

Optimizing s in an M/M/s waiting system

$$\psi = \frac{\alpha}{s\mu} \quad \{1\}$$

$$p0iQ = \frac{(s\psi)^s}{s!(1-\psi)} \quad p0iS = \sum_{n=0}^{s-1} \frac{(s\psi)^n}{n!} \quad \{2\}$$

$$p_0^{-1} = p0iQ + p0iS \quad \{3\}$$

$$L_q = p_0 \frac{s^s \psi^{s+1}}{s!(1-\psi)^2} \quad L = L_q + s\psi \quad \{4\}$$

GLOBAL cost

From s and {4b},

$$E(\text{ServerCost}) = C_s s \quad E(\text{SysWaitCost}) = C_w L \quad \{5\}$$

$$z_{\text{GLOBAL}} = E(\text{ServerCost}) + E(\text{SysWaitCost}) \quad \{6\}$$

WAIT cost

$$s_{\text{idle}} = s(1-\psi) \quad \{7\}$$

From {7} and {4a},

$$E(\text{IdleServerCost}) = C_s s_{\text{idle}} \quad E(\text{WaitCost}) = C_w L_q \quad \{8\}$$

$$z_{\text{WAIT}} = E(\text{IdleServerCost}) + E(\text{WaitCost}) \quad \{9\}$$

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