

## Reacting Differently to Adverse Ties Promotes Cooperation in Social Networks

S. Van Segbroeck,<sup>1,2,5</sup> F. C. Santos,<sup>2,5</sup> T. Lenaerts,<sup>3,5</sup> and J. M. Pacheco<sup>4,5</sup>

<sup>1</sup>COMO, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium

<sup>2</sup>IRIDIA, CoDE, Université Libre de Bruxelles, Avenue Franklin Roosevelt 50, 1050 Brussels, Belgium

<sup>3</sup>Switch Lab, VIB and Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium

<sup>4</sup>ATP-Group, Centro de Física Teórica e Computacional and Departamento de Física da Faculdade de Ciências, P-1649-003 Lisboa Codex, Portugal

<sup>5</sup>GADGET, Apartado 1329, 1009-001 Lisboa, Portugal

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We investigate how diversity in individual responses to unwanted interactions affects the evolution of cooperation modeled as a 2-person prisoner's dilemma. We combine adaptive networks and evolutionary game theory, showing analytically how the coevolution of social dynamics, network dynamics, and behavioral differences benefit the entire community even though myopic individuals still act in their own interest. As defectors are wiped out, surviving cooperators maintain the full diversity of behavioral types, establishing cooperation as a robust evolutionary strategy. The present framework can be used in other problems where the feedback between topology and dynamics affects the overall behavior of the system.

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Many physical processes are dynamical and take place on complex networks [1–6]. Recent evidence [7–10] supports the intuition that complex networks are also dynamical entities, such that the coevolution of the networks with the dynamical processes they host strongly influences the overall system behavior [3–15]. Unfortunately, up to now it has proven difficult to explore analytically any dynamics on coevolving networks (for exceptions cf. [4,6,13,15]). Here we explore the behavioral dynamics of a population interacting via an adaptive social network. Individual decision making is introduced in the framework of evolutionary game theory [16]. Often decisions are reduced to a black-and-white world in which cooperators ( $C$ 's) and defectors ( $D$ 's) interact, compete, and self-organize at increasing levels of complexity [7,8,13,14]. Up to now, the  $C$ 's (and  $D$ 's) exhibit no differences in the way they manage their contacts [7,8,13,14]. This situation contrasts with our everyday experience, where we recognize a continuous behavioral spectrum: Two  $C$ 's ( $D$ 's) may react differently when confronted with the same unfavorable situation. In which way does this variability in the spectrum of possible reactions influence cooperation among humans along the links of the social web?

Let us stick to  $C$ 's and  $D$ 's, engaging in a 2-player prisoner's dilemma (PD) with payoffs  $R$  for mutual cooperation,  $P$  for mutual defection,  $S$  when playing  $C$  against a  $D$ , and  $T$  when playing  $D$  against a  $C$ , satisfying  $T > R > P > S$ . Both  $C$ 's and  $D$ 's are satisfied when interacting with a  $C$  [14], in the sense that such an interaction is the most rewarding. An interaction with a  $D$ , on the other hand, is never the best option and can be interpreted as an adverse social tie. Here we introduce diversity in behavior by conferring individuals the possibility to treat adverse social ties differently.

We start from the model developed in [13] where network topology and strategy dynamics coevolve, and we

introduce diversity of individual behavioral types, leading to additional couplings with *both* topology *and* strategy dynamics. Individuals are assigned to the vertices of a dynamical graph, whose links define who interacts with whom. Here, individuals differ both in their strategy and in their aptness to break unwanted ties, falling into  $M$  (usually many) different types (behavioral classes). Once a link between two individuals has formed, its quality is assessed. Given each individual's aptness to break ties, unwanted links will be broken off at different rates, leading to a rich interplay between type, strategy, and network dynamics.

Let us assume for simplicity that all individuals have the same propensity  $\alpha$  to create new links. Individuals connected to  $C$ 's will be satisfied ( $C$ - $C$  or  $D$ - $C$ ). Links to  $D$ 's ( $C$ - $D$  or  $D$ - $D$ ) will be considered adverse. Both  $C$ 's and  $D$ 's will want to sever these interactions at different rates, type dependent. Breaking links depends usually on both individuals connected. We include this effect by defining the rate at which links are broken as  $k_{ij} = \frac{1}{2}(\gamma_{ij} + \gamma_{ji})$  for each link  $L_{ij}$ , where  $\gamma_{ij}$  is the rate at which individual  $i$  wants to break the link  $L_{ij}$  with individual  $j$  (other choices are also possible [13,14]). Those satisfied will want the link to last as long as possible, and hence we take those  $\gamma_{ij}$  to be given by the minimum value  $\gamma$  among all  $\gamma_{ij}$ . Naturally, adverse interactions will satisfy  $\gamma_{ij} \geq \gamma$ , and behavioral differences arise due to the plethora of  $\gamma_{ij}$  types present in the population. The corresponding linking dynamics of the network can be described by a set of ordinary differential equations [13]  $\dot{L}_{ij} = \alpha^2(N_{ij} - L_{ij}) - k_{ij}L_{ij}$ , where  $L_{ij}$  (respectively,  $N_{ij}$ ) is the number (respectively, maximum number) of links connecting individuals with strategies  $S_i$  and  $S_j$  (assuming a constant population size  $N$ ). These differential equations lead to a stationary distribution of links given by  $L_{ij}^* = N_{ij}\phi_{ij}$ , where  $\phi_{ij} = \alpha^2(\alpha^2 + k_{ij})^{-1}$  denotes the fraction of active  $ij$  links [13].

We start by introducing only two types of rates ( $M = 2$ )—fast and slow—obtaining four combined strategy-types: slow  $C$ 's (SC's) and  $D$ 's (SD's), whose adverse interactions last long (having a low breakup rate  $\gamma_S$ ), and fast  $C$ 's (FC's) and  $D$ 's (FD's), whose adverse interactions are short-lived (with a high breakup rate  $\gamma_F$ ).  $C$ 's and  $D$ 's engage in a PD (we reduce the game complexity making  $R = 2$ ,  $P = 1$ , varying  $T$  and  $S$  in the region defined by  $3 \geq T > 2 > 1 > S \geq 0$ ). As usual, the game only distinguishes between  $C$ 's and  $D$ 's—however, when individuals interact via an adaptive and coevolving network, the difference between types emerges as individuals break unwanted relations at different rates. In the following, we provide analytical insights into this tight coevolutionary dynamics between strategies, types, and networks in a finite population. To this end, we assume network dynamics to proceed faster than strategy dynamics, so that strategy updates occur under stationary network configurations. In other words, we assume that  $\chi \equiv T_S/T_N \ll 1$ , with  $T_S$  the characteristic time scale for strategy update and  $T_N$  that associated with network update. Numerical simulations, however, show that this approximation works very well whenever  $\chi < 10$  [13]. In the present case, this leads to a conservative estimate of  $\chi < 10\gamma$  for the approximation to remain valid.

In this regime, the evolutionary dynamics can be mapped onto that of a (simpler and analytically solvable) well mixed population with the same composition of  $C$ 's and  $D$ 's, engaging, however, in a different game, characterized by a payoff matrix which is rescaled compared to that defining the original PD. The network dynamics and number of types define the rescaling of the payoff matrix. Hence, in the case of  $2M = 4$  combined strategy-types, we get the following rescaled payoff matrix  $P_{ij}$ :

$$\begin{array}{c} \begin{array}{cc} & \begin{array}{cc} SC & FC \end{array} \\ \begin{array}{c} SC \\ FC \\ SD \\ FD \end{array} & \begin{pmatrix} R\phi_S & R\phi_S & S\phi_S & S\phi_S \\ R\phi_S & R\phi_S & S\phi_M & S\phi_M \\ T\phi_S & T\phi_M & P\phi_S & P\phi_M \\ T\phi_S & T\phi_M & P\phi_M & P\phi_F \end{pmatrix} \end{array} \quad (1)$$

where  $\phi_x = \alpha^2(\alpha^2 + k_x)^{-1}$  [ $k_S = \gamma_S$ ,  $k_M = \frac{1}{2}(\gamma_S + \gamma_F)$ , and  $k_F = \gamma_F$ ] reflects the impact of the characteristics of a given link (kind and associated lifetime dependent on the individuals connected) on its average abundance in the network of contacts of the community. In general, there will be more ( $M > 2$ ) types, which leads to a trivial generalization of the  $2M \times 2M$  payoff matrix above. Payoff matrix (1) shows the nontrivial coupling between individual type, game strategy, and network dynamics (given that different types imply different rates of breaking links), resulting in a complex multicomponent feedback interplay.

A complete solution for the case in which many types coevolve is unfeasible. However, it is possible to approximately determine the relative importance of each of the coevolving strategies by computing the stationary distributions in the so-called small-mutation limit [17,18]. In a

nutshell, we imagine that a mutant (using a strategy located at arrow end in Fig. 1) appears in an otherwise monomorphic population (using the strategy located at arrow start in Fig. 1). Thus, the population will contain only two strategies, and to the extent that the mutation probability is sufficiently small, the mutant will go extinct or fixates before a new mutation occurs. Adopting the so-called pairwise comparison rule [19]—in which the least fit mostly adopts the strategy of the most fit in the presence of noise, described via a Fermi function with inverse temperature  $\beta$ —allows us to calculate analytically the probability (fixation probability, FP) that a mutant with strategy  $S_i$  will take over a population of  $N - 1$  individuals with strategy  $S_j$ , for every  $i$  and  $j$  [19]:  $\rho_{ij} = (\text{erf}[\xi_1] - \text{erf}[\xi_0]) / (\text{erf}[\xi_N] - \text{erf}[\xi_0])$ , where  $\text{erf}(x)$  is the error function and  $\xi_k = \sqrt{\frac{2}{u}}(ku + v)$ . We have  $2u = P_{ii} - P_{ij} - P_{ji} + P_{jj}$  and  $2v = -P_{ii} + N(P_{ij} - P_{jj}) + P_{jj}$ . These FP's determine the transition matrix  $[\Delta_{ij}]_{i,j=1,\dots,2M}$ , with  $\Delta_{ii} = 1 - \sum_{k=1, k \neq i}^{2M} \rho_{ki} / (2M - 1)$  and  $\Delta_{ij} = \rho_{ji} / (2M - 1)$  ( $j \neq i$ ), of a Markov chain between the different monomorphic states [17,18]. The normalized left eigenvector of the unit eigenvalue of this matrix defines the stationary distribution, i.e., the fraction of time the population spends in each of the available strategy-type combinations. We have checked with numerical simulations that the stationary distributions obtained using this small-mutation approach also hold for larger mutation rates (of the order of  $1/N$ ), as also shown in [18].

Figure 1 shows that, in a region where  $D$ 's dominate, the population remains most of the time in the SD strategy. The arrows indicate which transitions are favored by natural selection, that is, those for which the FP is larger than  $\rho_N = 1/N$ , associated with neutral evolution, in which  $C$ 's

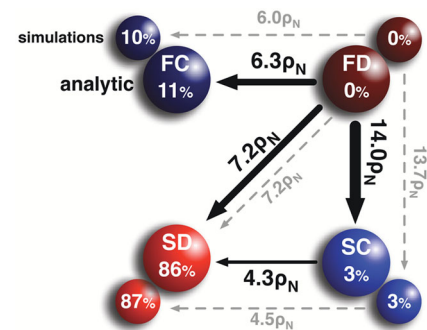


FIG. 1 (color online). Transition probabilities and stationary distributions for the PD with  $M = 2$  types of individuals. Solid black (dashed grey) arrows indicate those transitions, computed analytically (numerically), for which the fixation probability is greater than neutral fixation,  $\rho_N = 1/N$  (see main text for details). Large (small) circles indicate the analytical (numerical) values for the stationary distribution. FC's remain in the population while  $D$ 's dominate due to the direct FD-SD transition and the transient nature of SC ( $N = 100$ ,  $T = 2.1$ ,  $R = 2$ ,  $S = 0.9$ ,  $P = 1$ ,  $\alpha = 0.4$ ,  $\beta = 0.01$ ,  $\gamma_S = 0.25$ , and  $\gamma_F = 0.75$ ).

and  $D$ 's have the same fitness. Among  $C$ 's,  $FC$ 's are clearly favored. On the other hand, natural selection renders  $FD$ 's disadvantageous, whereas  $SC$ 's acquire a transient character by providing an alternative route for  $FD$  to evolve toward  $SD$ . In this specific case, the direct transition of  $FD$ 's into  $SD$ 's is the main factor suppressing  $C$ 's. As we will show below, the viability of  $C$ 's relies on the extent to which the transition  $FD \rightarrow SD$  is inhibited compared to transitions into  $FC$ 's, a feature which relies on the occurrence of a more continuous spectrum of behavioral types, as observed empirically [20]. Figure 1 also shows the excellent agreement between the analytical treatment and corresponding numerical simulations [21], which is robust to changes of parameters, except for those parameter regions leading to disconnected graphs.

In what follows we extend the analysis to an arbitrary number of types keeping the framework adopted. We consider a population containing  $M$  different types of individuals, whose rates at which they sever unwanted ties fall into the  $M$  subdivisions of the interval  $[0.5 - \delta, 0.5 + \delta]$ . We fix the range  $\delta = 0.25$ —we have checked that varying the range does not change the overall behavior of the population. Figure 2(a) shows the influence of the number

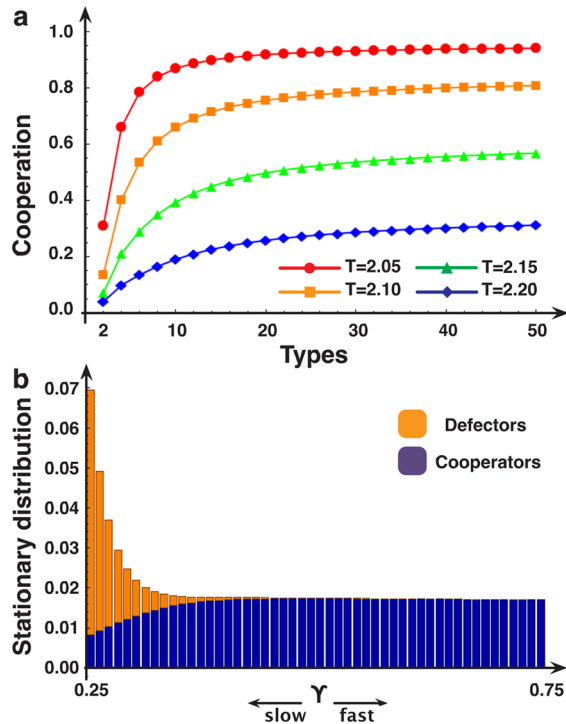


FIG. 2 (color online). Impact of behavioral differences. (a) Population spends more time in a cooperative state (of any type) when the number of types increases ( $M$ ), irrespective of the temptation to defect  $T$  in the PD ( $N = 100$ ,  $R = 2$ ,  $P = 1$ ,  $S = 3 - T$ ,  $\alpha = 0.4$ ,  $\beta = 0.01$ ,  $\delta = 0.25$ ). (b) Most types of  $C$ 's end up equally represented, whereas only the slower  $D$ 's survive. In contrast to Fig. 1, the surviving  $D$ 's exhibit behavioral differences, which acts to inhibit the dominance of their slowest type, providing an escape hatch for cooperation ( $M = 50$ ,  $T = 2.1$ ,  $R = 2$ ,  $P = 1$ ,  $S = 0.9$ ).

of types on the abundance of  $C$ 's in the population: Cooperation blooms with increasing number of types, a result that generally applies to most popular social dilemmas of cooperation [22].

What is the mechanism responsible for this remarkable performance of  $C$ 's? A first hint is provided in Fig. 2(b), where the fraction of time spent on each type is shown for the case in which there are  $M = 50$  types in the population and for fixed  $\delta$ ,  $T$ , and  $S$ . Figure 2(b) shows that all types of  $C$ 's are present in the population, whereas the  $D$ 's who survive are only of the slowest types. Importantly, however, by increasing the number of available types, the difference between the values of the rates associated with contiguous types is reduced, which provides a means for  $D$ 's other than the slowest to survive in the population. These are precisely the  $D$ 's who provide an escape hatch for  $C$ 's to survive (cf. Fig. 1), since many of them will be disadvantageous with respect to  $C$ 's. Indeed, Fig. 3 shows that increasing the number of possible types,  $D$ 's with breakup

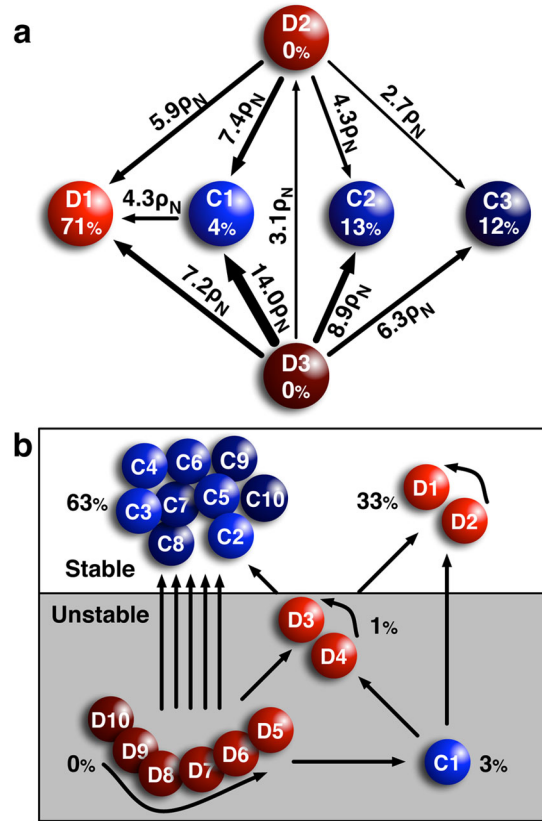


FIG. 3 (color online). Transition probabilities and stationary distributions for the PD game with  $M = 3$  (a) and  $M = 10$  (b) types of individuals.  $C$ 's and  $D$ 's are numbered according to increasing  $\gamma$ ,  $C1$  ( $D1$ ) being the slowest  $C$  ( $D$ ). We use the same notation of Fig. 1. Increasing the number of possible types splits the outflow of fast  $D$ 's toward other types among a wide range of different possibilities. Cooperation increases, since only few types of  $D$ 's are evolutionarily stable, whereas the majority of  $C$  types work as flow sinks. ( $N = 100$ ,  $T = 2.1$ ,  $R = 2$ ,  $S = 0.9$ ,  $P = 1$ ,  $\alpha = 0.4$ ,  $\beta = 0.01$ ,  $\delta = 0.25$ ).



rates higher than the minimum  $\gamma$  are now able to survive. Although these individuals do not dominate, they effectively promote the appearance of fast  $C$ 's, since the transition  $D \rightarrow C$  between these types is favored by natural selection. On the other hand, because all  $C$  types are neutral with respect to each other, they end up fairly equally distributed in the population, unlike  $D$ 's for whom natural selection favors the slower types (cf. Fig. 2). Figure 3 also shows that, by increasing the number of available types, the number of  $C$  types which are favored increases more than the corresponding number of  $D$  types. As a result, all but the slowest  $D$ 's are disadvantageous with respect to (most of the)  $C$ 's. Increasing the number of types efficiently inhibits the transition from the fastest to the slowest  $D$ 's, paving the way for cooperation to thrive, an effect which remains valid irrespective of the model parameters, inasmuch as  $T$  and  $S$  are such that cooperators manage to survive. Mathematically, a large number of different types allows individuals to engage in a wider range of games, and the complex set of different interactions allows the appearance of  $D$ 's from which  $C$ 's can profit.

The results of the present model convey an exciting message: Populations in which individuals exhibit diversity in handling their social contacts end up being more cooperative than those in which individuals exhibit no such diversity. The result is even more striking if we take into consideration that individuals always behave according to their own myopic preferences. In the framework of conventional evolutionary game theory, diverse behavioral preferences together with adaptive social dynamics make individuals perceive differently the same social dilemma; in doing so society as a whole benefits. Under static [2,5] or coevolving [13,14] network reciprocity, all individuals behave as if they engage in a transformed game—yet all individuals perceive the same game. Individual diversity enables all individuals who engage in the same game to perceive that game differently. To the best of our knowledge, this is the first time such an effect is introduced and studied. It is rewarding that behavioral diversity promotes cooperation, as cooperation will act to reinforce it, given the neutrality between cooperative strategies associated with diverse behaviors.

Diversity, in some of its forms [5,23,24] has been shown to play an important role in the social dynamics of cooperation. Thus, diversity not only is ubiquitous in all (free) societies but also appears as a fundamental mechanism toward the emergence of cooperative behavior. Similarly, diversity in physiology leads individuals to react differently to the same pathogen, and different software leads computers to exhibit diverse levels of protection to viruses. In such adaptive networks—the human web of social relations or the multilayered web of computer connections—individual diversity may profoundly alter their self-organization dynamics, calling for a coevolutionary process such as the one introduced here, given the unified way [6] in which dynamics on adaptive networks can be often

approached. The present formalism can be readily adapted to these problems. Work along these lines is in progress.

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