Supplemental Material (SM)

Emergence of Fairness in Repeated Group Interactions

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Summary

1. Evolutionary dynamics in infinite populations

1.1. Overview

Given that most mathematical terminology of population dynamics has been proposed for the case of infinite populations [1, 2], we shall ground our analysis of evolutionary dynamics resulting from repeated group interactions on the dynamics obtained from infinitely large populations. In section 2, this discussion will be extended to arbitrary population sizes.

Let us assume an infinitely large, well-mixed population. Groups of N individuals, sampled randomly from the population, interact in repeated **NPD**'s. There are two game strategies: AD and R_M. R_M players always contribute in the first round. Subsequently, they contribute only if at least M players did contribute in the previous round. ADs always opt for defection. We use x to denote the fraction of the population playing R_M. The expected average payoff associated with each strategy is given by [3-5]

$$\begin{aligned} f_{R_M}(x) &= \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-k-1} \Pi_{R_M}(k+1) \\ f_{AD}(x) &= \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-k-1} \Pi_{AD}(k) \end{aligned} \quad . \tag{1}$$

$\Pi_{R_M}(k)$ ($\Pi_{AD}(k)$) denotes the expected net payoff a R_M (AD) player acquires in a group with k R_M players and N-k AD players. These payoff values are defined as follows

$$\begin{aligned} \Pi_{AD}(k) &= \frac{Fkc}{N}\Big[1+\theta(k-M)\big(\langle r\rangle -1\big)\Big] \\ \Pi_{R_M}(k) &= \Pi_{AD}(k) - c\Big[1+\theta(k-M)\big(\langle r\rangle -1\big)\Big] \end{aligned} \tag{2}$$

where $\langle r\rangle$ is the average number of rounds and $\theta(x)$ the Heaviside step function ($\theta(x<0)=0$ and $\theta(x\geq 0)=1$).

The payoff of an individual measures the success of his/her strategy in the population. Successful strategies spread; the rest disappears. The replicator equation [1]

$$\dot{x} = x(1-x)(f_{R_M} - f_{AD}) \tag{3}$$

describes this dynamical process. The roots of the fitness difference $Q(x) \equiv f_{R_M}(x) - f_{AD}(x)$ determine the non-trivial *equilibria* of the replicator dynamics.

In the following section we detail the analysis of $Q(x)$ showing that it can be written as

$$Q(x) = c(\lambda - 1) + c(\langle r \rangle - 1)R(x), \tag{4}$$

with $\lambda = F/N$ and

$$R(x) = (\lambda M - 1)\binom{N-1}{M-1}x^{M-1}(1-x)^{N-M} + (\lambda - 1)\sum_{k=M}^{N-1}\binom{N-1}{k}x^k(1-x)^{N-k-1}. \tag{5}$$

We also prove several properties of the polynomial *R(x)* that allow us to come up with a detailed analysis of the roots of *Q(x)*. This analysis consists of a general part, valid for *1<M<N*, and the degenerate cases *M=1* and *M=N*.

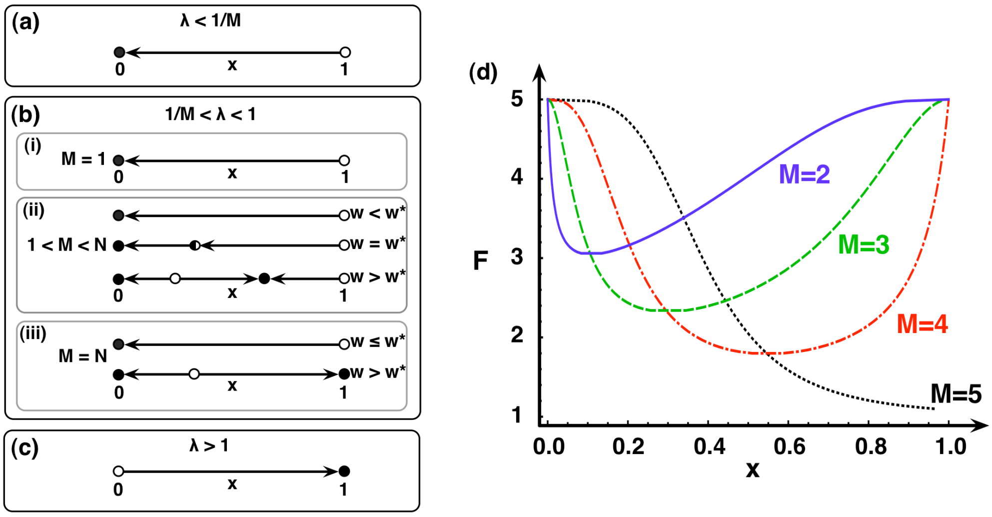

Fig. S1. Classification of all possible dynamical scenarios when evolving an infinitely large population of R_M and *AD* as a function of *M*, *N*, *w* and $\lambda = F/N$. A fraction *x* of an infinitely large population adopts the strategy R_M; the remaining fraction *1-x* adopts *AD*. The replicator equation describes the evolution of *x* over time. Solid (open) circles represent stable (unstable) *equilibria* of the evolutionary dynamics; arrows indicate the direction of selection. **(a)** Defection dominates if $\lambda < 1/M$, irrespective of the other parameters. **(b)** If $1/M < \lambda < 1$, there can be either zero, one or two interior equilibria, depending on the value of *w*, *M* and *N*. **(c)** Cooperation is dominant if $\lambda > 1$. **(d)** Each curve shows the position of the roots of the fitness difference $Q(x)$ as a function of *F* for a particular value of *M*. It illustrates the dynamical scenarios pictures on the left panels, and is qualitatively similar to those shown in Fig. 1 of the main text ($w = 0.8, N = 5$).

Fig. S1 illustrates the resulting classification of the different dynamical scenarios. If $1 < M < N$, $R(x)$ attains a maximum at $\bar{x} = \dfrac{\lambda M - 1}{\lambda N - 1}$. Furthermore, $R(x)$ increases monotonically between 0 and $\bar{x}$, and decreases monotonically between $\bar{x}$ and 1. Then: (i) for $\lambda > 1$, we have $Q(x) > 0$, for all $x \in [0,1]$; (ii) for $\lambda < 1/M$, we have $Q(x) < 0$, for all $x \in [0,1]$; (iii) for $1/M < \lambda < 1$ and $\langle r \rangle < 1 + \dfrac{1-\lambda}{R(\bar{x})} \equiv \bar{r}$, we have $Q(x) < 0$, for all $x \in [0,1]$; (iv) for $1/M < \lambda < 1$ and $\langle r \rangle = \bar{r}$, $Q(x)$ has a double root at $x = \bar{x}$; (v) for $1/M < \lambda < 1$ and $\langle r \rangle > \bar{r}$, $Q(x)$ has two simple roots $\{x_L^*, x_R^*\}$, $x_L^* \in]\,0, \bar{x}\,[$ and $x_R^* \in]\,\bar{x}, 1\,[$. x_L^* is unstable because $Q'(x_L^*) > 0$, x_R^* is stable because $Q'(x_R^*) < 0$.

If *M=N*, our analysis consists of five cases again. The first three are exactly the same as for *1<M<N*, discussed in the previous paragraph. The final two read as follows: (iv) for $1/M < \lambda < 1$ and $\langle r \rangle = \bar{r}$, $Q(1) = 0$ and $Q(x) < 0$ for all $x \in [0,1[$; (v) for $1/M < \lambda < 1$ and $\langle r \rangle > \bar{r}$, $Q(x)$ has one simple root in $x^* \in]0,1[$. $Q(x) < 0$ for $x \in [0, x^*[$ and $Q(x) > 0$ for $x \in]x^*, 1]$.

If *M=1*, R_M is essentially the same as unconditional cooperation, making the analysis independent of $\langle r \rangle$. There are only two cases: (i) for $\lambda > 1$, we have $Q(x) > 0$, for all $x \in [0,1]$; (ii) for $\lambda < 1$, we have $Q(x) < 0$, for all $x \in [0,1]$.

1.2. Detailed analysis of *Q*(x)

In section 1.1 we sketch the evolutionary dynamics of an infinitely large, well-mixed population of individuals playing the repeated *N*-person Prisoner's Dilemma (**NPD**). Here, we study in detail the direction of evolution by analyzing the fitness difference

$Q(x) \equiv f_{R_M}(x) - f_{AD}(x)$. The population is in equilibrium when $Q(x) = 0$. Evolution favors R_M (AD) players if $Q(x) > 0$ ($Q(x) < 0$). Following Equation (1), $Q(x)$ equals

$$Q(x) = \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-k-1} \left(\Pi_{R_M}(k+1) - \Pi_{AD}(k) \right). \tag{6}$$

The payoff differences in this equation are given by (see Equation (2))

$$\Pi_{R_M}(k+1) - \Pi_{AD}(k) = \Pi_{AD}(k+1) - \Pi_{AD}(k) - c\left(1 + (r-1)\theta(k+1-M)\right). \tag{7}$$

Note that we use r as a shorter notation for the average number of rounds $\langle r \rangle$. The payoff difference at the right-hand side of Equation (7) is given by

$$\Pi_{AD}(k+1) - \Pi_{AD}(k) = \lambda c + \lambda c(r-1)\left[(k+1)\theta(k+1-M) - k\theta(k-M)\right], \tag{8}$$

so that Equation (7) reduces to

$$\Pi_{R_M}(k+1) - \Pi_{AD}(k) = \begin{cases} cr(\lambda - 1) & \text{if } k > M-1 \\ \lambda c(1 + M(r-1)) - cr & \text{if } k = M-1 \\ c(\lambda - 1) & \text{if } k < M-1 \end{cases}. \tag{9}$$

This allows us to rewrite Equation (6) as follows

$$\begin{aligned} Q(x) = \quad & c(\lambda - 1) \sum_{k=0}^{M-2} \binom{N-1}{k} x^k (1-x)^{N-k-1} \\ & + c\left[\lambda(1 + M(r-1)) - r\right] \binom{N-1}{M-1} x^{M-1} (1-x)^{N-M} \\ & + cr(\lambda - 1) \sum_{k=M}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-k-1} \end{aligned}. \tag{10}$$

Since

$$\begin{aligned} 1 = \; & 1^{N-1} \\ = \; & (x + 1 - x)^{N-1} \\ = \; & \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-k-1} \\ = \; & \binom{N-1}{M-1} x^{M-1} (1-x)^{N-M} + \sum_{k=0}^{M-2} \binom{N-1}{k} x^k (1-x)^{N-k-1} + \sum_{k=M}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-k-1} \end{aligned}, \tag{11}$$

we have that

$$Q(x) = c(\lambda-1) + c(r-1)\left[(\lambda M-1)\binom{N-1}{M-1}x^{M-1}(1-x)^{N-M} + (\lambda-1)\sum_{k=M}^{N-1}\binom{N-1}{k}x^k(1-x)^{N-k-1}\right]. \tag{12}$$

By introducing the polynomial

$$R(x) \equiv (\lambda M-1)\binom{N-1}{M-1}x^{M-1}(1-x)^{N-M} + (\lambda-1)\sum_{k=M}^{N-1}\binom{N-1}{k}x^k(1-x)^{N-k-1}, \tag{13}$$

$Q(x)$ can be written as follows

$$Q(x) = c(\lambda-1) + c(r-1)R(x). \tag{14}$$

In the following section, we analyze the shape of $Q(x)$, assuming $1 < M < N$, later on followed by the analysis for the degenerate cases $M = N$ and $M = 1$. Together, these results prove the classification of all possible dynamical scenarios shown in Fig. S1.

1.2.1. Analysis for 1<M<N

The following lemma facilitates proving the main result of this section, comprised in Proposition a.1.

Lemma a.1 *Let* $1 < M < N$. *The polynomial* $R(x)$ *satisfies following properties*

1) $R(0) = 0$

2) $R(1) = \lambda - 1$

3) $R(x)$ *attains a maximum at* $\bar{x} = \dfrac{\lambda M - 1}{\lambda N - 1}$. *Moreover,* $R'(x) > 0$ *for all* $x \in \left]0, \bar{x}\right[$ *and* $R'(x) < 0$ *for all* $x \in \left]\bar{x}, 1\right[$.

Proof: The first two properties follow immediately from the definition of $R(x)$. We now prove the third property. First we rewrite $R(x)$ as follows

$$\begin{aligned} R(x) &= \lambda(M-1)\binom{N-1}{M-1}x^{M-1}(1-x)^{N-M} + (\lambda-1)\sum_{k=M-1}^{N-1}\binom{N-1}{k}x^{k}(1-x)^{N-k-1} \\ &= x^{M-1}\left[\lambda(M-1)\binom{N-1}{M-1}(1-x)^{N-M} + (\lambda-1)\sum_{k=M-1}^{N-1}\binom{N-1}{k}x^{k-M+1}(1-x)^{N-k-1}\right]. \end{aligned} \quad (15)$$

By substituting $N-k-1$ by k', we obtain

$$\begin{aligned} R(x) &= x^{M-1}\left[\lambda(M-1)\binom{N-1}{M-1}(1-x)^{N-M} + (\lambda-1)\sum_{k'=0}^{N-M}\binom{N-1}{k'}x^{N-M-k'}(1-x)^{k'}\right] \\ &= x^{N-1}\left[\lambda(M-1)\binom{N-1}{N-M}\left(\frac{1-x}{x}\right)^{N-M} + (\lambda-1)\sum_{k'=0}^{N-M}\binom{N-1}{k'}\left(\frac{1-x}{x}\right)^{k'}\right] \\ &\equiv x^{N-1}p(z) \end{aligned} \quad , \quad (16)$$

where $z = \dfrac{1-x}{x}$. The polynomial $p(z)$ is of the form $\sum_{i=0}^{N-M} a_i z^i$, with

$$\begin{aligned} a_i &= (\lambda-1)\binom{N-1}{i}, \qquad 0 \le i \le N-M-1 \\ a_{N-M} &= (\lambda M-1)\binom{N-1}{N-M} \end{aligned} \quad . \quad (17)$$

Noting that $z' = -\dfrac{1}{x^2} = -\dfrac{z+1}{x}$, we obtain the following expression for $R'(x)$

$$\begin{aligned} R'(x) &= (N-1)x^{N-2}p(z) - x^{N-2}p'(z)(z+1) \\ &= x^{N-2}\left[(N-1)p(z) - p'(z)(z+1)\right] \\ &= x^{N-2}\left[(N-1)\sum_{i=0}^{N-M}a_i z^i - \sum_{i=1}^{N-M} i a_i z^i - \sum_{i=1}^{N-M} i a_i z^{i-1}\right] \\ &= x^{N-2}\left[(N-1)a_0 - a_1 + (N-1)\sum_{i=1}^{N-M}a_i z^i - \sum_{i=1}^{N-M} i a_i z^i - \sum_{i=2}^{N-M} i a_i z^{i-1}\right] \\ &= x^{N-2}\left[(N-1)a_0 - a_1 + (N-1)\sum_{i=1}^{N-M}a_i z^i - \sum_{i=1}^{N-M} i a_i z^i - \sum_{i=1}^{N-M-1}(i+1)a_{i+1}z^i\right] \\ &= x^{N-2}\left[(N-1)\sum_{i=1}^{N-M}a_i z^i - \sum_{i=1}^{N-M} i a_i z^i - \sum_{i=1}^{N-M-1}(i+1)a_{i+1}z^i\right] \\ &\equiv x^{N-2}S(z) \end{aligned} \quad . \quad (18)$$

The polynomial $S(z)$ can be further simplified

$$\begin{aligned} S(z) &= (M-1)a_{N-M}z^{N-M} + \left[Ma_{N-M-1} - (N-M)a_{N-M}\right]z^{N-M-1} \\ &\quad + \sum_{i=1}^{N-M-2}\left[(N-i-1)a_i - (i+1)a_{i+1}\right]z^i \end{aligned} \quad . \quad (19)$$

For $1 \le i \le N-M-2$, the coefficients a_i satisfy the recurrence relation

$$\begin{aligned} a_{i+1} &= (\lambda-1)\binom{N-1}{i+1} \\ &= (\lambda-1)\frac{N-1-i}{i+1}\binom{N-1}{i}, \\ &= \frac{N-1-i}{i+1}a_i \end{aligned} \tag{20}$$

so that

$$\sum_{i=1}^{N-M-2}\left[(N-i-1)a_i-(i+1)a_{i+1}\right]z^i = 0 \tag{21}$$

and

$$\begin{aligned} S(z) &= (M-1)a_{N-M}z^{N-M}+\left[Ma_{N-M-1}-(N-M)a_{N-M}\right]z^{N-M-1} \\ &= \left[(\lambda M-1)\binom{N-1}{N-M}z-\lambda M\binom{N-1}{N-M-1}\right](M-1)z^{N-M-1} \end{aligned}. \tag{22}$$

Therefore, we can write the derivative of $R(x)$ as follows

$$\begin{aligned} R'(x) &= x^{N-2}\left[(\lambda M-1)\binom{N-1}{N-M}z-\lambda M\binom{N-1}{N-M-1}\right](M-1)z^{N-M-1} \\ &= M(M-1)\binom{N-1}{N-M-1}x^{M-1}(1-x)^{N-M-1}\left[\frac{\lambda M-1}{N-M}z-\lambda\right] \end{aligned}. \tag{23}$$

It is clear that $R'(x)$ vanishes at

$$\begin{aligned} \bar{z} &= \frac{\lambda(N-M)}{\lambda M-1} \\ &= \frac{\lambda N-1}{\lambda M-1}-1 \end{aligned}. \tag{24}$$

Since $z=\frac{1-x}{x}=\frac{1}{x}-1$, $\bar{z}$ corresponds to

$$\bar{x}=\frac{\lambda M-1}{\lambda N-1}. \tag{25}$$

Following Equation 23, $R'(x)>0$ for $z>\bar{z}$ and $R'(x)<0$ for $0<z<\bar{z}$. The function $z=\frac{1-x}{x}$ decreases monotonously and maps $]0,1[$ on $]0,\infty[$. Hence, the interval

$0 < z < \bar{z}$ corresponds to $\bar{x} < x < 1$. The region $z > \bar{z}$ corresponds to $0 < x < \bar{x}$. This proves the third property of Lemma a.1. $\square$

Proposition a.1 *Let* $1 < M < N$. $Q(x)$ *satisfies the following properties:*

1) If $1/M < \lambda < 1$, *then there is a critical number of rounds* $\bar{r} \equiv 1 + \frac{1-\lambda}{R(\bar{x})}$ *which determines the behavior of* $Q(x)$*:*

i) If $r < \bar{r}$, *then* $Q(x) < 0$ *for al* $x \in [0,1]$.

ii) If $r = \bar{r}$, *then* $Q(x)$ *has a double root at* $x = \bar{x}$.

iii) If $r > \bar{r}$, *then* $Q(x)$ *has two simple roots* $\{x_L^*, x_R^*\}$,

with $x_L^* \in]0, \bar{x}[$ *and* $x_R^* \in]\bar{x}, 1[$.

2) If $\lambda > 1$, *then* $Q(x) > 0$ *for al* $x \in [0,1]$.

3) If $\lambda < 1/M$, *then* $Q(x) < 0$ *for al* $x \in [0,1]$.

Proof:

1.i) Let $r < \bar{r}$. We obtain the following inequality by applying Lemma a.1

$$\begin{aligned} Q(x) &= c(\lambda - 1) + c(r-1)R(x) \\ &< c(\lambda - 1) + c(\bar{r}-1)R(\bar{x}) \\ &= c(\lambda - 1) + c(\bar{r}-1)\frac{1-\lambda}{\bar{r}-1}, \\ &= 0 \end{aligned} \tag{26}$$

for all $x \in]0,1[$. At the boundaries of the interval $[0,1]$, we have $Q(0) = c(\lambda - 1) < 0$ and $Q(1) = cr(\lambda - 1) < 0$. Hence, $Q(x) < 0$ for all $x \in [0,1]$.

1.ii) For $r = \bar{r}$, we have

$$\begin{aligned} Q(\bar{x}) &= c(\lambda - 1) + c(r-1)R(\bar{x}) \\ &= c(\lambda - 1) + c\frac{1-\lambda}{R(\bar{x})}R(\bar{x}). \\ &= 0 \end{aligned} \tag{27}$$

To prove that $\bar{x}$ is a double root, we show that $Q'(\bar{x}) = 0$ and $Q''(\bar{x}) \neq 0$. The existence of a root $Q'(\bar{x})$ in $\bar{x}$ follows directly from Equation 9 and Lemma a.1: $Q'(\bar{x}) = c(r-1)R'(\bar{x}) = 0$. To find the value of $Q''(\bar{x})$, we first derive an expression for $R''(x)$. We know from Equation 18 that $R'(x) = x^{N-2}S(z)$. The second derivative of $R(x)$ is therefore given by

$$R''(x) = (N-2)x^{N-3}S(z) - x^{N-3}S'(z)(z+1). \tag{28}$$

Calculating the derivative of $S(z)$ gives us

$$S'(z) = M(M-1)\binom{N-1}{N-M-1} z^{N-M-2}\left[(\lambda M - 1)z - \lambda(N-M-1)\right]. \tag{29}$$

Since $S'(\bar{z}) \neq 0$, it follows that

$$\begin{aligned} R''(\bar{x}) = & -\bar{x}^{N-3}S'(\bar{z})(\bar{z}+1) \\ \neq & \;\; 0 \end{aligned}, \tag{30}$$

which proves that $\bar{x}$ is a double root.

1.iii) For $r > \bar{r}$, we have

$$\begin{aligned} Q(\bar{x}) = & \;\; c(\lambda-1) + c(r-1)R(\bar{x}) \\ > & \;\; c(\lambda-1) + c(\bar{r}-1)R(\bar{x}). \\ = & \;\; 0 \end{aligned} \tag{31}$$

Since $Q(0) = c(\lambda-1) < 0$ and $Q(1) = cr(\lambda-1) < 0$, the Intermediate Value Theorem predicts that $Q(x)$ will have at least two roots: one root x_L^* in $\left]0,\bar{x}\right[$ and another root x_R^* in $\left]\bar{x},1\right[$. Since $R'(x)$ has only one root (see Lemma a.1), at $\bar{x}$, $Q(x)$ cannot have more than two roots. Hence, $Q(x)$ increases monotonically in $\left]0,\bar{x}\right[$ and decreases monotonically in $\left]\bar{x},1\right[$.

2) For $\lambda > 1$, $Q(x)$ is positive in 0, 1, and $\bar{x}$:

$$\begin{aligned} Q(0) = & \;\; c(\lambda-1) > 0 \\ Q(1) = & \;\; cr(\lambda-1) > 0 \\ Q(\bar{x}) = & \;\; c(\lambda-1) + c(r-1)R(\bar{x}) > 0 \end{aligned}. \tag{32}$$

Since $Q(x)$ increases monotonically in $]0,\bar{x}[$ and decreases monotonically in $]\bar{x},1[$, it follows automatically that $Q(x)>0$ for all $x\in[0,1]$.

3) For $\lambda<1/M$, then $\bar{x}<0$. Therefore, $Q(x)$ decreases monotonically between 0 and 1. Since $Q(0)=c(\lambda-1)<0$ and $Q(1)=cr(\lambda-1)<0$, it follows that $Q(x)<0$ for all $x\in[0,1]$. □

1.2.2. Analysis for M=N

Let us derive an expression for $Q(x)$, which is valid for $M=N$, starting from Equation (6). By calculating the fitness differences

$$\Pi_{AD}(k+1)-\Pi_{AD}(k)=\begin{cases}\lambda c & \text{if } k<N-1\\ \lambda c+\lambda c(r-1)N & \text{if } k=N-1\end{cases} \tag{33}$$

and

$$\Pi_{R_M}(k+1)-\Pi_{AD}(k)=\begin{cases}(\lambda-1)c & \text{if } k<N-1\\ \lambda c(1+(r-1)N)-cr & \text{if } k=N-1\end{cases}, \tag{34}$$

we arrive at the following expression for $Q(x)$:

$$\begin{aligned} Q(x) &= x^{N-1}\left[\lambda c(1+(r-1)N)-cr\right]+\sum_{k=0}^{N-2}\binom{N-1}{k}x^k(1-x)^{N-k-1}c(\lambda-1)\\ &= x^{N-1}\left[\lambda c(1+(r-1)N)-cr\right]+c(\lambda-1)-c(\lambda-1)x^{N-1}\\ &= (\lambda-1)c+c(r-1)(\lambda N-1)x^{N-1} \end{aligned}. \tag{35}$$

Proposition b.1 *Let* $M=N$. $Q(x)$ *satisfies the following properties:*

1) If $1/N<\lambda<1$, *then there is a critical number of rounds* $\bar{r}\equiv 1+\dfrac{1-\lambda}{\lambda N-1}$ *which determines the behavior of* $Q(x)$*:*

i) If $r<\bar{r}$*, then* $Q(x)<0$ *for al* $x\in[0,1]$.

ii) If $r=\bar{r}$*, then* $Q(1)=0$ *and* $Q(x)<0$ *for al* $x\in[0,1[$.

iii) If $r > \bar{r}$, then $Q(x)$ has one simple root x^* in $]0,1[$. $Q(x) < 0$ for $x \in [0,x^*[$ and $Q(x) > 0$ for $x \in]x^*,1]$.

2) If $\lambda > 1$, then $Q(x) > 0$ for al $x \in [0,1]$.

3) If $\lambda < 1/N$, then $Q(x) < 0$ for al $x \in [0,1]$.

Proof: The last two properties follow directly from Equation (35). We now prove the first property. Note that $Q(x)$ increases monotonously if $1/N < \lambda < 1$. Therefore, $Q(x)$ has one single root if and only if $Q(0) < 0$ and $Q(1) > 0$. This condition $Q(0) < 0$ is always true for $\lambda < 1$. The other condition, $Q(1) > 0$, holds in case $r > \bar{r} \equiv 1 + \dfrac{1-\lambda}{\lambda N - 1}$.

□

1.2.3. Analysis for M=1

For *M=1*, Equation (4) reduces to

$$\Pi_{R_M}(k+1) - \Pi_{AD}(k) = (\lambda - 1)cr. \tag{36}$$

Therefore,

$$\begin{aligned} Q(x) &= \sum_{k=0}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-k-1} (\lambda - 1)cr \\ &= (\lambda - 1)cr(x + 1 - x)^{N-1} \\ &= (\lambda - 1)cr \end{aligned}, \tag{37}$$

which proves the proposition below.

Proposition c.1 *Let M=1. Q(x) satisfies the following properties:*

1) If $\lambda > 1$, then $Q(x) > 0$ for al $x \in [0,1]$.

2) *If $\lambda < 1$, then $Q(x) < 0$ for al $x \in [0,1]$.*

2. Evolutionary dynamics in finite populations

2.1. Overview

We consider a well-mixed population of constant size Z. Suppose that only two strategies are present in the population: AD and R_M, M being fixed at one specific value. The expected payoff associated with each of these two strategies is given by [3, 4]

$$\begin{aligned} f_{R_M}(k) &= \binom{Z-1}{N-1}^{-1} \sum_{j=0}^{N-1} \binom{k-1}{j} \binom{Z-k}{N-j-1} \Pi_{R_M}(j+1) \\ f_{AD}(k) &= \binom{Z-1}{N-1}^{-1} \sum_{j=0}^{N-1} \binom{k}{j} \binom{Z-k-1}{N-j-1} \Pi_{AD}(j) \end{aligned} , \tag{38}$$

where k denotes the number of R_M players in the population.

Strategies evolve according to a mutation-selection process defined in discrete time. At each time step, the strategy of one randomly selected individual A is updated. With probability μ, A undergoes a mutation. He/she adopts a strategy drawn randomly from the space of available strategies, which includes the strategy AD and the N strategies R_M ($M \in \{1,...,N\}$). With probability $1-\mu$, another randomly selected individual B acts as a role model for A. The probability that A adopts the strategy of B equals $p = \left[1+e^{\beta(f_A - f_B)}\right]^{-1}$. A sticks to his/her former strategy with probability $1-p$. This update rule is known as the pairwise comparison rule [6, 7]. We used f_A and f_B to denote the fitness of individual A and B, respectively. The parameter $\beta \geq 0$, in **EGT** called the *intensity of selection*, measures the contribution of fitness to the update process. In the limit of strong selection ($\beta \to \infty$), the probability p is either zero or one, depending on f_A and f_B. In the limit of weak selection ($\beta \to 0$), p is always equal to $1/2$, irrespective of the fitness of A and B.

If the mutation probability μ is sufficiently small, the population will never contain more than two different strategies simultaneously. The time between two mutation events is usually so large that the population will always evolve to a homogeneous state, i.e., to a state in which all individuals adopt the same strategy, before the next mutation occurs. The dynamics can now be approximated by means of an embedded Markov chain whose states correspond to the different homogeneous states of the population [8-11]. Let us denote the list of available strategies as S_i ($i \in \{1,...,N+2\}$). The transition matrix $\Lambda = \left[\Lambda_{ij}\right]_{i,j=1,\ldots,N+2}$ collects the different (transition) probabilities for the population to move from one state to the other. Specifically, Λ_{ij} is the probability that a population in state S_i will end up in state S_j after the occurrence of one single mutation. This probability is given by $\Lambda_{ij} = \frac{\rho_{ji}}{N+1}$ $(j \neq i)$, where ρ_{ij} is the probability that a S_j mutant takes over a resident population of S_i individuals. The diagonal of the transition matrix is defined by $\Lambda_{ii} = 1 - \frac{1}{N+1}\sum_{\substack{k=1 \\ k \neq i}}^{N+2} \rho_{ki}$. The normalized left eigenvector associated with eigenvalue 1 of matrix Λ determines the stationary distribution, i.e, the fraction of time the population spends in each of the homogeneous states of the population [12, 13].

The fixation probability ρ_{ij} can be calculated analytically as follows. Let us assume a population with k S_i individuals and $Z\text{-}k$ S_j individuals. The probability that the number of S_i individuals increases/decreases by one is given by

$$T^{\pm}(k) = \frac{k}{Z}\frac{Z-k}{Z}\left[1 + e^{\mp\beta(f_{S_i} - f_{S_j})}\right]^{-1}, \qquad (39)$$

establishing also the probability ρ_{ij} [6, 7, 14]

$$\rho_{ij} = \left[1 + \sum_{l=1}^{Z-1}\prod_{k=1}^{l}\frac{T^{-}(k)}{T^{+}(k)}\right]^{-1}. \qquad (40).$$

Finally, it is important to highlight the difference between behavioural mutations and execution or implementation errors. In this context, the latter turns the analytical computation of the fitness values of each strategy cumbersome (see Eqs. (2) and (38)), being however easy to compute numerically. In fact, all results portrayed in Fig. 2 of the main text remain qualitatively unaltered if one adopts a small probability (ε) associated with a limited fraction of implementation errors.

2.2. Computer simulations

All individual-based computer simulations start from a randomly initialized population of size $Z = 100$. The population dynamics is implemented following the rules described above. We calculate the fitness of an individual by averaging the payoff he/she acquires in 1000 **NPD**'s with randomly selected partners. We have checked that the obtained values provide a good approximation of the actual expected payoff values, which are given by Equation (38) in case of just two strategies. The stationary distributions shown in Fig. 3 of the main text are computed as the configuration of the population averaged over the entire simulation time (10^9 iterations). We do 30 independent runs for each value of μ. The value of $\beta=0.05$ reflects a convenient choice in terms of computation time, but similar results are obtained if we adopt the selection of strength ($\beta=1.0$) used in the rest of the figures (see stationary distributions in Fig. 2 and dashed lines in Fig. 3b).

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