



**BIRS Workshop  
Evolutionary Game Dynamics  
Saturday, June 10, 2006 to Thursday, June 15, 2006**

**MEALS**

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday  
 \*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday  
 \*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday  
 Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall  
 \*Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.

**MEETING ROOMS**

All lectures are held in the main lecture hall, Max Bell 159. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

**SCHEDULE**

	Sunday	Monday	Tuesday	Wednesday	Thursday
7:00-9:00	Continental Breakfast, 2nd floor lounge, Corbett Hall <sup>1</sup>				
9:00-9:30	Berger	Lieberman	Apaloo	Leimar	Riedel
9:30-10:00	Hofbauer	Hauert	Eshel	Vincent	Iwasa
10:00-10:30	Coffee Break, 2nd floor lounge, Corbett Hall				
10:30-11:00	Abrams	Liekens	Hopkins	Ohtsuki	Pacheco
11:00-11:30	Cohen	Krivan	Sandholm	Sorin	Nowak
11:30-13:30	Buffet Lunch, Donald Cameron Hall				
13:30-15:00	Open Forum <sup>2</sup>	Guided Tour <sup>3</sup>	free afternoon	Discussion Session	X
15:00-15:30	Coffee Break, 2nd floor lounge, Corbett Hall (except Tues.) Group Photo <sup>4</sup> (Monday)				X
15:30-16:00	Szabo	Brown	free afternoon	Dieckmann	X
16:00-16:30	Traulsen	Imhof	free afternoon	Skyrms	X
17:30-19:30	Buffet Dinner, Donald Cameron Hall <sup>5</sup>				X

<sup>1</sup>On Sunday June 11th at 8:50, the BIRS Station Manager will make introduction and welcome remarks.

<sup>2</sup>Open forum speakers include: Antal, Cressman, Garay, Santos, C. Taylor, P. Taylor, Wild.

<sup>3</sup>A free guided tour of The Banff Centre is offered to all participants and their guests on Monday starting at 1:00 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.

<sup>4</sup>A group photo will be taken on Monday at 3:20 pm, just before the first lecture of the afternoon. Please meet on the front steps of Corbett Hall.

<sup>5</sup>We will have a banquet on the evening of Wednesday June 14 at 6:30PM in Function room # 4-5. Participants who wish to attend the banquet will need to RSVP by June 12 the latest. They will be billed the cost of the banquet upon checkout from BIRS at the end of the workshop.



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**ABSTRACTS**  
**(in alphabetic order by speaker surname)**

Speaker: **Peter Abrams** (University of Toronto)

Title: *Causes of instability in the behavioral and evolutionary dynamics of the traits of interacting species*

Abstract: This talk will examine the roles of fitness functions, ecological interactions, and the form of the adaptive dynamics as potential causes of cyclic dynamics in the traits of interacting species. The talk will also try to identify conditions under which it is likely that the dynamics of multiple traits in multiple species will lead to a stable equilibrium that is attained relatively rapidly.

Speaker: **Tibor Antal** (Boston University)

Title: *Fixation properties of  $2 \times 2$  matrix games on complete and uncorrelated graphs*

Abstract: On a complete graph (well mixed population) the fixation probability and the mean fixation time of a single mutant are known exactly in finite populations. Their asymptotic behaviors in the large population limit are given by relatively simple expressions. These formulas then can be compared to the (also exact) formulas obtained from the widely used diffusion approximation. It turns out that the diffusion approximation results are not only inaccurate in some cases, but the formulas themselves are less appealing than the correct results. On general graphs, on the other hand, the fixation properties are not known exactly, however, a remarkably accurate approximation can be obtained for uncorrelated graphs with arbitrary degree distribution for frequency independent games.

Speaker: **Joseph Apaloo** ( St. Francis Xavier University)

Title: *Matrix Games and Optimization Theory*

Abstract: The application of optimization theory to evolution has a long history. The evolutionary stability concept of unbeatable strategy benefited from this application. This concept was later formalized as evolutionary stable strategy (ESS) which has become the main mathematical tool of evolutionary game theory. Specific to matrix games, optimization theory has not played any major role. This is rather unfortunate given the role of optimization theory in evolution. This situation arises from the nature of the fitness function used in matrix games which is commonly linear in the strategy adopted by rare mutants. Using the neighborhood invader strategy (NIS) concept in conjunction with the ESS concept, we propose an algorithm for identifying ESS and/or NIS in matrix games based purely on optimization theory rather than the explicit expected payoff comparisons as is commonly done. Thus we show that optimization theory is applicable to matrix games.

Speaker: **Ulrich Berger** (Vienna University of Economics Business)

Title: *Brown's Original Fictitious Play*

Abstract: What modern game theorists describe as "fictitious play" is not the learning process George W. Brown defined in his 1951 paper. His original version differs in a subtle detail, namely the order of

belief updating. In this note we revive Brown’s original fictitious play process and demonstrate that this seemingly innocent detail allows for an extremely simple and intuitive proof of convergence in an interesting and large class of games: non-degenerate ordinal potential games.

Speaker: **Joel Brown** (University of Illinois at Chicago)

Title: *Evolution of cooperation with non-linear costs and benefits*

Abstract: Doebeli et al. (2004, Science 306:859-862) examine the snow drift game in the context of non-linear costs and benefits. We follow their approach and consider a broad class of private cost/public benefit models. We apply a G-function approach to identify ESS candidate solutions, convergent stable points, and points that maximize fitness on the adaptive landscape. The snow drift game, where benefits are public and costs are private, can produce single strategy ESSs of extreme cooperation, extreme non-cooperation, an intermediate level of cooperation, and a mix of extreme strategies. The game also exhibits non-ESS points such as a convergent stable minimum, convergent unstable minimum and convergent unstable maximum. Variations of the Prisoners Dilemma produce a single ESS of extreme non-cooperation, but extreme cooperation can be a non-ESS, convergent stable maximum. Games of partial altruism (benefits and costs have private and public elements) produce opportunities for single and two-strategy ESSs. They cannot, however, produce a bifurcation point at a convergent stable minimum. The ESS and evolutionary dynamics emerge from an interplay between the type of game and the non-linearities in the cost and benefit functions. The emergence of cooperative behavior in this model represents a form of enlightened self interest whereas non-cooperative solutions produce a form of tragedy of the commons.

Speaker: **Yosef Cohen** (University of Minnesota)

Title: *A framework for evolutionary ecology and the evolutionary distributions of host-pathogen.*

Abstract: Smooth evolutionary games are modeled usually with ordinary differential equations (ODE). Accordingly, the players are identified with species and their strategies with values of their adaptive traits. These traits are inherited and different phenotypes (trait values) undergo different mortality rates (i.e. natural selection). The rules are usually interspecific interactions such as competition and predation. One outcome of the game dynamics might be a set of strategy values such that any small population with strategy values different from this set will not survive. This set of values is called Evolutionary Stable Strategies (ESS). At ESS, the fitness of all surviving species must be zero. Models of such games require derivation of the strategy dynamics (usually with ODE). The strategy itself represents some summary value (e.g., mean) of the distribution of values within a population at any particular time. In other words, at any instant, the strategy of a species population is distributed among phenotypes.

Here I show the link between the smooth evolutionary games approach and what I call evolutionary distributions (ED). In ED, we follow the dynamics of the distribution of strategy values (phenotypes) rather than some single value representation of this distribution. Consequently, if the dynamics lead to a stable distribution, then the latter is ESS. Furthermore, unless the distribution is homogeneous in the strategy-phenotypic density space (which I call the evolutionary space), phenotypes with all possible fitness values (including  $\pm\infty$ ) can coexist at the “ESS”. I then outline a generic approach for modeling evolutionary games with ED and apply the approach to a number of familiar host-pathogen models. The models are extended to include host-pathogen coevolution and lead to some surprising conclusions.

Speaker: **Ross Cressman** (Wilfrid Laurier University)

Title: *Intrinsic noise in evolutionary game dynamics*

Abstract: A one-step (birth-death) process is used to investigate stochastic noise in an elementary two-phenotype evolutionary game model based on a payoff matrix. In this model, we assume that the population size is finite but not fixed and that all individuals have, in addition to the frequency-dependent fitness given by the evolutionary game, the same background fitness that decreases linearly in the total population size. Although this assumption guarantees population extinction is a globally attracting absorbing barrier of the Markov process, sample trajectories do not illustrate this result even for relatively small carrying capacities. Instead, the observed persistent transient behavior can be analyzed using steady state statistics

of our model of intrinsic noise based on random pairwise interactions between individuals. It is shown that there is good agreement between the theory of these statistics and the simulation results. Furthermore, the ESS of the evolutionary game can be used to predict the mean steady state.

Speaker: **Ulf Dieckmann** (International Institute for Applied Systems Analysis)

Title: *Unfolding the fundamental degeneracy of matrix games*

Abstract: Whenever applications of evolutionary game theory consider mixed strategies in matrix games, a peculiar degeneracy raises its ugly head. This degeneracy, which gives rise to the Bishop-Cannings theorem, directly follows from the bilinear form through which matrix-game payoffs of mixed strategies are determined. Here we show how a more extensive population dynamical embedding and the addition of salient elements of ecological realism help to unfold this degeneracy. We thus reveal how the classical treatment, straddling two important bifurcation curves at once, serves as the organizing center of a rich bifurcation structure. By overcoming this fundamental structural instability, more conclusive predictions of evolutionary outcomes can be made, with the interplay between population-level polymorphisms and individual-level mixed strategies becoming amenable to analysis. Based on these findings, we suggest that the only features of an evolutionary game likely to be biologically relevant are those that stay intact under the unfolding.

Speaker: **Ilan Eshel** (Tel Aviv University)

Title: *Different Questions In Evolution Correspond To Different Sorts Of Population Dynamics And Topology, Hence To Different Concepts Of Evolutionary Stability.*

Abstract: The concept of ESS, as well as its various ramifications, is traditionally defined in terms of a conflict or, more specifically, of its appropriate game structure. We therefore speak then of a strategic stability. Yet if evolution indeed converges to equilibrium, this equilibrium is most likely to be dynamically stable. Dynamic stability of an equilibrium depends on the choice of a specific dynamics (or, hopefully in our case, a family of dynamics, corresponding in a specific way to a given game structure), and, not less importantly, on that of an appropriate topology. We shall see that different biological questions, even when corresponding to the same structure of a conflict, correspond either to different dynamics or to different topologies, maybe to both. They, therefore, lead to different stable equilibria. We see, moreover, that they may naturally correspond to different concepts of strategic stability. The case of ESS, CSS and NIS will be given as an example.

Speaker: **Jozsef Garay** (Eötvös Loránd University)

Title: *An ecological game-theoretical model for the foraging problem of bees*

Abstract: In many ecological situations individuals change their behaviour strategies according to the actual state of the population system, maximizing their benefits. Thus, all individuals can be considered as real players. In a possible mathematical model the individual behaviour changes can be described by a game dynamics, since the actual individual strategies are determined by time-dependent densities (and strategies). In ecological dynamics the densities of players change. As a concrete case we consider two bee species and two plant species. If the plant densities are fixed then in the framework of our model we obtain the following conclusions: 1. The stability of the shortest time scale (foraging) depends only on morphological characteristics determined by the longest-term co-evolutionary process. 2. Stability of the shortest time scale does not automatically imply the ecological coexistence.

Speaker: **Christoph Hauert** (Harvard University)

Title: *Evolutionary Dynamics on Graphs*

Abstract: Evolutionary dynamics in finite populations represent a particular balance between Darwinian selection and random drift. The two effects are nicely captured by the Moran process. Based on the Moran process the fixation probability of a single mutant in a resident population can be calculated, i.e. the probability that eventually the entire population adopts the mutant strategy. A mutant is favored if its fixation probability exceeds  $1/N$ , i.e. the fixation probability of a neutral mutant in a population of

size  $N$ . The Moran process assumes panmictic populations and neglects population structures that lead to limited local interaction and proliferation. Population structures can be modeled by identifying each individual with the node of a graph and by restricting interaction and proliferation to adjacent nodes, i.e. to nodes that are connected by an edge. For a long time it was generally believed that population structures do not affect the balance between selection and drift, provided that the individuals' fitness is frequency independent, i.e. the fitness of a mutant does not depend on its neighbors. Generalizations of the Moran process to spatially structured populations show that this only holds for a certain (although large) class of structures. Other structures can tilt the balance to the extent that either selection is eliminated and random drift rules or drift is eliminated and only selection matters. The frequency dependent case, where the fitness of an individual is affected by interactions with neighbors, leads to complex dynamics such as critical phenomena and is generally not amenable for analytic investigations. However, based on pair approximation a surprisingly simple and robust condition is obtained for selection to favor cooperation in prisoner's dilemma interactions:  $b > ck$ , i.e. if the cooperative benefits  $b$  exceed the  $k$ -fold costs of cooperation  $c$ , where  $k$  denotes the (average) number of neighbors of each individual. Whenever  $b > ck$  holds, a single mutant cooperator has a higher chance to take over the entire population than a neutral mutant through random drift. This rule represents an excellent predictor for cooperation on diverse types of graphs ranging from lattice geometries to scale-free networks.

Speaker: **Josef Hofbauer** (University College London)

Title: *Innovative dynamics and survival of dominated strategies*

Abstract: For the replicator dynamics and other monotone selection dynamics, strictly dominated strategies are eliminated. For innovative dynamics this need not be true.

Speaker: **Ed Hopkins** (University of Edinburgh)

Title: *Learning in Games with Unstable Equilibria*

Abstract: We propose a new concept for the analysis of games, the TASP, which gives a precise prediction about non-equilibrium play in games whose Nash equilibria are mixed and are unstable under fictitious play-like learning processes. We show that, when players learn using weighted stochastic fictitious play and so place greater weight on more recent experience, the time average of play often converges in these "unstable" games, even while mixed strategies and beliefs continue to cycle. This time average, the TASP, is related to the best response cycle first identified by Shapley (1964). Though conceptually distinct from Nash equilibrium, for many games the TASP is close enough to Nash to create the appearance of convergence to equilibrium. We discuss how these theoretical results may help to explain data from recent experimental studies of price dispersion.

Speaker: **Lorens Imhof** (Universität Bonn)

Title: *Evolutionary game dynamics in finite populations*

Abstract: We consider a class of stochastic models that describe the evolution of a finite population. The dynamics are determined by a selection mechanism based on an underlying game and by mutations. The main topics of the talk are the long-run behavior of these stochastic processes and a comparison with classical replicator dynamics for infinite populations. While similar stochastic models have been studied in the literature mainly for 2x2 games, the present approach allows to analyze evolutionary dynamics for games with an arbitrary finite set of pure strategies.

By way of illustration, we consider the repeated prisoner's dilemma game and the evolution of the strategies always cooperate, always defect and tit-for-tat. It turns out that the stochastic model can explain cooperation in populations of moderate size, whereas under the replicator dynamics cooperative behavior would not persist.

Speaker: **Yoh Iwasa** (Kyushu University)

Title: *Social norms that can maintain cooperation by indirect reciprocity: role of errors*

Abstract: Theory of indirect reciprocation explains the evolution of cooperation among unrelated individuals, engaging in one-shot interaction. Using reputation, a player acquires information on who are worth

cooperating and who are not. In a previous paper, we formalized the reputation dynamics, a rule to assign a binary reputation (good or bad) to each player when his action, his current reputation, and the opponent's reputation are given. We then examined all the possible reputation dynamics, and found that there exist only eight reputation dynamics named "leading eight" that can maintain the ESS with a high level of cooperation, even if errors are included in executing intended cooperation and in reporting the observation to the public. In this paper, we study the nature of these successful social norms. First we characterize the role of each pivot of the reputation dynamics common to all of the leading eight. We conclude that keys to the success in indirect reciprocity are to be nice (maintenance of cooperation among themselves), retaliatory (detection of defectors, punishment, and justification of punishment), apologetic, and forgiving. Second we prove the two basic properties of the leading eight, which give quantitative evaluation of the ESS condition and the level of cooperation maintained at the ESS. I also add new results on the errors causing reputation and the coexistence of multiple strategies.

Speaker: **Vlastimil Krivan** (Institute of Entomology, Czech Republic)

Title: *The Ideal Free Distribution*

Abstract: The Ideal Free Distribution is a crucial theoretical concept that describes animal distribution in a heterogeneous environment consisting of habitat patches. The original concept of the IFD was defined for a single species and it was shown that the IFD corresponds to the ESS of the game theory (which was defined later than the IFD). The concept of the IFD does not address the mechanism of reaching the IFD, i.e., it does not associate any dynamics with the IFD. It also does not deal with multiple species. In my talk I will address these two questions. In particular, I will show that the dynamics that corresponds to the assumptions behind the IFD is the best response dynamics. I will also discuss some other dynamics that do not assume completely omniscient animals. I will give an extension of the IFD for two species and discuss some possible consequences for animal population dynamics.

Speaker: **Olof Leimar** (Stockholm University)

Title: *The concept of adaptive phenotypic polymorphism in evolutionary theory*

Abstract: I will attempt to resolve and clarify a fifty-year-old debate in ecological genetics on the adaptive nature of polymorphism. In 1951, Theodosius Dobzhansky claimed that genetic polymorphism as such can be adaptive. He argued that polymorphic populations are more efficient at exploiting diverse environments and are thus better adapted to varied circumstances. This led to controversy, involving arguments that selection at the individual level need not improve the efficiency of populations. In 1958, R. A. Fisher concluded the debate, generally supporting Dobzhansky's claim and putting forward the idea that an organism can use the segregation of alleles at polymorphic loci to randomize a strategy. Fisher's treatment can be regarded as an early suggestion of an evolutionary game theory. In my analysis, an organism's plastic development is represented as a switching device, with potentially informative developmental cues as input and alternative phenotypes as output. If environmental cues can inform developing individuals about the likely success of phenotypic alternatives, then these should be taken into account, resulting in adaptive phenotypic plasticity (a conditional strategy). I will present the new idea that alleles present at polymorphic loci can also play the role of informative cues for developmental switching. When selection leads to gene frequency differences between population segments, such genetic cues can show statistical correlation with selective conditions. In this way, the original debate is resolved by incorporating forms of genetic polymorphism under the general heading of adaptive developmental plasticity, i.e. as instances of adaptive conditional strategies.

Speaker: **Erez Lieberman** (Harvard University)

Title: *Evolutionary Graph Theory*

Abstract: Evolutionary processes may be dramatically amplified or completely suppressed by the effects of spatiotemporal structure. In this talk, we will discuss the Isothermal Theorem and the Structure Theorems of Evolutionary Graph Theory, and introduce the notion of selection amplifiers and suppressors. Using the methods of statistical mechanics, we will derive a first-order theory of neutral strategies in the case of

frequency dependent evolution on graphs. Evolution on cascade graphs and "magnetization" results will be introduced as time allows.

Speaker: **Anthony Liekens** (Technische Universiteit Eindhoven )

Title: *Finite populations playing Rock-Paper-Scissors*

Abstract: Discrete time Markov chains are constructed to model finite populations evolving strategies for the Rock-Paper-Scissors game. In the presence of variational pressure, the models provide us with a unique long term equilibrium behavior. Using high performance computers, we have determined the limit behavior for relatively large populations. The expectations for finite and infinite models allow us to study the effects of stochastic pressure on genetic drift and punctuated equilibria in the context of the Rock-Paper-Scissors game.

Speaker: **Martin Nowak** (Harvard University)

Title: *Five Rules for the Evolution of Cooperation*

Abstract: Cooperation means a donor pays a cost,  $c$ , for a recipient to get a benefit  $b$ . In Evolutionary Biology, cost and benefit are measured in terms of fitness. While mutation and selection represent the main "forces" of evolutionary dynamics, cooperation is a fundamental principle that is required for every level of biological organization. Individual cells rely on cooperation among their components. Multi-cellular organisms exist because of cooperation among their cells. Social insects are masters of cooperation. Most aspects of human society are based on mechanisms that promote cooperation. Whenever evolution constructs something entirely new (such as multi-cellularity or human language), cooperation is needed. Evolutionary construction is based on cooperation. I will present five basic principles for the evolution of cooperation, which arise in the theories of kin selection, direct reciprocity, indirect reciprocity, graph selection and group selection.

Speaker: **Hisashi Ohtsuki** (Kyushu University)

Title: *The Replicator Equation on Graphs*

Abstract: We study evolutionary games on graphs. Each player is represented by a vertex of the graph. The edges denote who meets whom. A player can use any one of  $n$  strategies. Players obtain a payoff from interaction with all their immediate neighbors. We consider three different update rules, called 'birth-death', 'death-birth' and 'imitation'. We use pair-approximation to describe the evolutionary game dynamics on regular graphs of degree  $k$ . In the limit of weak selection, we can derive a differential equation which describes how the average frequency of each strategy on the graph changes over time. Remarkably, this equation is a replicator equation with a transformed payoff matrix. Therefore, moving a game from a well-mixed population (the complete graph) onto a regular graph simply results in a transformation of the payoff matrix. The new payoff matrix is the sum of the original payoff matrix plus another matrix, which describes the local competition of strategies.

Speaker: **Jorge Pacheco** (Lisboa University)

Title: *Co-evolution of strategy and structure leads to the emergence of cooperation*

Abstract: Recent empirical evidence has confirmed that social networks of contacts are strongly heterogeneous, reflecting the fact that different individuals engage in different numbers of interactions at different rates. When mapped onto a graph, one observes broad-scale patterns of connectivity, resembling scale-free graphs. Here we study the evolution of the network topology entangled with the evolution of cooperation, the latter modeled in terms of the most popular social dilemmas describable by symmetric two-person games. We find a detailed interplay between the characteristic time-scales associated with the evolutionary dynamics of the network structure and that associated with the individual strategies. In all cases the temptation to cheat induces the emergence of network topologies which contrast with those induced by the fear of being cheated, with the overall emergence of cooperation as a viable evolutionary trait. We find that in more adaptive communities, in which individuals swiftly update their network of contacts, cooperators are able to wipe-out defectors without developing strong heterogeneities, whereas slowly adapting communities tend to lead to much stronger heterogeneities, associated with broad-scale graphs.

Speaker: **Frank Riedel** (Bonn University)

Title: *Evolution with a Continuum of Traits*

Abstract: I discuss recent progress for the replicator dynamics with a continuum of traits based on joint work with Ross Cressman and Josef Hofbauer. Moreover, the talk explores a recent approach by Ruijgrok who proposes a way of introducing mutation into the replicator dynamics with a continuum of traits. I show invariance for the class of normal distributions and study the ergodic limit distribution.

Speaker: **Bill Sandholm** (University of Wisconsin-Madison)

Title: *Logit Evolution and Large Deviations in Potential Games*

Abstract: We use techniques from large deviations theory to investigate the long run behavior of a population whose members play a potential game, making choices according to the logit choice rule. The large deviations methods allow us to compute exit times from basins of attraction of stable equilibria, allowing us to obtain a detailed description of the nature of infinite horizon play. We also use geometric techniques to characterize the rate of convergence of the evolutionary process to its stationary distribution.

Speaker: **Francisco Santos** (Universite Libre de Bruxelles)

Title: *A multi-level selection model for the emergence of social norms*

Abstract: We develop a multi-level selection model in the framework of indirect reciprocity. Using two levels of selection, one at the individual level and another at the group level, we propose a competitive scenario among social norms, in which all individuals in each group undergo pairwise interactions, whereas all groups also engage in pairwise conflicts, modeled by different games. Norms evolve as a result of groups' conflicts whereas evolution inside each group promotes the selection of best strategies for each ruling social norm. The proposed evolutionary model leads to the emergence of one of the 'leading eight' social norms, obtained recently by Ohtsuki and Iwasa, irrespective of the type of conflict between groups employed. This reputation assignment rule gives rise to a stern and unambiguous response to each individual behavior, where prompt forgiving coexists with implacable punishment.

Speaker: **Brian Skyrms** (UC Irvine)

Title: *Learning to Signal*

Abstract: The central question is whether reinforcement learning always converges to signaling system equilibria. A little motivation, some simulations, a little analytical result, some generalized models and open problems.

Speaker: **Sylvain Sorin** (Université Pierre et Marie Curie)

Title: *Differential inclusions, stochastic approximation and applications*

Abstract: The dynamical system approach to stochastic approximation is extended to differential inclusions and a limit set theorem is achieved. Applications are given to dynamics in games.

Speaker: **György Szabó** (Research Institute for Technical Physics and Materials Science)

Title: *The effect of noise, payoff, and regular connectivity structures on cooperation for the evolutionary prisoner's dilemma games*

Abstract: The two-strategy (AllD+AllC) evolutionary prisoner's dilemma games are studied with a noisy dynamics meanwhile varying the payoff  $b$  (temptation to defect) suggested by Nowak and May on many different regular ( $z=4$ ) connectivity structures, such as e.g., the square-, Kagome-, and Bethe-lattices or random regular graphs. The analyses are performed by using Monte Carlo simulations and the dynamical cluster techniques determining the configuration probabilities on clusters up to size of 9 or 11 sites.

It is turned out that two different behaviors can be distinguished in the zero noise limit. The cooperators (AllC) survive in a region of  $b > 1$  for those connectivity structures where the overlapping triangles span the whole system. This feature is not restricted to the spatial connectivity structures. For low noises the highest frequency of cooperators is found for those structure where triangles are substituted for the nodes of a Bethe-lattice or random regular graphs with a degree of  $z=3$ .



If the connectivity structure is free of triangles (e.g., square- and Bethe-lattice) or the overlapping triangles do not span the whole system (e.g., square lattice of four-site cliques), then the cooperation vanishes in the zero noise limit if  $1 < b < b_{cr}$ . In these cases the system exhibits an optimum temperature (on the analogy of stochastic resonance) where the frequency of cooperators reaches the maximum value. Besides it the system shows two subsequent transitions [AllD  $\rightarrow$  (AllD+AllC)  $\rightarrow$  AllD] when increasing the amplitude of noise [1].

Surprisingly, for sufficiently large noise levels the highest frequency of cooperators is observed for the locally tree-like structures (Bethe-lattice and random regular graphs).

Recently we study this evolutionary game on the one-dimensional lattice with first- and second-neighbor interactions ( $z=4$ ) because the preliminary results indicate a third type of behaviors.

Speaker: **Christine Taylor** (Harvard University)

Title: *Can envy promote cooperation?*

Abstract: Variations in traits and fitness abound in natural and social realms. Natural selection ensures survival of the fittest and extinction of the least fit. However, in cooperative dilemmas, the more cooperative and less fit survive against the odds of natural selection. There are many explanations for the survival of the less fit. In this talk, we focus on the effect of envy, which brings about punishment for those who are fitter. We explore how punishment might promote and maintain cooperation. We propose two mechanisms of punishment. Under institutional punishment, those who are fitter than the population mean are punished; under individual punishment, everyone is punished by those who are less fit. When punishment is strong, cooperation evolves and is boosted by heterogeneity. Without mutation, the entire population eventually conforms to a single trait determined by the initial composition of the population and level of punishment. Mutation adds diversity and suppresses cooperative effect of strong punishment.

Speaker: **Peter Taylor** (Queen's University)

Title: *Inclusive fitness equivalents in finite populations*

Abstract: We will present a number of equivalent measures of fitness in a structured finite population, including measures of fixation probability, average allele frequency change, and inclusive fitness. In some cases the inclusive fitness measure seems to provide a significantly simpler calculation.

Speaker: **Arne Traulsen** (Harvard University)

Title: *A simple equation for group selection*

Abstract: A new approach to group selection is presented in which selection on a higher level emerges from individual selection. This higher level selection can favor cooperation, while selection on the lower level works in the opposite direction. Consider a population that is subdivided into a finite number of groups. Individuals reproduce proportional to their fitness. When a group reaches a certain size it can split into two groups while another group is eliminated. Selection on two levels arises naturally from this population structure. The fixation probability of newly introduced mutants can be calculated for constant selection as well as for frequency dependent selection. Conditions under which the population structure favors the evolution of cooperation by group selection are derived.

Speaker: **Thomas L. Vincent** (University of Arizona)

Title: *Evolutionary design vs. optimal design: an assessment*

Abstract: The population dynamics of biological systems are defined in terms of fitness. Fitness is a function of strategies used by the organisms. When these strategies are chosen so as to maximize fitness then this is referred to as optimal design. When the strategies are determined by means of an evolutionary process, then this is referred to as evolutionary design. Evolutionary design may lead to an evolutionarily stable strategy (ESS). These two methods for determining strategies will generally yield different results. Organisms using optimal design are usually unable coexist with an organisms employing an ESS. However there are some interesting situations in which optimal design may have a role to play. Two such situations are examined here. The first situation is one in which both optimal design and evolutionary design lead

to the same results. This situation becomes interesting when an ESS does not exist. The second situation is when an ESS strategy exists, but where it is possible for optimal design to mimic the ESS.

Speaker: **Geoff Wild** (Queen's University)

Title: *Inclusive fitness equivalents in finite populations*

Abstract: We will present a number of equivalent measures of fitness in a structured finite population, including measures of fixation probability, average allele frequency change, and inclusive fitness. In some cases the inclusive fitness measure seems to provide a significantly simpler calculation.