\ \quad +(\beta_1 - 1)\dfrac{X_M^* D(Q)Q}{r-\alpha}(1 + e^{-(r-\alpha)T}(1 - N(d_1)) \\ \quad -\beta_1\left(\dfrac{\theta Q}{r}e^{-rT}(1 - N(d_0)) + I\right) = 0 & X_M^* \geqslant X_\theta \end{cases} \tag{26}$$

3 Social welfare

This section presents the key contribution of our work, whereby we derive the optimal subsidies that induce a monopolistic firm to invest at the social planner's optimal investment timing. Consequently, the optimal subsidies attain optimal total surplus (i.e, optimal social welfare), which is the sum of the producer and consumer surpluses at the social planner's optimal threshold.

We start deriving the social planner's optimal investment threshold. We then show that the firm invests inefficiently late when comparing the firm's threshold and the social planner's threshold. Lastly, we derive the optimal subsidies to induce the firm to invest at the social planner's optimal investment timing.

The instantaneous total surplus is equal to:

$$\int_0^Q X_t D(q)dq = X_t \int_0^Q D(q)dq \tag{27}$$

Hence, the expected present value of the total surplus is:

$$TS(X) = E\left[\int_0^{+\infty} X_t \left(\int_0^Q D(q)dq\right) e^{-rt}dt | X_0 = X\right] = \frac{X}{r-\alpha}\int_0^Q D(q)dq \tag{28}$$

The social planner would choose the optimal investment timing that maximizes the social welfare net of the investment cost I:

$$SW(X) = \sup_{\tau} E\left[\frac{X_\tau}{r-\alpha}\int_0^Q D(q)dq - Ie^{-r\tau}|X_0 = X\right] \tag{29}$$

Proposition 3. *The social planner's value function is:*

$$SW(X) = \begin{cases} (TS(X^*) - I)\left(\dfrac{X}{X^*}\right)^{\beta_1} & X < X^* \\ \\ TS(X) - I & X \geqslant X^* \end{cases} \tag{30}$$

where X^* is the optimal threshold:

$$X^* = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\int_0^Q D(q)dq} I = \frac{D(Q)Q}{\int_0^Q D(q)dq} X_W^* < X_W^* \tag{31}$$

A monopolistic firm invests inefficiently late in terms of social welfare ($X_W^* > X^*$). Thus, a government may use a price subsidy support scheme to induce the firm to invest sooner. We study next whether the social planner's solution is attainable.

By offering a price subsidy scheme S, with level θ, a government can induce the firm to invest at a different threshold, $X_S^*(\theta)$. The present value of the total surplus for $X < X_S^*(\theta)$ becomes:

$$TS_S(X,\theta) = \left(\frac{X_S^*(\theta)}{r-\alpha}\int_0^Q D(q)dq - I\right)\left(\frac{X}{X_S^*(\theta)}\right)^{\beta_1} \tag{32}$$

In order to maximize social welfare, the government chooses the optimal level of the price subsidy (θ_S^*), that can be found by solving the following optimization problem:

$$\sup_\theta TS_S(X,\theta) = \sup_\theta \left[\left(\frac{X_S^*(\theta)}{r-\alpha}\int_0^Q D(q)dq - I\right)\left(\frac{X}{X_S^*(\theta)}\right)^{\beta_1}\right] \tag{33}$$

This optimization problem is equivalent to (29)[4]. Therefore, the optimal level of the price subsidy is such that $X_S^*(\theta_S^*) = X^*$, and the following proposition holds:

Proposition 4. *A government can optimally choose the level of a finite-lived price subsidy (θ_S^*) to induce a monopolistic firm to invest at the social planner's optimal timing, maximizing social welfare. All subsidies optimally chosen produce the same welfare.*

When a government implements a price subsidy, it incurs a cost and transfers value to the firm. However, since investment timing changes, the consumer surplus also changes to:

$$CS_S(X,\theta) = \left(\frac{X_S^*(\theta)}{r-\alpha}\int_0^Q \left(D(q) - D(Q)\right)dq - I\right)\left(\frac{X}{X_S^*(\theta)}\right)^{\beta_1} \tag{34}$$

The value of the producer surplus is the value of the investment opportunity ($F_S(X,\theta) = F_S(X)$ in (8)). The cost of incurred by the government corresponds to the difference between

[4]See proof of Proposition 4 in Appendix A.

the total surplus and the consumer and producer surpluses:

$$PE_S(X,\theta) = TS_S(X,\theta) - CS_S(X,\theta) - F_S(X,\theta) \tag{35}$$

Proposition 5. *Solving the optimization problem in 33, or equivalently solving $X_S^*(\theta_S^*) = X^*$, the optimal subsidy level is:*

for the fixed-price policy:

$$\theta_F^* = \left(1 - e^{-(r-\alpha)T} \frac{D(Q)Q}{\int_0^Q D(q)dq}\right) \frac{I\,r}{(1-e^{-rT})\,Q}; \tag{36}$$

for the premium-price policy:

$$\theta_P^* = \left(1 - \frac{D(Q)Q}{\int_0^Q D(q)dq}\right) \frac{I\,r}{(1-e^{-rT})\,Q}; \tag{37}$$

and for the minimum price guarantee policy obtained by solving the numerically the following equations:

$$\begin{cases} -(\beta_1-\beta_2)B_2(\theta_M^*)X^{*\beta_2}N(d_{\beta_2}(\theta_M^*)) \\ \quad +(\beta_1-1)\dfrac{X^*D(Q)Q}{r-\alpha}e^{-(r-\alpha)T}(1-N(d_1(\theta_M^*))) \\ \quad +\beta_1\left(\dfrac{\theta Q}{r}(1-e^{-rT}(1-N(d_0(\theta_M^*)))) - I\right) = 0 & X^* < \theta_M^*/D(Q) \\ (\beta_1-\beta_2)B_2(\theta_M^*)X^{*\beta_2}(1-N(d_{\beta_2}(\theta_M^*))) \\ \quad +(\beta_1-1)\dfrac{X^*D(Q)Q}{r-\alpha}(1+e^{-(r-\alpha)T}(1-N(d_1(\theta_M^*))) \\ \quad -\beta_1\left(\dfrac{\theta_M^*Q}{r}e^{-rT}(1-N(d_0(\theta_M^*))) + I\right) = 0 & X^* \geqslant \theta_M^*/D(Q) \end{cases} \tag{38}$$

4 Comparative statics

In this section, we perform a comparative static analysis of the main drivers of our model. In particular, we study the influence of some parameters on social welfare and optimal subsidy design. For the numerical study, we use the base-case parameters from Nagy et al. (2021), as shown in Table 1. The parameters were estimated from a dataset that has 214 small hydropower projects in Norway. In addition, we assume that the firm faces a linear demand function[5] with a multiplicative shock, as in Nagy et al. (2021), where:

$$D(Q) = a - bQ \tag{39}$$

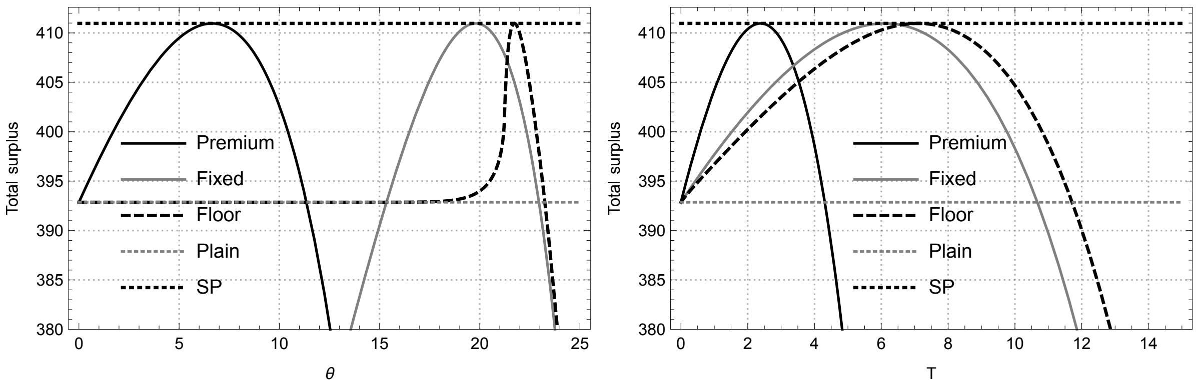
Figure 1 shows that the three subsidy policies induce the same social welfare outcome when the subsidy is set optimally (Proposition 5). For each subsidy policy, there is a maximum social welfare attained by changing the price subsidy θ (Figure 1a) or the contract duration T (Figure

[5]We have also tested the isoelastic demand function and it presents the same qualitative results.

Table 1: Base-case parameters.

Parameter	Description	Value
a	Constant in linear demand function	1.0
b	Slope of the linear demand function	0.01
α	Risk-neutral drift rate	0.02
σ	Volatility	0.05
r	Risk-free rate	0.06
X	Current level of the demand shock	10
I	Investment cost	€350/MWh
Q	Output quantity	25 MWh
θ	Price subsidy	€22.5
T	Duration of the contract	10

1b), which corresponds to the social optimum (SP). As expected, the total surplus reaches the maximum value as we increase the price subsidy or the contract duration. When a fixed price subsidy level is set too low, it can be harmful in terms of welfare, i.e., it reduces the social welfare compared to the plain no-subsidy case. For the same subsidy level, the optimal floor policy has a longer duration (T) than the fixed policy, and, obviously, longer than the premium policy.



(a) θ (b) T

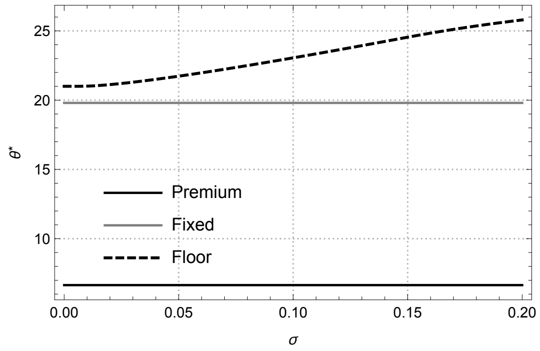
The figures plot the total surplus for the plain no-subsidy project (Plain in (32) with $S = W$ and $\theta = 0$), the social planner (SP in (30)) and the three subsidy policies for different levels of the price subsidy (Premium, Fixed and Floor in (32) with $S = P$, $S = F$, and $S = M$, respectively). Base-case parameter values as in Table 1.

Figure 1: Total surplus as a function of the subsidy parameters.

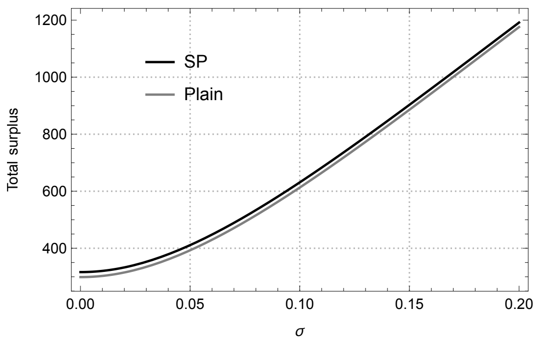
The parameters of the stochastic shock play a central role in these models. Higher uncertainty increases the investment thresholds, therefore affecting social welfare. However, after investment, only the price floor policy is affected by uncertainty. In fact, the volatility of the stochastic process influences the probability of activating the minimum price guarantee when the market price is lower than the price floor.

Proposition 6. *A higher uncertainty induces a higher optimal subsidy level of the minimum price guarantee, has no effect on the optimal fixed and premium subsidy levels, increases social welfare, and increases the optimal investment threshold.*

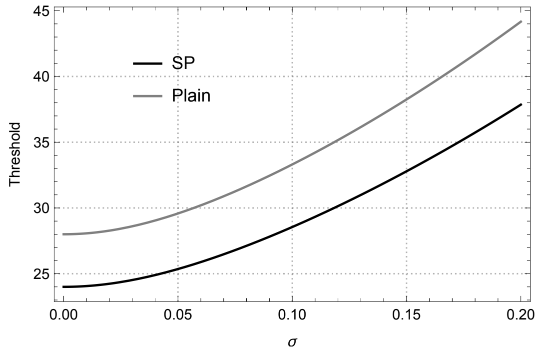
Figure 2 illustrates this proposition. For low volatility, the optimal floor level approaches the fixed price level (Figure 2a). The other two optimal subsidy levels are not affected by uncertainty, as their use is independent of the market price level. Additionally, higher uncertainty increases the investment thresholds (Figure 2c), and increases social welfare (Figure 2b). Social welfare increases due to optimal entry occurring at a higher level of the exogenous shock (resulting in a higher payoff for the social planner), which dominates the effect of a higher discount factor.



(a) Optimal subsidy



(b) Total Surplus



(c) Threshold

The optimal subsidies are found using equations in Proposition 5. The social planner's (SP) and plain's total surpluses and thresholds are in (30) and (31), and (32), with $S = W$ and $\theta = 0$, and (13), respectively. Base-case parameter values as in Table 1.

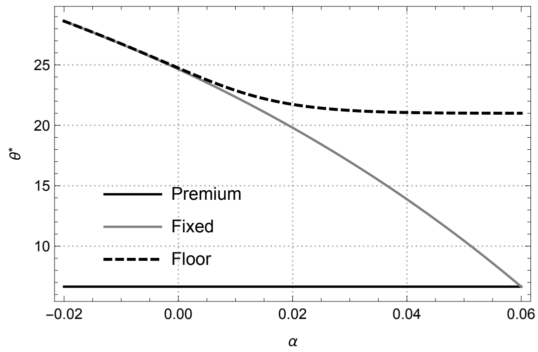
Figure 2: The effect of uncertainty on optimal policies.

Regarding the drift rate of the exogenous demand shock X, the following proposition holds:

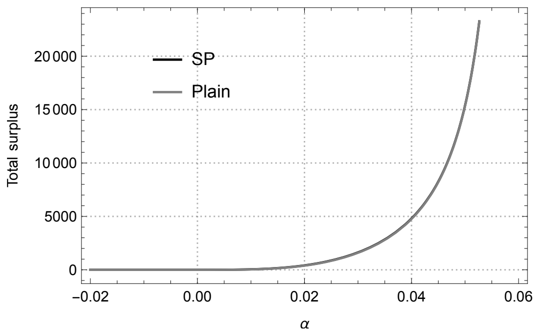
Proposition 7. *A higher drift rate of the exogenous shock induces lower optimal subsidy levels of the minimum price guarantee and fixed-price policies, has no effect on the optimal premium subsidy levels, increases social welfare, and reduces the optimal investment threshold.*

Figure 3 illustrates these results. As expected from the real options theory, we observe a reduction in the investment thresholds (Figure 3c) and an increase in option values (social welfare) (Figure 3b). The effects on the optimal subsidy levels are shown in Figure 3a. The optimal premium subsidy level is independent of the drift rate because the firm receives, besides the subsidy, the market price throughout its life (during and after the contract). For the other two subsidy policies, the firm's revenue is (for the fixed price subsidy) or might be (for the price floor) independent of the market price, which explains why the optimal subsidy levels decrease with the drift rate. Notice that the optimal fixed price level converges to the optimal premium price level as the risk-neutral drift rate converges to the risk-free interest rate ($\alpha \to r$).

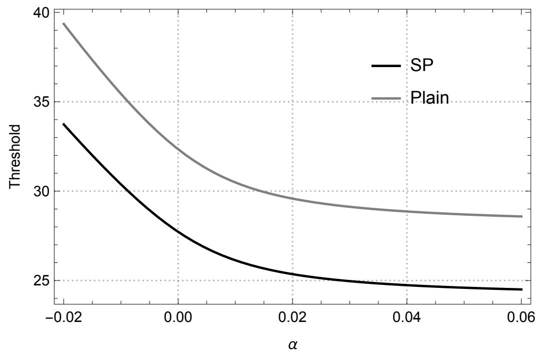
The minimum price is optimally set above the fixed price, given the possibility of escaping the limitation of selling the output for the fixed price, when only a floor price is offered.



(a) Optimal subsidy



(b) Total Surplus



(c) Threshold

The optimal subsidies are found using equations in Proposition 5. The social planner's (SP) and plain's total surpluses and thresholds are in (30) and (31), and (32), with $S = W$ and $\theta = 0$, and (13), respectively. Base-case parameter values as in Table 1.

Figure 3: The effect of the drift rate on optimal policies.

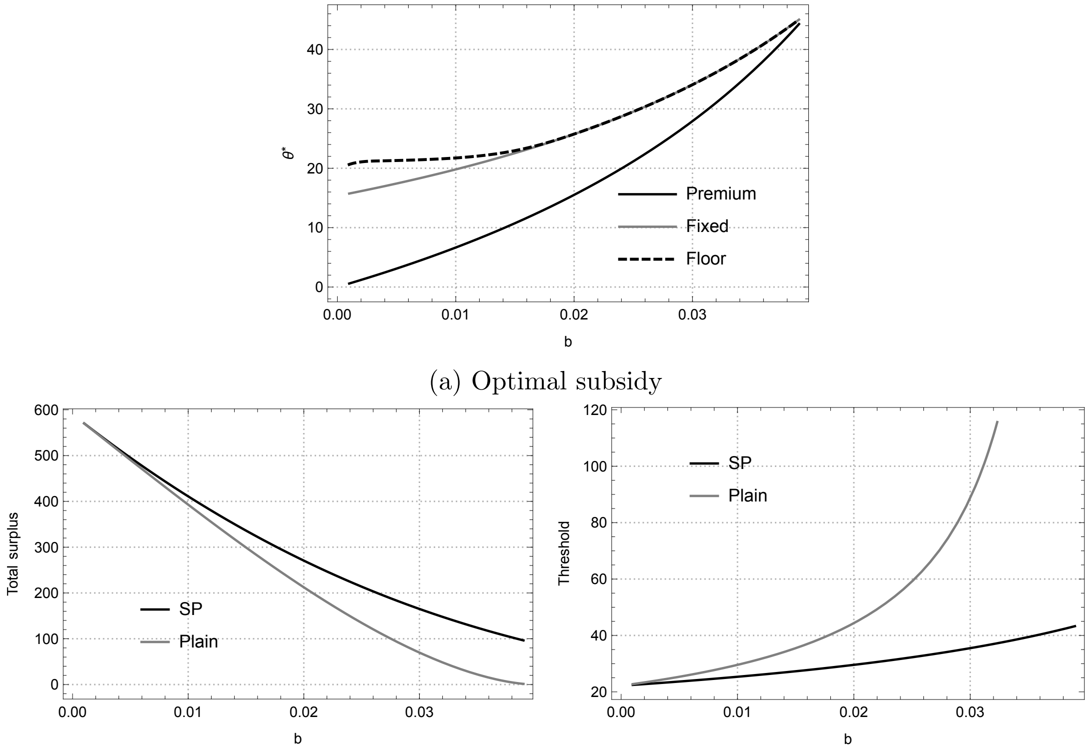
Finally, we study how the slope of the linear demand function affects optimal subsidy policies, optimal investment threshold, and social welfare. The following proposition holds:

Proposition 8. *A higher slope of the demand function induces higher optimal subsidy levels, higher optimal investment thresholds, and reduces social welfare.*

In Figure 4, we see that all the three optimal subsidy levels increase with a higher slope and converge to the same level at a certain slope value (near 0.04 in Figure 4a). The slope also affects the plain and social planner thresholds, whereby a higher slope deters investment (Figure 4c). The increased under-investment problem of the monopolist requires higher subsidies (Figure 4a). Social welfare decreases with a higher slope for both the plain and social planner cases (Figure 4b).

5 Concluding remarks

This work analyzes the effect of three finite-lived subsidy policies under market uncertainty, namely a fixed price, a fixed premium, and a minimum price guarantee (price floor) offered by a government to a monopolistic firm. There are many examples of monopoly rights given in



(a) Optimal subsidy

(b) Total Surplus

(c) Threshold

The optimal subsidies are found using equations in Proposition 5. The social planner's (SP) and plain's total surpluses and thresholds are in (30) and (31), and (32), with $S = W$ and $\theta = 0$, and (13), respectively. Base-case parameter values as in Table 1.

Figure 4: The effect of the slope on optimal policies.

concessions by a government, such as infrastructure projects like airports, ports, or roads. For a generic demand function with an exogenous multiplicative shock, we show how investment timing and social welfare are affected by subsidies. Without subsidies, a monopolistic firm under-invests, because it invests too late. In contrast, a policymaker may use any of the three subsidy policies to eliminate the under-investment inefficiency. Moreover, there is an optimal subsidy level for each policy that induces the firm to invest at the optimal social threshold.

The optimal fixed and premium subsidy levels do not depend on demand uncertainty because they are independent of the price levels. The optimal price floor increases with uncertainty, while the investment threshold increases, requiring a higher price floor to effectively eliminate the under-investment inefficiency. Furthermore, the optimal premium subsidy level is independent of the expected growth of the demand shock because the firm receives the market price throughout its life (during and after the contract). In contrast, the optimal fixed price and price floor decrease with the growth rate, as a higher growth rate penalizes their value.

Possible extensions could consider optimal capacity choice by the firm, price ceilings combined with price floors, and non-monopolistic markets.

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A Proofs

Proofs of Propositions 1-3. The optimal thresholds and option values are found with the value-matching and smooth-pasting conditions. □

Proof of Proposition 4. We need to prove that the optimization problem (33) is equivalent to (29). The solution to (33) is found with:

$$\frac{\partial\, TS_S(X,\theta)}{\partial\theta} = \frac{\partial TS_S(X,\theta)}{\partial X^*}\frac{\partial X^*}{\partial\theta} = 0 \tag{40}$$

Since $\partial X^*/\partial\theta$ is not zero, the solution is found with:

$$\frac{\partial TS_S(X,\theta)}{\partial X^*} = 0, \tag{41}$$

which is the smooth-pasting that solves (29).

Since all optimal subsidies induce investment at the same threshold (X^*), they all produce the same social welfare. □

Proof of Proposition 5. θ_F^* is obtained by setting $X_F^* = X^*$, using expressions in (24) and (31). θ_P^* is obtained by setting $X_\theta^* = X^*$ using expressions in (25) and (31). Finally, equations in (38) are the same in (26) with X^* replaced for X_M^* and noting that B_2 and the d_β are functions of θ_M^*. □

Proof of Proposition 6. First note that $\partial\beta_1/\partial\sigma < 0$ (Dixit and Pindyck, 1994).

$$\frac{\partial\theta_F^*}{\partial\beta_1} = 0 \Rightarrow \frac{\partial\theta_F^*}{\partial\sigma} = \frac{\partial\theta_F^*}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma} = 0 \tag{42}$$

$$\frac{\partial\theta_P^*}{\partial\beta_1} = 0 \Rightarrow \frac{\partial\theta_P^*}{\partial\sigma} = \frac{\partial\theta_P^*}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma} = 0 \tag{43}$$

We could not obtain a proof for the sign of $\partial\theta_M^*/\partial\sigma$. However, extensive numerical simulations allowed us to conclude that it is positive.

Regarding investment timing and social welfare:

$$\frac{\partial X^*}{\partial\beta_1} = -\frac{1}{(\beta_1-1)^2}\frac{r-\alpha}{\int_0^Q D(q)dq} I < 0 \Rightarrow \frac{\partial X^*}{\partial\sigma} = \frac{\partial X^*}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma} > 0 \tag{44}$$

Notice that for $X < X^*$,

$$SW(X) = \left(\frac{X^*}{r-\alpha}\int_0^Q D(q)dq - I\right)\left(\frac{X}{X^*}\right)^{\beta_1} = \frac{I}{\beta_1-1}\left(\frac{X}{X^*}\right)^{\beta_1}. \tag{45}$$

Then:

$$\frac{\partial SW(X)}{\partial\beta_1} = \ln\left(\frac{X}{X^*}\right)\frac{I}{\beta_1-1}\left(\frac{X}{X^*}\right)^{\beta_1} < 0 \Rightarrow \frac{\partial SW(X)}{\partial\sigma} = \frac{\partial SW(X)}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma} > 0. \tag{46}$$

□

Proof of Proposition 7.

$$\frac{\partial \theta_F^*}{\partial \alpha} = -Te^{-(r-\alpha)T}\frac{D(Q)Q}{\int_0^Q D(q)dq}\frac{I\,r}{\left(1-e^{-rT}\right)Q} < 0 \tag{47}$$

$$\frac{\partial \theta_P^*}{\partial \alpha} = 0 \tag{48}$$

We could not obtain a proof for the sign of $\partial\theta_M^*/\partial\alpha$. However, extensive numerical simulations allowed us to conclude that it is negative.

Regarding investment timing and social welfare:

From (9):

$$\frac{\partial \beta_1}{\partial \alpha} = -\frac{\frac{\partial Q}{\partial \alpha}}{\frac{\partial Q}{\partial \beta_1}} = -\frac{\beta_1}{\frac{1}{2}\sigma^2(2\beta_1-1)+\alpha} < 0 \tag{49}$$

$$\frac{\partial X^*}{\partial \alpha} = -\frac{1}{(\beta_1-1)^2}\left(\beta_1(\beta_1-1)+(r-\alpha)\frac{\partial \beta_1}{\partial \alpha}\right)\frac{1}{\int_0^Q D(q)dq}I \tag{50}$$

Using (49) and (9), and given that $\beta_1 > 1$, we obtain:

$$\frac{\partial X^*}{\partial \alpha} = -\frac{1}{\beta_1-1}\left(1-\frac{\frac{1}{2}\sigma^2\beta_1+r}{\frac{1}{2}\sigma^2\beta_1^2+r}\right)\frac{1}{\int_0^Q D(q)dq}I < 0 \tag{51}$$

Additionally:

$$\frac{\partial SW(X)}{\partial \alpha} = -\frac{\beta_1}{\beta_1-1}\frac{I}{X^*}\left(\frac{X}{X^*}\right)^{\beta_1}\frac{\partial X^*}{\partial \alpha} > 0 \tag{52}$$

□

Proof of Proposition 8. For the linear demand function, $D(Q) = a - bQ$, the total surplus is:

$$\int_0^Q D(q)dq = aQ - \frac{bQ^2}{2} \tag{53}$$

$$\frac{\partial \theta_F^*}{\partial b} = e^{-(r-\alpha)T}\frac{2a}{(2a-bQ)^2}\frac{I\,r}{\left(1-e^{-rT}\right)Q} > 0 \tag{54}$$

$$\frac{\partial \theta_P^*}{\partial b} = \frac{2a}{(2a-bQ)^2}\frac{I\,r}{\left(1-e^{-rT}\right)Q} > 0 \tag{55}$$

We could not obtain proof for the sign of $\partial\theta_M^*/\partial b$. However, extensive numerical simulations allowed us to conclude that it is also positive.

Regarding investment timing and social welfare:

$$\frac{\partial X^*}{\partial b} = \frac{\beta_1}{\beta_1-1}\frac{2}{(2a-bQ)^2}(r-\alpha)I > 0 \tag{56}$$

$$\frac{\partial SW(X)}{\partial b} = -\frac{\beta_1}{\beta_1-1}\frac{I}{X^*}\left(\frac{X}{X^*}\right)^{\beta_1}\frac{\partial X^*}{\partial b} < 0 \tag{57}$$

□