Evolutionary games in self-organizing populations

Arne Traulsen, Francisco C. Santos, and Jorge M. Pacheco

Abstract Social networks are dynamic: We make new friends and loose touch with old ones, depending on the interactions with them. Most analytic studies of social networks assume that links remain unchanged at all times. In this case, individuals have no control over the number, frequency or duration of their interactions with others. Here, we discuss analytical and numerical models in which individuals can break links and create new ones. Interactions are modeled as general symmetric two-player games. Once a link between two individuals has formed, the productivity of this link is evaluated. Links can be broken off at different rates. In the limiting cases where linking dynamics is much faster than evolutionary dynamics or vice-versa, the system can be tackled analytically. We show how the individual capacity of forming new links or severing inconvenient ones can change the nature of the game. If the linking rules are local, numerical simulations show that networks emerge that have several features of real-world social networks.

1 Evolutionary game dynamics

Game theory describes systems in which the success of an individual depends on the action of others. The classical approach focused on the determination of optimal

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strategic behavior of rational individuals in such a static setting [1]. Evolutionary game theory places this framework into a dynamical context [2]. Successful behaviors spread in a population, either by genetical reproduction or by social learning. The generic mathematical description of evolutionary game dynamics is the replicator equation [3, 4, 5]. This system of nonlinear ordinary differential equations describes how the relative abundances (frequencies) of strategies change over time. The expected payoff from the game is a function of the frequencies of all strategies. It is conventionally interpreted as biological fitness: Individuals reproduce proportional to their fitness. A second interpretation is cultural evolution: Successful behaviors are imitated with a higher probability, leading to the same mathematical description.

The assumption underlying the replicator equation is that individuals meet each other at random in infinitely large, well-mixed populations. But it also emerges in other cases, e.g. if the interaction rates between individuals are not random [6] or from a large-population approximation of evolutionary game dynamics in finite populations [7].

However, in reality we often do not have the same probability to interact with anyone else. Interactions occur on social networks which form the basis of such cultural dynamics. Initially, this line of research has focused on regular lattices [8, 9, 10, 11, 12]. More recently, more complex topologies derived from lattices [13, 14] and general networks have been considered in great detail [15, 16, 17, 18, 19, 20]. While the theoretical advances in this field are tremendous, there is so far a lack of experimental data. Designing and implementing such experiments has proven difficult and, so far, only general statements as "the probability to be generous is correlated with the number of social links of an individual" can be made [21]. This statement corresponds perfectly with observations of the evolutionary dynamics in theoretical models of social network dynamics [18, 20]

Observing such data from real-world systems is also problematic. One important property of social networks that is seldom addressed in theoretical studies is that real world social networks are not static. Instead, we make new friends and loose touch with old ones, depending on the kind of interaction we have with them. This makes social networks an example of an adaptive network. The basic idea is that interactions which benefit both partners last longer than interactions where one partner is exploited by the other. Here, we discuss such an approach, which leads to analytical results in certain limits. These serve as important starting points for further developments.

2 Active linking

We break down the model into two parts: Evolutionary dynamics of strategies (or behaviors) of the agents associated with nodes in a network whose links describe social interactions. The adaptive nature of the social interactions leads to a network linking dynamics. We consider a game between two strategies, A and B. The network is of constant size with N nodes. The number of links, however, is not constant and changes over time. There are N_A individuals who use strategy A and N_B individuals who use strategy B. We have $N = N_A + N_B$.

2.1 Linking dynamics

An interaction between two players occurs if there is a link between these players. Links are formed at certain rates and have specific life-times. We denote by $X(t)$ the number of AA links at time t. Similarly, $Y(t)$ and $Z(t)$ are the number of AB and BB links at time t. The maximum possible number of AA, AB and BB links is respectively given by

$$\begin{aligned} X_m &= N_A(N_A - 1)/2 \\ Y_m &= N_A N_B \\ Z_m &= N_B(N_B - 1)/2 \end{aligned}$$

Suppose A players form new links at rate α_A and B players form new links at rate α_B. Thus, AA links are formed at a rate α_A^2, AB links are formed at a rate $\alpha_A \alpha_B$ and BB links are formed at a rate α_B^2. The death rates of AA, AB and BB links are given by β_{AA}, β_{AB} and β_{BB}, respectively. Thus, the average life-times of links are $\tau_{AA} = 1/\beta_{AA}$, $\tau_{AB} = 1/\beta_{AB}$ and $\tau_{BB} = 1/\beta_{BB}$. If the number of nodes and links is large, we can model the dynamics of links by differential equations. We obtain a system of three ordinary differential equations for the number of links

$$\begin{aligned} \dot{X} &= \alpha_A^2(X_m - X) - \beta_{AA}X \\ \dot{Y} &= \alpha_A \alpha_B(Y_m - Y) - \beta_{AB}Y \\ \dot{Z} &= \alpha_B^2(Z_m - Z) - \beta_{BB}Z \end{aligned}$$

Rescaling α and β in an appropriate way (note that the equation contains squares of α and linear terms of β) does not change the fixed points of the system, but affects the overall timescale of active linking. When this process is coupled with strategy dynamics, such changes can be crucial.

While the above is probably the simplest possibility to model linking dynamics, more sophisticated choices are possible, taking for example the number of existing links of a node into account. However, to address some general properties of the coevolution between links and strategies, we concentrate on the simplest choice first. In the steady state, the number of links of the three different types is given by

$$\begin{aligned} X^* &= X_m \frac{\alpha_A^2}{\alpha_A^2 + \beta_{AA}} = X_m \phi_{AA} \\ Y^* &= Y_m \frac{\alpha_A \alpha_B}{\alpha_A \alpha_B + \beta_{AB}} = Y_m \phi_{AB} \end{aligned}$$

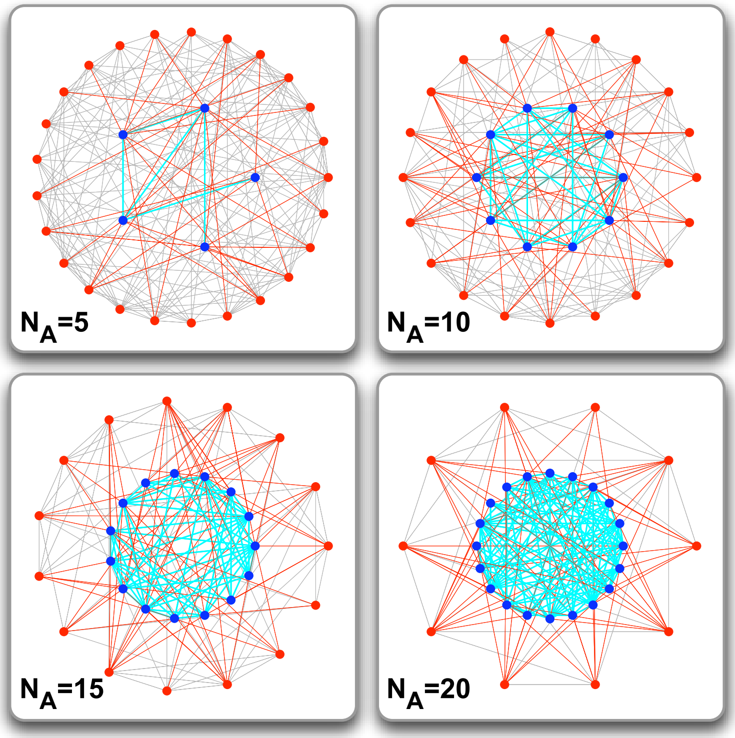

Fig. 1 Frequency dependent steady state dynamics. Results of active linking dynamics for a population size of $N = 30$ individuals. A-players are located in the "inner-rim", and are represented by blue circles, whereas B-players are located in the "outer-rim", and are represented by red circles. In this way, AA-links (solid cyan lines) live only within the "inner-rim", whereas AB-links (solid red lines) occupy the space between the rims while BB-links (solid grey lines) cross the entire region of the figure. Each panel depicts a snapshot in the steady state of the active-linking dynamics, associated with a different (and fixed) frequency of A and B players. The population size is $N = 30$, and the parameters determining the active linking dynamics are: $\alpha_A = \alpha_B = 0.5$, $\beta_{AA} = 0.5$, $\beta_{AB} = 0.25$ and $\beta_{BB} = 0.5$.

$$Z^* = Z_m \frac{\alpha_B^2}{\alpha_B^2 + \beta_{BB}} = Z_m \phi_{BB}$$

Here, ϕ_{AA}, ϕ_{AB}, and ϕ_{BB} are the fractions of active AA, AB and BB links in the steady states. Examples of population structures attained under steady-state dynamics for three different combinations of (N_A, N_B) are shown in Fig.1.

2.2 Strategy dynamics

Next, we address the dynamics of the strategies at the nodes. We consider the stochastic dynamics of a finite population, i.e. we restrict ourselves to finite networks. We consider a game between A and B given by the payoff matrix

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array}. \tag{1}$$

Thus, an A individual interacting with another A obtains the payoff a. A against B obtains b, whereas the B individual obtains c in such an interaction. Finally, B individuals obtain d from interactions with other B individuals.

We have to distinguish three generic cases of 3×3 games:

- Dominance. If $a > c$ and $b > d$, strategy B is dominated by strategy A. Thus, strategy A always obtains a higher payoff. The outcome does not have to be a social optimum: For $a < d$, the individuals playing strategy A end up with a non-optimum payoff. Similarly, B dominates A for $c > a$ and $d > b$.
- Coordination games: $a > c$ and $b < d$ leads to coordination games, in which it is always good to follow the strategy of the majority in the population. In the generic case, one strategy has a larger basin of attraction. This strategy is called risk dominant strategy. For $a + b > c + d$, strategy A is risk dominant.
- Coexistence games: In the case of $a < c$ and $b > d$, a small minority is favored. This means that the ultimate outcome in a population of players is a mixture of strategies A and B.

From the payoff matrix, we can calculate the payoffs of the individuals, depending on the number of interactions they have with the different types. On a complete network, the payoffs are

$$\pi_A = a(N_A - 1) + bN_B \tag{2}$$

and

$$\pi_B = cN_A + d(N_B - 1). \tag{3}$$

Often, the payoffs are scaled by $1/(N-1)$, such that the payoffs do not increase with the population size. For the strategy update process defined below, this corresponds simply to a rescaling of the intensity of selection, i.e. changing the noise intensity, if all individuals have the same number of interactions. If the number of interactions is not the same for all players, the heterogeneity between players can lead to new effects [20, 22].

Reproduction can be genetic or cultural. We adopt the pairwise comparison rule [11, 23], which has been recently shown to provide a convenient framework of game dynamics at all intensities of selection [24, 25]. According to this rule, two individuals from the population, A and B are randomly selected for update (only the selection of mixed pairs can change the composition of the population). The strategy of A will replace that of B with a probability given by the Fermi function

$$p = \frac{1}{1 + e^{-\beta(\pi_A - \pi_B)}}. \tag{4}$$

The reverse will happen with probability $1 - p$. The quantity β, which in physics corresponds to an inverse temperature, controls the intensity of selection. In the limit $\beta \to \infty$, the individual with the lower payoff will always adopt the strategy of the other individual. For $\beta \ll 1$, we recover the weak selection limit of the frequency dependent Moran process, which can be viewed as a high temperature expansion of the dynamics [26].

The quantity of interest in finite population dynamics is the fixation probability ρ, which is the probability that a single mutant individual of type A takes over a resident population with $N-1$ individuals of type B.

2.3 Separation of timescales

The system of coevolving strategies and links is characterized by two timescales: One describing the linking dynamics (τ_a), the second one describing strategy dynamics (τ_e). We can obtain analytical results in two limits, where both timescales are separated. Defining the ratio $W = \tau_e/\tau_a$, separation of time scales will occur for $W \ll 1$ and $W \gg 1$.

2.3.1 Fast strategy dynamics

In this case, active linking does not affect strategy dynamics. Thus, the dynamics is identical to the evolutionary game dynamics on a fixed network. Such systems have been tackled by many authors for a long time [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. The difficulty of an analytical solution for such systems is determined by the topology of the network, which corresponds to an initial condition in our case. Only for few topologies, analytical solutions are feasible. One important limiting case leading to analytical solutions are complete networks corresponding to well-mixed systems. In this case, the fixation probability can be approximated by [25]

$$\rho_A = \frac{\mathrm{erf}[\xi_1] - \mathrm{erf}[\xi_0]}{\mathrm{erf}[\xi_N] - \mathrm{erf}[\xi_0]}, \tag{5}$$

where $\mathrm{erf}(x)$ is the error function and $\xi_k = \sqrt{\frac{\beta}{u}}\,(ku + v)$. We have $2u = a - b - c + d$ and $2v = -a + bN - cN + c$. For $u = 0$, this simplifies to

$$\rho_A = \frac{1 - e^{-2\beta v}}{1 - e^{-2\beta v N}}. \tag{6}$$

A second example are Cayleigh trees. In this case, analytical solutions are only possible for weak selection, $\beta \ll 1$. For example, the fixation probability of a single

A individual under death-birth update can be calculated. For this update process, one individual selected at random is removed (death) and one of its neighbors is selected proportional to payoff to fill the empty space (birth). The fixation probability then reads [19]

$$\rho_A = \frac{1}{N} + \beta \frac{N-1}{N} \left[\frac{\alpha}{N} + \alpha + 3\beta \right], \tag{7}$$

where the parameters are $\alpha = (k+1)(k-1)(a-b-c+d)$, $\beta = (k+1)a + (k^2 - k - 1)b - c - (k^2-1)d$ and k is the degree of the homogeneous graph.

Consequently, whenever $W \ll 1$ the linking dynamics only becomes relevant in states where the system can no longer evolve from strategy dynamics alone, but changing the topology allows to escape from these states.

2.3.2 Fast linking dynamics

At the steady state of the linking dynamics, the average payoff of A and B individuals is respectively given by

$$\pi_A = a\phi_{AA}(N_A - 1) + b\phi_{AB}N_B \tag{8}$$

and

$$\pi_B = c\phi_{AB}N_A + d\phi_{BB}(N_B - 1). \tag{9}$$

Note that the effective number of interactions of an A player and a B player can become very different if $\phi_{AA} \gg \phi_{BB}$ or vice versa. Eqs. (8) and (9) suggest that the linking dynamics introduces a simple transformation of the payoff matrix. We can study standard evolutionary game dynamics using the modified payoff matrix

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a\phi_{AA} & b\phi_{AB} \\ c\phi_{AB} & d\phi_{BB} \end{pmatrix} \end{array} = \begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} \end{array} \tag{10}$$

This is an important observation: Linking dynamics can change the nature of the game [27]. So far, we have only shown this in the limit where linking dynamics is much faster than strategy dynamics ($W \gg 1$). However, the result is expected to hold even when the two time scales are comparable (see below and also Refs. [28, 27]. In general, all generic transformations are possible:

- A dominance game with $a > c$ and $b > d$ can change into a coordination game with $a' > c'$ and $b' < d'$ or into a coexistence game with $a' < c'$ and $b' > d'$.
- A coordination game with $a > c$ and $b < d$ can be transformed into a dominance game with $a' < c'$ and $b' < d'$ (or $a' > c'$ and $b' > d'$) or into a coexistence game with $a' < c'$ and $b' > d'$.
- A coexistence game with $a < c$ and $b > d$ can be transformed into a dominance game with $a' < c'$ and $b' < d'$ (or $a' > c'$ and $b' > d'$) or into a coordination game with $a' > c'$ and $b' < d'$.

The transition points can be determined as follows: Strategy A is a Nash equilibrium for $a > c$. This property changes to $a' < c'$ when

$$\frac{a}{c} < \frac{\phi_{AB}}{\phi_{AA}} = \frac{\alpha_B}{\alpha_A} \frac{\alpha_A^2 + \beta_{AA}}{\alpha_A \alpha_B + \beta_{AB}} \tag{11}$$

For example, ϕ_{AB} can be increased by reducing the death rate of AB links, β_{AB}. With increasing ϕ_{AB}, the condition is fulfilled at some point. At the transition point, A is either transformed into a Nash equilibrium or loses this property. An equivalent transition for B is given by the condition

$$\frac{d}{b} < \frac{\phi_{AB}}{\phi_{BB}} = \frac{\alpha_A}{\alpha_B} \frac{\alpha_B^2 + \beta_{BB}}{\alpha_A \alpha_B + \beta_{AB}}. \tag{12}$$

However, the conditions are not entirely independent, since at least two parameters have to be varied. Usually, it is enough to vary the three link-death rates β and fix the birth-link rates α to observe these transitions. Also the risk dominant strategy, i.e. the strategy with the larger basin of attraction, in coordination games can change. Thus, active linking can lead to a wide range of scenarios that effectively change the character of the game. Thus, self-organising network structures and the evolutionary game dynamics on the network are intimately entangled.

3 Individual based linking dynamics

In the model discussed in Section 2, we have a fluctuating number of links and analytical results in the two limits where the time scale of linking dynamics and strategy dynamics are well separated, allowing for the mean-field tretament considered. We now introduce an alternative description in which the number of links is conserved, but in which decision to maintain or rewire a link results both from individual preference in the choice of partners and negotiation between individuals linked [29]. Such an individual based decision making cannot be dealt with at a mean-field level and calls for a numerical implementation.

Let us start by restricting the space of possible games by fixing $a = 1$ and $d = 0$, while $-1 \leq b \leq 1$ and $0 \leq c \leq 2$.

$$\begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 1 & b \\ c & 0 \end{pmatrix} \end{array}. \tag{13}$$

This spans the four dynamical outcomes introduced before: a) dominance of A over B ($b > 0$ and $c < 1$); b) coexistence game ($b > 0$ and $c > 1$); c) coordination game ($b < 0$ and $c < 1$) and d) dominance of B over A ($b < 0$ and $c > 1$) (see Section 2.2).

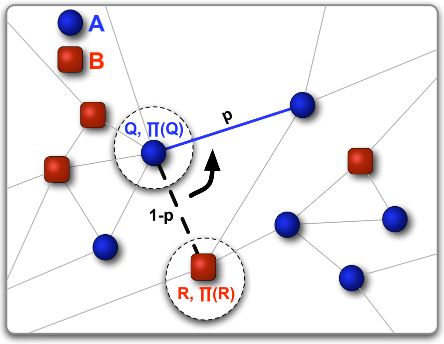

Fig. 2 Readjusting social ties. A and B individuals interact via the links of a network. R (A) is satisfied, since Q has a strategy B. On the other hand, Q is unsatisfied with this situation. Therefore, Q wants to change the link whereas R does not. The action taken is contingent on the fitness π_Q and π_R of Q and R, respectively. With probability p (see Eq. 4), Q redirects the link to a random neighbor of R. With probability $1-p$, Q stays linked to R. Finally, if both players are dissatisfied, the same methodology is used to decide who keeps the connection.

Because $b \leq 1$ and $c \geq 0$, the payoff against an A individual is never smaller than the payoff against a B individual, cf. Eq. 13. Thus, interacting with an A-player is always the best possible option. Consequently, every individual will be satisfied when connected to a A and dissatisfied otherwise. Keeping the total number of links constant, all individuals are now able to decide, on an equal footing, those ties that they want to maintain and those they want to change. The co-evolution between strategy and network structure is therefore shaped by individual preferences towards interacting with one of the two strategies [29]. Figure 2 illustrates the process. If Q is satisfied, she will decide to maintain the link. If dissatisfied, then she may compete with R to rewire the link (see Fig. 2), rewiring being attempted to a random neighbor of R. The intuition behind this reasoning relies on the fact that agents, equipped with limited knowledge and scope, look for new social ties by proxy [30]. Such a procedure can only be treated numerically and does no longer lead to a simple rescaling of a payoff matrix as the mechanism discussed in Sec. 2. On the other hand, it leads to some features of realistic social networks.

The fact that all individuals naturally seek to establish links with individuals with strategy A, creates possible conflicts of interests as illustrated in Fig. 2. For instance, R is satisfied, because it can profit from Q. Obviously, Q is not satisfied and would prefer to seek for a individual A. Decision is contingent on the payoff π_Q and π_R of Q and R, respectively. With the probability $p = [1 + e^{-\beta[\pi_Q - \pi_R]}]^{-1}$ (also used in the strategy update), Q redirects the link to a random neighbor of R. With probability $1-p$, Q stays linked to R. Whenever both Q and R are satisfied nothing happens. When both Q and R are unsatisfied, rewiring takes place such that the new link keeps attached to Q with probability p and attached to R with probability $1-p$. Thus, the more successful individual keeps the link with higher probability.

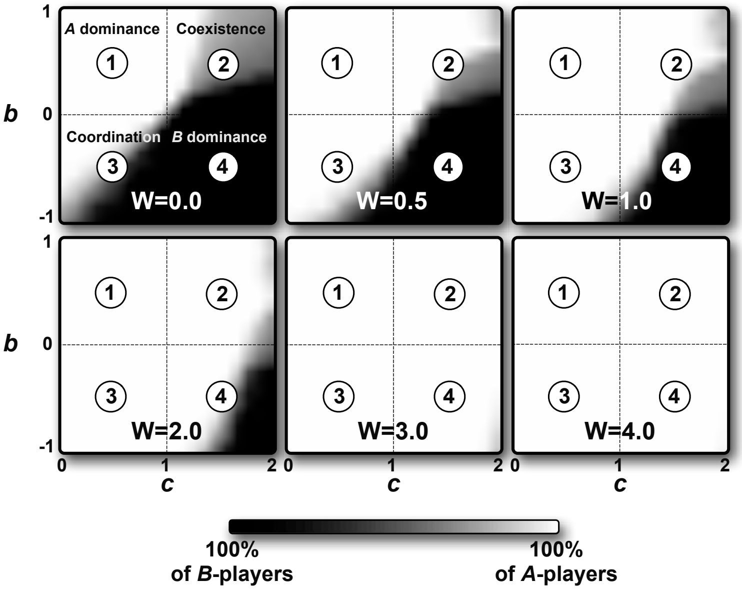

Fig. 3 Final frequency of strategy A in all games for different time-scale ratios between strategy and structure dynamics. Results for the fraction of successful evolutionary runs ending in 100% of individuals with strategy A for different values of the time scale ratio W, starting from an equal frequency of each strategy. We study the four different games in the area $2 \geq c \geq 0$ and $1 \geq b \geq -1$: (1) A dominates B; (2) coexistence game; (3) coordination game; (4) B dominates A (see Section 2.2). For $W = 0$ ($N = 10^3$, $z = 30$ and $\beta = 0.005$), the results fit the predictions from well-mixed populations, although individuals only interact with a small subset of the population. With increasing W (faster structure dynamics), the rate at which individuals readjust their ties increases, and so does the viability of strategy A. Above a critical value $W_{critical} \sim 4.0$ (see also Fig. 4), individuals with a strategy A efficiently wipe out Bs. For the strategy evolution dynamics adopted here (pairwise comparison, see section 2.2), and according to [19], A would never be favored in static networks.

As previously, this model establishes a coupling between individual strategy and population structure leading necessarily to a time scale associated with strategy evolution, τ_e and a second associated with structure evolution, τ_a. When the ratio $W = \tau_e/\tau_a$ equals 0 we recover the fast strategy dynamics of section 2.3.1. On the other hand, with increasing W, individuals become apt to adapt their ties with increasing efficiency.

The contour plots in Fig. 3 illustrate the final fraction of individuals which adopt strategy A for different values of the ratio W in networks with average connectivity $z = 30$ (this value reflects the mean value of the average connectivities reported in [31] for socials networks). We plot the fraction As who survive evolution, averaged over 100 independent realizations for the same values of the game payoff entries (b, c) and the time scale ratio W. For $W = 0$ the results reproduce, as expected [20], the predictions for finite, well-mixed populations. Yet, with increasing W, rewiring

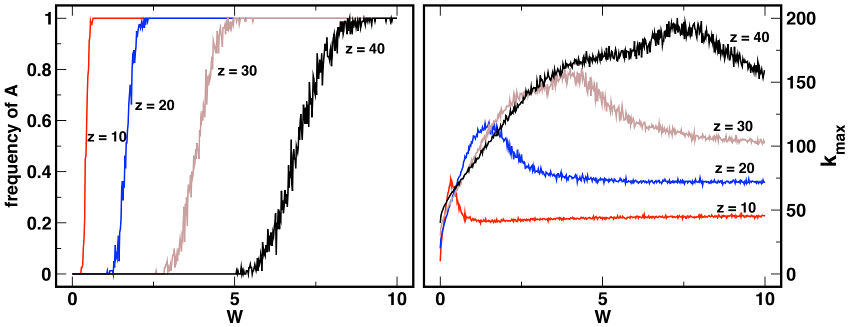

Fig. 4 Co-Evolution of strategies and links in the game region in which B should dominate for different time-scales. Left panel: Final frequency of A strategy at end as a function of W for different average connectivity z. For each average connectivity z, there is a critical value of the time scale ratio W - $W_{critical}$ - above which A-players wipe out B-players. Right panel: Connectivity k_{max} of the largest hub in the network, as a function of the time scale ratio W. With increasing z, $W_{critical}$ increases. In all cases, the heterogeneity of the associated network becomes maximal at $W_{critical}$. For higher values of W, the heterogeneity decreases again when type B decreases in frequency. For high values of W, type B is wiped out and only the heterogeneity generated by the rewiring mechanism in a neutral system prevails (Payoffs $a = 1$, $c = 2$, $b = -1$ and $d = 0$. Intensity of selection $\beta = 0.005$.).

changes the strategy dynamics and paves the way for a radically distinct evolutionary outcome in which As are now able to dominate for the entire range of games. Under structural dynamics, A individuals can cut their links to B individuals, which gives them an advantage compared to the situation on a static network. The swifter the response of individuals to the nature of their ties, the easier it gets for As to wipe out Bs. Note further that A already dominates B for $W = 4$, corresponding to a situation far from the time-scale separation conditions defined in section 3.2.

Additional insight is provided in Fig. 4 (left panel), where we show how A dominates B as a function of W when $c = 2$ and $b = -1$ and different values of the average connectivity z. For small W, As have no chance. Their fate changes as W approaches a critical value $W_{critical}$ - which increases monotonically with z - As wiping out Bs above $W_{critical}$ (the increase of $W_{critical}$ with z is expected, since there are more links to be rewired; in practice, $W_{critical}$ is determined as the value of W at which As reaches 50%). Thus, the evolutionary outcome and effective game at sake relies on the capacity of individuals to adjust to adverse ties.

Figure 4 also provides evidence of the detailed interplay between strategy and structure. On one hand, strategy updating promotes a local assortment of strategies, since As *breed* As and Bs *breed* Bs. On the other hand, under structural updating, one is promoting local assortative interactions between A-players (that is, AA-links) and disassortative interactions between B and A-players (that is, AB-links), which constitute *favorable steps* from an individual point of view. Clearly, when simultaneously active, strategy update will reinforce assortativity among As, but will inhibit disassortativity between B and A-players, which overall will promote the dominance of A over B.

For any $W > 0$, individual choices lead to heterogeneous graphs in which some individuals interact more, and more often than, others. The overall onset of increase of heterogeneity qualitatively follows the wave of A dominance shown in Fig. 3 [29]. In fact, the overall heterogeneity of the graph increases as W increases reaching a maximum at $W_{critical}$, above which heterogeneity decreases again down to a stationary value[29]. The results shown suggest that the adaptive dynamics of social ties introduced here coupled with social dilemmas accounts for the heterogeneities observed in realistic social networks [32].

4 Discussion

Our analysis has been limited to one-shot games. In other words, individuals interact once during the lifetime of a link as if they have never met before. But in repeated interactions, more possibilities exist. If I only take into account your behavior in the last interaction, there are already $2^2 = 4$ strategies. Since the number of strategies grows rapidly with memory [33], one often considers so called trigger strategies in which individuals keep their behavior unchanged until they are faced with an unsatisfactory partner for the first time. Such strategies can be implemented into our active linking framework, assuming that individuals act repeatedly as long as a link between them is present. This procedure leads to analytical results for evolutionary stability under active linking even in the context of repeated games [34].

To sum up, by equipping individuals with the capacity to control the number, nature and duration of their interactions with others, we introduce an adaptive network dynamics. Thus leads to surprising and diverse new game dynamics and realistic social structures. We have presented two approaches how to implement this network dynamics. The first one, active linking, allows to define differential equations for the numbers of links, which leads to analytical results. The second approach, individual based linking dynamics, is implemented numerically and leads to network features of empirical social networks.

The consideration of adaptive social networks is an important step towards more realistic models of social interactions in structured populations. Coupling the dynamics on networks with the dynamics of networks leads to emergent new phenomena outside the classical considerations of social dynamics on static networks.

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