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Escaping the tragedy of the commons via directed investments

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ABSTRACT

Cooperation is ubiquitous in the world surrounding us, from bacteria to Human interactions. In Humans, cooperation is often associated with various group decisions, resulting from their complex web of interrelated interests, associations or preferences. The existence of such social structures not only opens the opportunity of having diverse behaviors depending on the individuals' social position, but also for a dynamical allocation of contributions depending on the returns obtained from each group. Here, we address these issues by studying the evolution of cooperation under Public Goods Games in the framework of Evolutionary Game Theory where cooperative players are able to distribute their donations to their liking. As a result, cooperation is greatly enhanced when the community structure is described by homogeneous graphs, as cooperators become able to support cooperative groups and retaliate against those with poor achievements by withdrawing donations from them. Whenever the underlying network becomes complex enough to add diversity to the distribution of group sizes, directed investments do not optimize the emergence of cooperation, but they do enhance its robustness against the invasion of a minority of free-riders. We define a robustness index and show that directed investments expand the robustness of cooperation by about 50%.

1. Introduction

Cooperation is a key feature of self-organized systems, occurring at all scales and levels of complexity (Hardin, 1968; Taylor, 1982; Axelrod, 1984; Trivers, 1985; Maynard-Smith and Szathmáry, 1995; Sigmund, 1995; Barrett, 2007; Sigmund, 2010). Despite this, the reasons behind its ubiquity remain an open and challenging quest in several areas of science. To address this issue, different models were built in the framework of evolutionary game theory (Maynard-Smith, 1982; Sigmund, 2010) to try to reproduce the emergence of cooperation amongst selfish individuals, using different mechanisms to achieve this goal. In this quest, the role of higher levels of individual cognition has remained elusive. In the present paper, we show that an additional layer of individual complexity may provide a major contribution to the emergence and robustness of cooperation and investigate how the evolutionary advantage of such complexity is tightly connected with the way in which the population is structured.

For this purpose, we study the N-person Prisoner's Dilemma, better known as a Public Goods Game (**PGG**) of cooperation (Hardin, 1968; Kollock, 1998; Barrett, 2007; Sigmund, 2010). **PGG**s constitute the primary tool in evolutionary game theory to investigate the emergence of cooperation in group interactions. In this game, N participants can decide to donate or not an amount to the public good. An individual is considered to be a cooperator (**C**), if she donates; otherwise she is a defector (**D**). The donations are collected in a common pot and multiplied by a factor r ($r > 1$). The resulting sum is subsequently shared equally among the members of the group independently of their contribution. Hence, in a mixed group of N individuals, refusing to contribute to a common good assures the highest individual payoff. Thus, if all participants are rational, individuals refuse to donate, falling into the Tragedy of the Commons (Hardin, 1968).

Among the many mechanisms (Nowak, 2006; West et al., 2007) suggested to avoid this negative outcome, such as repeated interactions (Trivers, 1971), reward and punishment mechanisms (Sigmund et al., 2001; Fehr and Gachter, 2002; de Quervain et al., 2004; Sigmund et al., 2010; Szolnoki and Perc, 2010), reputation systems (Nowak and Sigmund, 2005; Ohtsuki and Iwasa, 2006), voluntary participation (Brandt et al., 2006), etc., most assume large populations and a well-mixed interaction pattern in which every player interacts equally likely with everyone else. While the well-mixed limit may be valid for small populations, spatial constraints or complex networks of contacts often shape the interactions within large-scale societies. This feature has been initially addressed by

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means of regular lattices and graphs, exploring the role of space in the emergence of collective behaviors (Nowak and May, 1992; Nowak et al., 1994; Szabó and Hauert, 2002; Szabó et al., 2005; Ohtsuki et al., 2006; Szabó and Fáth, 2007; Taylor et al., 2007). More recently, our increasing understanding of real interactions structures (Doreian and Stokman, 1997; Barabási and Albert, 1999; Watts, 1999; Amaral et al., 2000; Dorogotsev and Mendes, 2003; Newman, 2003) has led to a general analysis of evolutionary dynamics in a broad range of topologies (Santos and Pacheco, 2005; Vukov and Szabó, 2005; Santos et al., 2006; Gómez-Gardeñes et al., 2007; Szabó and Fáth, 2007; Santos et al., 2008).

In a networked population, nodes represent individuals, whereas links represent shared goals, investments or exchanges. In an N-person interaction setting, neighborhoods define a network of overlapping groups (Szabó and Hauert, 2002; Santos et al., 2008), defining not only who interacts with whom, but also the universe of possible role models of each individual. With the help of this powerful and general population structure metaphor, many different communities can be modeled and the outcome of the strategies' evolution is highly dependent on the underlying topology. During the evolutionary process, every player is involved in $(k+1)$ game-interactions, where k is the number of acquaintances (neighbors) of the given player. The group interactions take place in the $(k+1)$ groups centered on the neighbors and on the focal player (see Fig. 1A). The total payoff of a player is gained from these $(k+1)$ games (Szabó and Hauert, 2002; Santos et al., 2008).

In the simplest setting (Santos et al., 2008), cooperators donate a fixed cost c to every **PGG** they participated in. However this assumption may be unrealistic in situations where players participate in a large number of interactions, as it is very unlikely that players have such a huge amount of resources at their disposal at any time. Limited resources may add the limitation that all players have the same amount to invest, which will be equally shared amongst all the groups (Santos et al., 2008; Pacheco et al., 2009). This means that cooperators donate $c/(k+1)$ to every group. This modification has a big impact when the interaction graph is heterogeneous, as we will discuss later. However for regular networks where the number of neighbors is the same for every node, this principle is equivalent to the traditional case with a rescaling of the cost c by a factor of $1/(k+1)$. Nevertheless fixing the available resources for the players raises new opportunities: what if cooperators could decide themselves how to distribute their donations amongst the groups they interact with?

Such a cooperator opens up a whole avenue of new strategies, from random ones where cooperators just randomly contribute to the different groups, to strategies where cooperators can take past events, decisions or incomes into account before deciding about the amount to contribute to each collective endeavor. Using this idea, we shall address the role played by this additional speck of complexity, and consider cooperators that donate to different groups proportionally to the income previously received from each given group (see Fig. 1B). Individuals assess how large is the share they obtained from each group and, in the next generation, they donate the corresponding fraction of c to this group. As detailed in Section 2, this strategy is reactive and inherently assumes that players can keep track of their payoffs from immediate past events, i.e., they have some kind of short-term memory. As group profits are generated solely from donations of cooperators, this strategy rewards groups with higher cooperative standards. From this point of view, this strategy can be seen as a form of direct reciprocity (Trivers, 1971) in group interactions. For this reason, we shall refer to this type of behavior as *reactive strategies*.

2. Methods

To have a clearer understanding of the results, here we give a more thorough description of the model details. Players are located on the nodes of a graph. The edges of the graph define who interacts with whom and who can imitate whom. Each individual engages in $k+1$ **PGG** games where k is the number of her neighbors. The **PGG** groups are defined by the central player and her neighbors, i.e., a given player is member of his own and his neighbors' group (Fig. 1A). Players gain their accumulated payoff from these interactions in each generation. There are two available strategies: defectors (**D**) do not donate to the public good, while cooperators (**C**) donate the cost c. For the different simulation scenarios, cooperators use slightly different strategies. In the unconditional, unconstrained case (UUC), they donate the cost c to every group they participate in. In the unconditional, constrained case (UCC), when the amount of donation per player is fixed to c, they donate to all their groups equally, i.e., all groups' pot receive $c/(k+1)$. In the conditional, reactive case (CRC), cooperators are allowed to redistribute their donations, and they donate proportional to the payoffs they received from the given group in the previous simulation step. Hence, if a cooperator received payoff p_i from his ith group at a given time (see Introduction (Section 1) and Fig. 1) then she will donate $(cp_i)/P$ in the next round, where $P=\sum_{j=1}^{k+1} p_j$. In the first round of the simulation, cooperators donate equally to the groups. The same happens if a defector imitates a cooperator and she had zero total payoff ($P=0$) in the previous round.

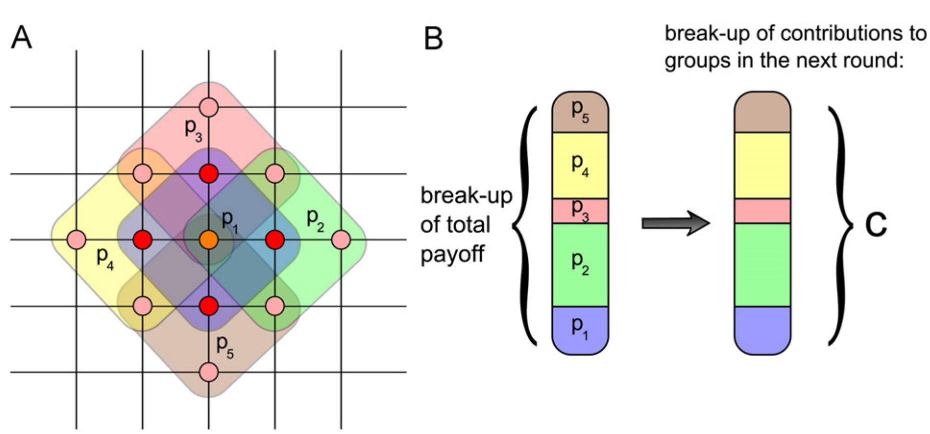

Fig. 1. Public Goods Game on graphs. (A) Players are participating in $(k+1)$ games in every generation. Colored bubbles show the PGG groups in which the central (orange) player is participating while the p_i values show the payoff she gains from the corresponding groups. (B) Reactive behavior of CRC cooperators: colored bars show the orange player's payoff-share gained from each group (with a given color) compared to the player's total payoff. In the next generation, cooperators divide their contribution to the public good according to their previous income. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Simulations start from a random initial condition where the concentration of cooperators and defectors is equal. Having different initial conditions (e.g. fewer cooperators at the start) does not really influence the stationary distributions. The average values are decreased slightly because of the cases when cooperator clusters cannot form and cooperation vanishes at the initial transitional period due to the low initial cooperator frequency. We use synchronous update: in each simulation time step, we update the payoff for every player, and then randomly pick a neighbor (y) for every player (x). Player x can adopt the strategy of player y with a probability given by the so called pair-wise comparison rule: $W(x \leftarrow y) = 1/(1+e^{(P_x - P_y)/K})$, where P_x and P_y are the total payoff of players x and y while K is characterizing the amount of errors in decision making. After calculating the possible strategy imitations, we update the strategy of every player at the same time.

The final outcome of evolution and cooperation is obtained from extensive computer simulations performed on the following network topologies: we consider paradigmatic examples of homogeneous and heterogeneous network structures. For the former class we consider a square lattice with von Neumann-neighborhood structure and periodic boundary conditions. For the latter class, we explore the effects of topological heterogeneity using Barabási–Albert scale-free networks (Barabási and Albert, 1999), generated by the combination of growth and linear preferential attachment. This leads to distributions of group sizes and number of games played by each player that follows the degree distribution of the network, i.e., a power-law. After 2000 initial generations, we average the strategy concentrations over the population during 10,000 generations. Each result is obtained from 100 runs from different random initial conditions and in the case of heterogeneous networks, from 10 different network realizations. We investigated the strong selection regime and used $K=0.04$. The qualitative behavior of the system is the same for higher K values (we tested it up to $K=1.0$), the ranking among the different strategies remains the same so the conclusions are valid for those parameter values too. In the case of the homogeneous networks, higher K shifts the threshold above which cooperation can be maintained to higher multiplication values as among these conditions defectors can break into the cooperator cluster more easily due to the higher noise. For heterogeneous networks, the thresholds are about the same but the transition from the full defector state to full cooperation is sharper, isolated, small islands of the minority strategy are consumed due to the higher noise.

3. Results and discussion

Fig. 2 shows the results for the unconditional cooperative strategy (UUC) and the reactive cooperative strategy (CRC) on the square lattice (see Fig. 1A) as a homogeneous interaction network (note that, in this case, UUC and UCC lead to the same results). The fraction of cooperators is plotted as a function of the normalized multiplicative factor $\eta = r/(k+1)$, where $k=4$ for the square lattice with von Neumann-neighborhood. In infinite, well-mixed populations, full defection is replaced by full cooperation at $\eta \geq 1$, as in this case, a single cooperator can provide positive payoff for the whole group. Under spatial reciprocity, the threshold happens for significantly lower values of η. There is also formally a lower threshold at $\eta \leq 1/(k+1)$, in which case even full cooperation results in negative payoffs, that is, cooperation becomes impossible among these conditions.

The results in Fig. 2 show that reactive cooperative strategy (CRC, red circles) successfully outcompetes defectors for a wide range of parameters when compared with unconditional cooperators (blue squares), while managing to achieve mixed, dynamical

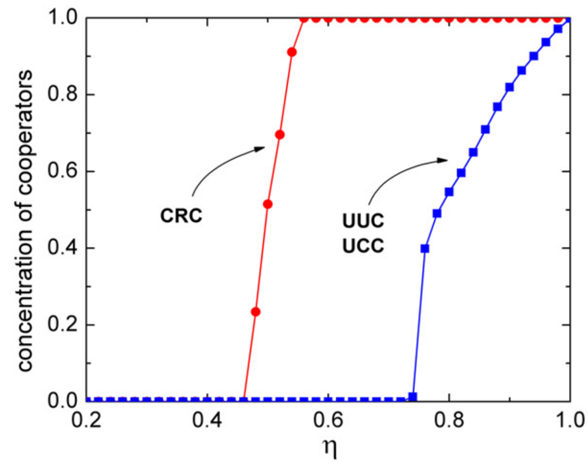

Fig. 2. Results of additional cooperator complexity on the square lattice: the fraction of cooperators as a function of the normalized multiplicative factor (η) for the different cooperator strategies. The incipient cognitive abilities of CRC cooperators make it possible to prevail under much worse conditions. Red circles show the concentration of CRC cooperators when the donations are given proportional to the payoff from the groups, while blue squares show the results of the traditional case (UUC or, equivalently, UCC), where the donations are shared equally. We used a square lattice of size $Z = 100 \times 100$ as an example of a homogeneous interaction structure, with nearest neighbor (von Neumann) interactions ($k=4$) and with periodic boundary conditions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

coexistence with them under rather unfavorable conditions. By dividing their donations among successful groups, cooperators are able to support the emergence of cooperative clusters by "directed" contributions to fellow cooperator groups. Moreover, these individuals are able to withdraw help from the most defective groups, located at the edges of the cooperative clusters. So the reactive cooperative strategy could be considered as a mix of two mechanisms: *reciprocity* towards good, generous neighbors/groups and *punishment* with the withdrawal of donations from defective neighbors. As a result, whenever the conditions are not too adverse, the cooperative clusters can grow and they will almost always take over the whole community. In the coexistence region, due to the lower multiplication factor, cooperator clusters gain and loose territories dynamically while the strategy concentrations slightly fluctuate around the average. We have investigated other regular networks, such as the kagome-lattice (Syôzi, 1951) and the one-dimensional ring-graph (Watts, 1999) to filter out possible "square lattice"-specific features but the results were qualitatively the same on all of them: the overall level of cooperation was very significantly increased with the introduction of a small level of complexity (CRC). Similar qualitative results are also obtained for other values of the intensity of selection, here associated with the parameter K.

The picture is different when the interaction graph is described by a heterogeneous network. In this case different players will have, in general, different number of neighbors. Consequently, fixing the maximum amount of contribution for unconditional cooperators is no longer a mere rescaling issue—in fact, it has a huge impact on the final outcome of evolution (Santos et al., 2008; Pacheco et al., 2009). Blue squares in Fig. 3 show the results on a scale-free network when cooperators donate c to every group they are part of (UUC). Cooperation becomes viable in a considerably wide range even if cooperators in a central role have to invest a big amount. The key of the success of cooperation is that the complex interaction network made the payoff distribution heterogeneous and this gave an opportunity for cooperators to outplay defectors: central cooperators can collect a high income due to the many groups they are part of and can turn most of their neighbors to cooperators while if a defector ends up in a central role, she will turn her followers to defectors, decreasing her own payoff and after a while losing the "leading" position. However for lower η, the establishment of cooperation is hindered by the fact that highly connected individuals (hubs) have to invest a huge amount, which can be non-remunerative in a partly defective environment. Fixing the total amount of donation of each individual (UCC) can be of assistance to this problem, as shown with the green triangles. Indeed, in (Santos et al., 2008; Pacheco et al., 2009) it has been argued that it is not the amount given what is important but the act of giving.

Unlike the situation observed in homogeneous networks, the introduction of reactive cooperators (CRC) does not boost cooperation further (red circles in Fig. 3). Apparently, the additional complexity in the strategy does not add up to the effects already induced by the scale-free interaction network, associated with the heterogeneous payoff distribution In other words, the network structure may by itself dispense the need to develop highly cognitive capabilities. Differently, a heterogeneous allocation of

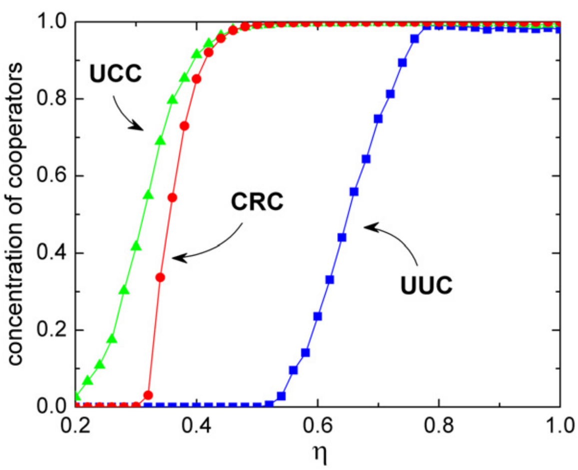

Fig. 3. Cooperation level with scale-free networks as interaction structures. The different symbols show the fraction of cooperators with different model setups as a function of the normalized multiplicative factor (η). For heterogeneous, scale-free networks, η is calculated using the average connectivity ($\langle k \rangle = 4$) of the graph. Blue squares stand for the UUC cooperators, green triangles for the UCC cooperators, while red circles indicate the CRC cooperators. The definition of the strategies is given in the Methods. Scale-free networks (size of $Z=1000$ and average degree of $\langle k \rangle = 4$) were generated using the Barabási–Albert algorithm (Barabási and Albert, 1999). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

donations may open a window to smarter ways of cheating in a heterogeneous network context.

On the other hand one expects that additional skills may enhance the aptitude of Cooperators to protect themselves against Defectors, which may play a different role depending on the composition of the population. Up to now we have been discussing the viability of cooperation to emerge from an equal distribution of cooperators and defectors. But how stable is cooperation once established? In the following we investigate the robustness of the cooperative community against defector invasion attempts. To this end, and after an initial transient period, in every generation we replace a given amount of the population by defectors. It turns out that reactive cooperators can withstand defector attacks far more successfully. Fig. 4 shows that even for extreme defector inflow values as high as 10% of the population in every generation, cooperation survives with the help of this more sophisticated strategy. We can define a numerical index to compare the robustness of cooperation in different scenarios by calculating the integral below the surfaces in Fig. 4. The ratio of the integrals $\Omega_{CRC}/\Omega_{UCC}=1.46$ shows that the reactive cooperative strategy (CRC) is almost 1.5 times more successful in defending itself than the unconditionally equal cooperators (UCC).

Finally, it is also noteworthy that different cognitive skills and levels of complexity can have an impact in several emerging features of the population beyond the levels of cooperation. In Fig. 5 we portray the wealth (here understood as fitness) distribution of the population in a fully cooperative community, that is, we compute how the total income is divided among the individuals. It is known (Santos et al., 2008) that donating a fixed cost per individual results in less poor and more rich people than in the case of donating a fixed cost per game. With the advent of reactive cooperators, society becomes more "fair", individuals are shifted from the poor regions to the "middle class". This can be also shown by the Gini coefficient G (Gini, 1912), which measures inequality of a distribution ($G=0$ for maximum equality and $G=1$ for total inequality): G is 0.30 for the reactive cooperators (CRC) and 0.38 for the UCC. The few, very poor individuals are victims of the randomly built scale-free interaction network: they belong to an unfortunate neighborhood that condemns them to lower payoffs.

4. Conclusions

We investigated the emergence of cooperation in Public Goods Games from the point of view of individual complexity. We found that increasing the complexity of the cooperator strategy can help

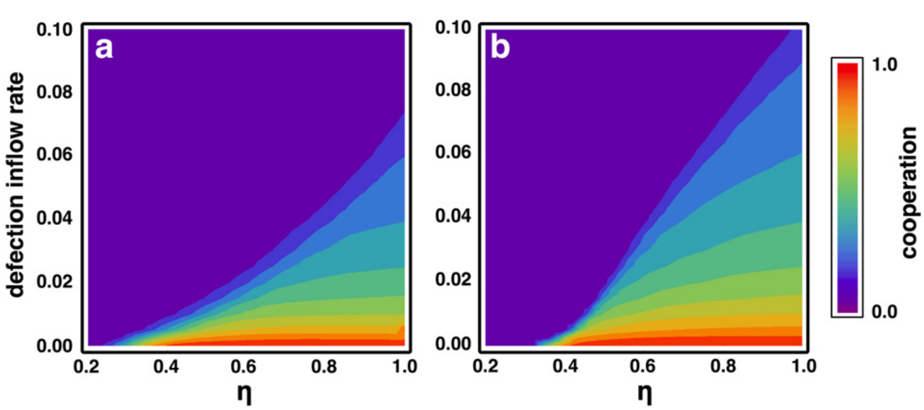

Fig. 4. Robustness against defector inflow on scale-free networks. Color codes show the cooperator concentration for different defector inflow rates as a function of η. After a transient period, a given amount of players were randomly replaced by defectors in every generation and the concentration values were calculated during the subsequent 10^4 generations under continuous defector inflow. The left panel (4A) displays the results for the case when cooperators share their donations equally (UCC), while the right panel (4B) shows them for the more complex, reactive cooperator strategy (CRC). CRCs are more robust against the invasion of a minority of free-riders, specially for large η and defectors inflow rates. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

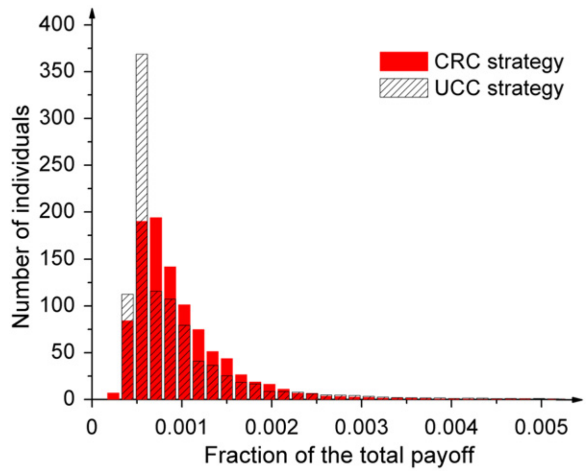

Fig. 5. Wealth distribution in fully cooperative populations for the different cooperative strategies. Red bars stand for the cognitive cooperator strategy (CRC) while striped bars show the distribution for the "equal" cooperators (UCC). CRC cooperators lead to less poor and more rich individuals. Both distributions were obtained from an average over 10 different network realizations with a size of $Z=10^3$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

to establish and maintain cooperation in different environments. When the interaction network is homogeneous, described by regular graphs, the introduction of a more complex, reactive cooperator strategy (CRC) helped to improve the performance of cooperators to a great extent. However if the interaction network itself is complex and heterogeneous, as in the case of a scale-free graph, then the additional complexity in strategy (CRC) does not positively take effect on the spreading range but renders established cooperation more robust against defector invasion.

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