Hole-burning effects in conventional and modified distributed feedback laser structures

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ABSTRACT
Above-threshold analysis of various distributed-feedback (DFB) laser structures using the transfer matrix method (TMM) is presented. Special attention is given to the possibility of obtaining single mode stability, regarding their applications in the field of coherent optical communication systems. A summary of the main laser figures of merit in modified DFB laser diodes and their variation with the injected current will be referred.

Keywords: Distributed feedback lasers, hole-burning, transfer matrix method.

1. INTRODUCTION
InGaAsP/InP DFB lasers have been known as suitable sources for high bit-rate optical communication systems due to the possibility of achieving stable conditions for single longitudinal mode (SLM) operation. In conventional DFB structures the reflectivities and phase-shifts at both facets represent the critical factors for the mode selection, mainly because these parameters are strongly influenced by the tolerances inherent to the fabrication processes 1. To avoid their influence it is quite usual to consider laser diodes with anti-reflexive coatings 2. In conventional mirrorless DFB lasers the symmetry of the structure is associated with an emission pattern that is doubly degenerate. Therefore, the SLM operation is not achieved. To overcome this problem several solutions have been adopted, generally related to more complex DFB structures where the symmetry has been broken. These modified DFB structures may include alterations in the periodicity of the corrugation, either by introducing one or several phase-shifts (PS) 3, or by considering non-uniform grating depths, such as the distributed coupling coefficient (DCC) structure 4. When properly chosen, these alterations may induce important additional improvements in the laser performance 5. For the corresponding analysis, the eigenvalue approach based on the coupled-wave theory is no longer adequate, since it will generally involve a great amount of boundary conditions to match. On the contrary, those methods based on matricial techniques 6 are very flexible, since different device structures can be represented by the same general matrix equation. In the present analysis, making use of the transfer matrix method (TMM) 7 plus the photon and carrier rate equations has carried on the above-threshold regime of lasers. The numerical model and the associated simulation procedures adopted in the calculations will be described in section 2. Investigation on the main laser figures of merit will conclude about the improvement arising from an adequate choice of the laser structural parameters. The results will be summarized in section 3, in addition to some conclusions concerning a comparative analysis of the structures under study.

2. THE TRANSFER MATRIX MODELING
Instead of trying to formulate and solve complicated analytical eigenvalue equations, the model uses the transfer matrix formalism. This means that each arbitrary one-dimensional structure may be modeled by a 2×2 complex matrix [M]:

\[ [A] = [M][B] \]  \hspace{1cm} (1)

where \([A]\) and \([B]\) are two column matrices and \([M]\) acts as a transfer function which allows the calculation of the electric field distribution along the longitudinal structure.
In the TMM formalism both elements of each of the matrices \([A]\) and \([B]\) refer to the same spatial location and correspond to the 2 counter-running waves \(u\) and \(v\) associated to the electric field distribution (Fig. 1(a)):

\[ [A] = \begin{bmatrix} u_i \\ v_i \end{bmatrix} ; \quad [B] = \begin{bmatrix} u_{i-1} \\ v_{i-1} \end{bmatrix} \]  \hspace{1cm} (2)
All boundary conditions inside the cavity are taken into account once a transfer matrix properly defines the associated interfaces. The combination of these partial matrices makes the analysis of any \( N \)-section laser cavity straightforward if we take into account that \( [M] \) is simply the product of the successive matrices related to each cell:

\[
[M] = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = [M_R][M_{cor}][M_L]
\]  

(3)

where \( [M_R] \), \( [M_{cor}] \) and \( [M_L] \) are the matrices related to the right facet, the corrugation and the left facet, respectively. The corrugation may be uniform, it may have phase-shift discontinuities or it may be related to non-uniform grating depths. The corrugation corresponds to a periodic variation of the refractive index with period \( \Lambda = 2l \), with eventual phase discontinuities of value \( \phi \).

In the present analysis the multiple phase-shift structures have \( \phi_1 \) and \( \phi_3 = \phi_1 \) symmetrically placed respective to the center of the cavity, where a phase-shift \( \phi_2 \) exists. The phase-shift position is defined by the parameter \( p_\phi = 2L_1/L \) (Fig. 1 (b)). In the DCC structure the coupling change position is defined by the parameter \( p_K = 2L_1/L \) and by the ratio \( r = K_C/K_{C2} \) (Fig. 1 (c)). For comparative analysis between different DFB structures, an average coupling coefficient \( < K_C > \) is defined for DCC lasers, according to:

\[
<K_C> = K_{C1} p_K + K_{C2} (1 - p_K)
\]  

(4)

The matrices \( [M_R] \) and \( [M_L] \) depend on their reflectivities \( R_{R,L} \) and phase shifts \( \phi_{R,L} \). The oscillation condition corresponds to the vanishing of the incoming waves \( u_L \) and \( v_R \) and leads to the following requirement:

\[
M_{22} (\delta \beta, \gamma) = 0
\]  

(6)
Equation (6) is solved by a Newton-Raphson method in the complex plane\(^7\), the solutions corresponding to the detuning \(\delta \beta\) and to the modal gain \(\gamma\) for each mode that propagates inside the cavity. The normalized gain selectivity \(\Delta \gamma L\) corresponds to the difference between the normalized gains related to the lasing mode and to the most probable side mode of the DFB laser. Another important figure of merit is the parameter \(F\) (for flatness) which quantifies the flatness of the field distribution. For a general \(N\)-section DFB laser cavity it is given by:

\[
F = \frac{1}{L} \int_{z_0}^{z_N} \left( \frac{I(z)}{I_{av}} - 1 \right)^2 \, dz
\]  

(7)

where \(I(z)\) is the longitudinal electric field distribution and \(I_{av}\) is its mean value.

Photon and carrier distributions are inter-related by the rate equations. For the stationary situation and for a given current density \(J\), the photon density is given by:

\[
n_{ph} = \frac{(J - J_{th})}{\Gamma q d [\nu g \, g_{th} - (J - J_{th}) \, \varepsilon / q d]}
\]  

(8)

where \(\varepsilon\) represents the non-linear gain coefficient, \(\Gamma\) is the optical confinement factor, \(d\) is the active layer width, \(J_{th}\) is the current density at threshold and \(g_{th}\) is the material gain at threshold. Considering a parabolic model it is given by:

\[
g_{th} = A_0 (n_{th} - n_0) - A_1 \left[ \lambda - \left( n_{th} - n_0 \right)^2 \right]
\]  

(9)

where \(A_0\) is the differential gain, \(n_0\) is the transparency carrier concentration and \(\lambda_0\) is the wavelength of the peak gain for the transparency.

Non-uniformities on photon and carrier distributions induce important changes in the refractive index according to:

\[
n_k = n_0 + \Gamma \frac{dn_r}{dn} (n_k - n_{th})
\]  

(10)

where \(n_k\) represents the refractive index of the \(k\)th cell, \(n_t\) its carrier density and \(n_{th}\) is the threshold carrier density.

The emission properties of the laser diode are altered by the non-uniformity of \(n_r\), which often leads to a quick degradation of the laser performance in the above-threshold regime. It represents the major drawback in DFB laser structures being especially disadvantageous when \(F\) is high. It is generally accepted for lasers with hundreds Angstrom length that a normalized gain selectivity greater than 0.5 will ensure a SLM operation and besides that, a flatness less than 0.05 will keep stable the SLM operation in the high power regime.

A block diagram with the procedure adopted in the calculations is shown in Fig. 2. The above-threshold regime recurs to the rate equations, and for the respective calculations, the program stops when the current achieves a maximum preset value or whenever the multi-mode operation is reached.

3. RESULTS

In this work numerical results obtained for the threshold and above-threshold regimes of an AR coated index-guided Q-1.55 InGaAsP will be presented. The laser parameters are summarized in table I.

3.1 Threshold regime

The impact of structural changes in DFB lasers will be investigated by optimizing the gain selectivity and the field uniformity when operating in the threshold regime. Multi-phase shifted DFB, distributed coupling coefficient DFB or combinations of both will be analyzed. Fig. 3 corresponds to a 3PS (\(\phi_1, \phi_2, \phi_3\)) DFB laser with \(KCL=2\). The dotted zone corresponds to all combinations of phase shifts \(\phi_1\) and \(\phi_2\) in the range \([0, \pi/2]\) satisfying the conditions \(\Delta \gamma L > 0.5\) and
When $p_\phi=0.5$, the results enhance the great flexibility in the choice of structural parameters in order to ensure a stable SLM operation.

### Table I: Parameters used in the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
<tr>
<td>Bimolecular Recombination Coefficient</td>
<td>$B$</td>
<td>$10^{-16}$</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>Auger Coefficient</td>
<td>$C$</td>
<td>$3\times10^{-41}$</td>
<td>$m^6/s$</td>
</tr>
<tr>
<td>Cavity Length</td>
<td>$L$</td>
<td>500</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>Differential Gain</td>
<td>$A_0$</td>
<td>$2.7\times10^{-20}$</td>
<td>$m^2$</td>
</tr>
<tr>
<td>Internal Losses</td>
<td>$\alpha_{int}$</td>
<td>40</td>
<td>$cm^{-1}$</td>
</tr>
<tr>
<td>Effective Refractive Index</td>
<td>$n_0$</td>
<td>3.41</td>
<td></td>
</tr>
<tr>
<td>Carrier Concentration at transparency</td>
<td>$n_0$</td>
<td>$1.5\times10^{24}$</td>
<td>$m^3$</td>
</tr>
<tr>
<td>Active Layer Width</td>
<td>$d$</td>
<td>0.3</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>Active Layer Thickness</td>
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<td>$\mu m$</td>
</tr>
<tr>
<td>Confinement Factor</td>
<td>$\Gamma$</td>
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<td></td>
</tr>
<tr>
<td>Differential index</td>
<td>$dn_0/dn$</td>
<td>$-1.8\times10^{-26}$</td>
<td>$m^3$</td>
</tr>
<tr>
<td>Group velocity</td>
<td>$v_g$</td>
<td>$3\times10^9/3.7$</td>
<td>$m.s^{-1}$</td>
</tr>
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</table>

Fig. 2: Block diagram of the simulation model.
Fig. 3: Gain margin and flatness variations with respect to phase shifts in 3PS (ϕ₁, ϕ₂, ϕ₃) DFB lasers with \( K_{CL} = 2 \) and \( p_ϕ = 0.5 \).

Fig. 4 (a) and (b) show the selectivity and flatness, respectively, versus \( r \) and \( p_K \) concerning the DCC DFB laser with \( \phi_2 = \pi/2 \) and \( (K_{CL})_{av} = 2 \). Results show that no combination of \( r \) and \( p_K \) ensure both conditions necessary for the stable SLM operation. Notice that \( r = 1 \) corresponds to a special DFB diode, generally known as quarterly wavelength shifted (QWS) DFB laser diode. Although associated with high values for the gain margins when \( \phi_2 \) is near the center of the cavity, the strong peak of the electric field in that zone restricts its use to a relatively low power operation. Despite a small modification in the value of flatness, the coupling change \( (r \neq 1) \) leads to a strong increase in the laser gain selectivity, thus favoring the possibility of single mode oscillation. Thus, the DCC laser may be seen as an improvement face to the QWS case.

Fig. 4: Gain selectivity (a) and flatness (b) versus \( p_K \) and \( r \) in a DCC DFB laser diode with a phase shift of \( \pi/2 \) at the center of the cavity.
Finally, we have studied the combined influence of multiphase shifts included in a corrugation associated with a non-uniform coupling coefficient. Fig. 5 shows the influence of the phase shift and corrugation change locations, $p_\phi$ and $p_K$, on the values of gain margin and flatness for 3PS ($\pi/3$, $\pi/2$, $\pi/3$) DCC lasers with $(K_{CL})_{av}=2$ and $r=2$. Fig. 6 shows the gain margin and flatness of 3PS ($\phi_1$, $\pi/2$, $\phi_1$) DCC laser versus $r$ and $\phi_1$ when $p_\phi=p_K=0.5$ and $(K_{CL})_{av}=2$. In both cases the dotted areas cover all the combinations that make possible a stable SLM operation.

Fig. 5: Stable SLM operation criteria for 3PS ($\pi/3$, $\pi/2$, $\pi/3$) DCC DFB diodes. $(K_{CL})_{av}=2$ and $r=2$ were assumed.

Fig. 6: Stable SLM operation criteria for 3PS ($\phi_1$, $\pi/2$, $\phi_1$) DCC DFB diodes when $(K_{CL})_{av}=2$ and $p_K=p_\phi=0.5$. 

It is worth noticing that the introduction of phase shifts in DCC structures has improved the field uniformity inside the cavity. So, best laser performances are foreseen in the high power regime.

### 3.2 Above-threshold regime

In the present work we combine the TMM model with the photon and carrier rate equations in order to evaluate the influence of the hole-burning corrections on the laser oscillation conditions and on the photon, carrier and effective refractive index longitudinal profiles. To maintain a single mode oscillation, a phase shift has been introduced at the center of the cavity in all structures. The DFB laser structures under evaluation were labeled as follows:

- **Laser I** – 3PS ($\pi/3$, $\pi/3$, $\pi/3$) DFB with $p_\phi=0.5$ and $K_C L=2$;
- **Laser II** – 3PS ($\pi/18$, $7\pi/18$, $\pi/18$) DFB with $p_\phi=0.5$ and $K_C L=2$;
- **Laser III** – QWS DFB with $K_C L=2$;
- **Laser IV** – DCC DFB with $K_C L=2$, $p_\phi=0.5$ and $r=0.5$;
- **Laser V** – 3PS ($\pi/3$, $\pi/2$, $\pi/3$) DCC DFB with $(K_C L)_av=2$, $p_\phi=p_\psi=0.5$ and $r=0.5$;
- **Laser VI** – 3PS ($\pi/18$, $\pi/2$, $\pi/18$) DCC DFB with $(K_C L)_av=2$, $p_\phi=p_\psi=0.5$ and $r=0.8$.

The laser performances are described in terms of typical laser figures of merit, namely the gain selectivity, the side mode suppression ratio (SMSR), the emitted power, the threshold current, the spectral linewidth, etc. Fig. 7 shows the gain margin variation with respect to current injection for the previously referred structures. Notice that all these lasers satisfy the condition $\Delta \gamma L>0.5$ at threshold. Nevertheless only lasers I and V satisfy both selection criteria for stable SLM operation. It is apparent from Fig. 7 that:

i) the structures associated with lower values of $F$ are less influenced by the carrier injection (compare curves I and II or V and VI). This may be obtained by adequate discontinuities along the corrugation in multiphase shift structures;

ii) the introduction of coupling changes in DCC structures may increase the threshold gain margins (compare curves III and IV), thus enlarging the current range where the single mode oscillation is made possible.

![Mode selectivity as a function of the current injection for several modified DFB laser structures.](image)

For comparative reasons the table II summarizes the results obtained with the present simulation numerical model. The evaluation of the laser spectral linewidth has only taken into account the contribution due to the spontaneous emission. This one has been calculated assuming a spontaneous emission factor $\beta_{SP}=10^{-4}$, which has been often referred as a typical value for index-guided InGaAsP lasers. The influences of photon and carrier fluctuations have been neglected.
4. CONCLUSIONS

An exhaustive investigation on the performances of modified DFB laser structures is a complex task, regarding the number of structural parameters involved. Besides, one should have always in mind that when trying to optimize the structure, the improvement of one characteristic doesn’t necessarily means (and often doesn’t) a benefit for all the others. Nevertheless, some remarks may be emphasized. First, the flexibility of the TMM approach makes it especially adequate to deal with the design of laser structures. Regarding the results, we may conclude that an adequate choice of modifications in the corrugation may be advantageously used to ensure the stability of the SLM operation with the current injection. On one hand, the inclusion of multiphase shift discontinuities may be used to flatten the electric field distribution, making the structures almost unaffected by the SHB effect. On the other hand, some coupling changes in the corrugation may improve the mode selectivity, extending the injection range suitable for single mode oscillation.

REFERENCES