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Review

Climate change governance, cooperation and self-organization

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Abstract

When attempting to avoid global warming, individuals often face a social dilemma in which, besides securing future benefits, it is also necessary to reduce the chances of future losses. In this manuscript, we introduce a simple approach to address this type of dilemmas, in which the risk of failure plays a central role in individual decisions. This model can be shown to capture some of the essential features discovered in recent key experiments, while allowing one to extend in non-trivial ways the experimental conditions to regions of more practical interest. Our results suggest that global coordination for a common good should be attempted by segmenting tasks in many small to medium sized groups, in which perception of risk is high and uncertainty in collective goals is minimized. Moreover, our results support the conclusion that sanctioning institutions may further enhance the chances of coordinating to tame the planet's climate, as long as they are implemented in a decentralized and polycentric manner.

Keywords: Collective action; Complex systems; Global warming; Governance of the commons; Environmental agreements; Evolutionary game theory

1. Introduction

The most recent report of the Intergovernmental Panel on Climate Change (IPCC) accumulates evidence that Human activity is responsible for most of the Global Warming we have witnessed since the 50s. The recent World Summits set up to work out a solution to Global Warming have added up to a (now) long list of unsuccessful attempts to solve the Climate Change problem. Despite *i)* the actual risk of collective disaster, *ii)* the scientific consensus that anthropogenic greenhouse gas (**GHG**) emissions perturb global climate patterns with negative consequences for many

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ecosystems [1–3] and *iii*) the predictions of *early warning signals* and *severe climate change consequences* that are already in place, such as increased occurrence of heat waves and droughts [1], country leaders insist in discounting [4] the severity of the problem, given the scientific uncertainty regarding the impacts of climate change [5–7].

Similar to other public goods dilemmas of collective action [8], any participant that curbs emissions pays a cost while the benefits are shared between everyone. As a result, individuals, regions or nations often unilaterally opt to be *free riders*, thus benefiting from the efforts of others, while making no effort themselves. Mechanisms that act to promote and maintain cooperation based on joint decisions made by groups involving more than two individuals have been thoroughly investigated [8–12], leading to the formulation of N-person Public Goods games **(PGG)**, in which collective action often depends on the coordination, into cooperation, of a threshold number of group members.

Given the inevitable uncertainties regarding the schedule and consequences of **GHG**-induced climate change, collective action cannot be dissociated from the overall perception of risk that climate change imparts. This feature was confirmed by actual experiments [13]. In addition, the global nature of the problem, combined with the increasing levels of globalization, begs the question of who should participate in the summits, whether world citizens, cities [14], regions or country leaders, and to which extent the chances of overall cooperation depend on it. Another problem relates to the lack of sanctioning mechanisms to be imposed on those who do not contribute (or stop contributing) to the welfare of the planet [15–18]. Naturally, agreeing on the way punishment should be implemented is also far from reaching a consensus, given the difficulty in converging on the *pros* and *cons* of some procedures against others.

Each of the issues mentioned above may play a role of its own, as well as when combined with other issues, and the study of these effects and their interactions poses severe constraints from an experimental standpoint, calling for a theoretical framework in which such issues may be addressed in a unifying way. Here, we provide such a theoretical framework, which can be shown to capture some of the essential features discovered in recent key experiments, while allowing one to extend in non-trivial ways the experimental conditions to regions of more practical interest. To this end, we model N-person group interactions as a threshold Public Goods Game (**PGG**) in the presence of risk (introduced as an exogenous parameter), and allowing individual behaviors to change (evolve) in time [19,20], taking into consideration decisions and achievements of others, which influence one's own decisions [21–24]. This dynamical process is conveniently described in the framework of Evolutionary Game Theory (**EGT**). Moreover, we shall realistically consider the dynamics of finite (and small) populations, and allow for the fitness driven dynamics to occur in the presence of errors [25] and spontaneous exploration of the possible strategies [26], adding to the overall stochasticity of the dynamical process. Incidentally, this will also allow us to assess the validity of infinite population approximations often used in the literature in related contexts.

In the following section we introduce what we haughtily designate as "the standard model of Climate Change" we shall employ throughout this work. We investigate the effect of group size and risk perception on the chances of coordinating to save the planet's climate. We shall also investigate the role of sanctioning, when combined with risk, addressing the important point of whether to set up local or global sanctioning institutions. In this already complex scenario, we shall also study the effect of uncertainties regarding the targets required to reach collective cooperative action. We conclude that overall risk perception constitutes the most important ingredient determining collective cooperative action. In the presence of risk, sanctioning needs to be neither high nor does it require a global institution to supervise abidance (or opposition) to the agreement. Instead, multiple local institutions [15,27] may provide a solution to this "game that concerns all of us, and we cannot afford to lose" [28]. Threshold uncertainty, in turn, acts to obliterate the coordination towards collective action. Depending on the overall conditions, the consequences can vary from mild to disastrous. We close the text with conclusions and further discussion on future prospects.

2. Methods

The intricacies and variability of Human behavior are so complex that there is little hope that one is able to model theoretically, in detail, every aspect of a human population of decision makers. This problem is well-known to Physicists, where systems with too many degrees of freedom are more the rule rather than the exception. Hence, what at first seems to be an apparent disadvantage, however, is amenable to constitute an advantage from a Physics point of view. In fact, the interference of such a diverse plethora of complex behaviors may actually render the description of average salient properties and their evolution not only feasible but also they may turn out to be governed by considerably simpler laws [29–32]. For this reason, we make use of a variety of concepts related to the statistical mechanics of non-equilibrium stochastic processes. We start by describing how individuals interact in Section 2.1, whereas in Section 2.2. we describe the stochastic process which conveniently frames the strategy revision process in finite populations of individuals.

2.1. The game we cannot afford to lose

Individuals of a population of size Z are organized into groups of size N. Each individual has an initial endowment b (viewed as the asset value at stake) and individuals are classified according to their behavior regarding climate issues. In the simplest scenario, they are either Cooperators (**C**), who contribute a fraction cb of their endowment, or Defectors (**D**), who do not contribute anything. A successful agreement is reached if the overall number of contributions to the public good in the group exceeds a certain threshold ($n_{pg}cb$). In that case, all participants will keep whatever they have. Otherwise, if the goal is not met, with a probability r (the risk of collective disaster [13,33]) everyone in the group will lose whatever she/he had. In other words, failure to reach a given minimum contribution may imply – also depending on the risk (r) of disaster – that cooperators invest in vain and all endowments are lost. By imposing such a coordination threshold into a N-person cooperation dilemma [11,12,33–36] we are mimicking situations common to most of human public endeavors, including international environmental agreements [16,37–39], which demand a minimum number of ratifications to come into practice [40,41].

Overall, the payoff of **D**s and **C**s in a group of size N with j_C **C**s and $N - j_C$ **D**s can be summarized as

$$\begin{aligned} P^D(j_C) &= b\Theta(j_C - n_{pg}) + (1-r)b\big(1 - \Theta(j_C - n_{pg})\big) \\ P^C(j_C) &= P^D(j_C) - cb \end{aligned} \tag{1}$$

where $\Theta(k)$ is the Heaviside function (that is, $\Theta(k) = 1$ whenever $k \geqslant 0$, being zero otherwise).

The simplicity of this core model allows us to easily include additional strategies or more complex interactions. Indeed, in Section 6 we extend this model to incorporate a new strategy in connection with the study of emergence of sanctioning institutions, whereas in Section 7 n_{pg} is replaced by a probability distribution, allowing one to investigate the role played by (scientific, for instance) uncertainties in defining the coordination targets required to avoid a collective disaster. All variations introduced on this *standard model* will be detailed in the appropriate section.

2.2. Strategy revision and population dynamics

Unlike common treatments based on economic theory [16,37], we shall not rely on individual or collective rationality. Instead, we adopt a dynamical approach, in which individuals revise their strategies through *peer-influence*, copying others whenever these appear to be more successful. Such social learning (or evolutionary, in the sense of cultural evolution) approach allows policies to change in time [19,20,42] as individuals are influenced by the behavior (and achievements) of others, something one actually witnesses in the context of donations to public goods [21–23]. This also takes into account the fact that agreements may be vulnerable to renegotiation [16,37–39,43,44]. Moreover, we also include what is known as *random exploration of strategies*, or *strategy mutation*, to take into account all those additional circumstances that may lead individuals to change their behavior.

Formally, this setup can be conveniently implemented as a birth-death process with mutations [45], here combined with the pairwise comparison rule [25], also known as the Fermi process (see below). In a nutshell, at each (discrete and asynchronous) time-step, a random individual i will adopt the strategy of a randomly selected member of the population k with a probability p, which increases with the fitness difference between the two (where we employ the Fermi distribution function [25]).

In a stochastic mean field description, all those who behave as **C**s will have the same fitness, the same happening to all those behaving as **D**s. In other words, groups of N individuals are assembled at random from a population of size $Z \geqslant N$, such that each individual potentially interacts with any of the other $Z - 1$ players. Thus, individual fitness will be given by the average payoff resulting from all possible group interactions. Under such mean-field description, we are able to characterize a given state of the population by determining the fraction of individuals who (for instance) adopt a cooperative behavior at any time. In the following paragraphs we describe the details of the method, where we write down the equations in a general framework which will prove adequate to deal with the generalizations introduced in the following sections.

Let us consider a population of Z individuals, each of whom can be in one of s states, corresponding to different strategies or behaviors: $S_1, \ldots, S_s$, and let us study the time evolution of this population. Let $i_k(t)$ be, at a given time t, the number of individuals with strategy S_k, satisfying $\sum_{k=1}^{s} i_k(t) = Z$. At the mean-field level of description, the set of s integers will specify what we designate by a configuration of the population. Since individuals revise their strategy solely based on theirs and others present fitness, the evolutionary dynamics of the population constitutes a Markov process embedded in a phase space of dimension $d = s - 1$, where s is the number of strategies; transitions take place between different configurations of the population characterized by the vector $\mathbf{i}(t) = \{i_1, \dots, i_d\}$. Consequently, the Probability Density Function $p_{\mathbf{i}}(t)$ (**PDF**) associated with the process $\mathbf{i}(t)$ obeys the discrete time Master-Equation, Eq. (2), for some delta-shaped initial condition [46]. This allows one to compute $p_{\mathbf{i}}(t)$ given the transition probability from the configuration $\mathbf{i}$ to the configuration $\mathbf{i}' = \mathbf{i} + \mathbf{\Delta}$, $T_{\mathbf{i}}^{\mathbf{\Delta}}$, in the time interval τ:

$$p_{\mathbf{i}}(t+\tau) - p_{\mathbf{i}}(t) = \sum_{\mathbf{\Delta}} \left(T_{\mathbf{i}+\mathbf{\Delta}}^{-\mathbf{\Delta}} p_{\mathbf{i}+\mathbf{\Delta}}(t) - T_{\mathbf{i}}^{\mathbf{\Delta}} p_{\mathbf{i}}(t) \right) \tag{2}$$

The stationary solution for $p_{\mathbf{i}}$ is obtained by making the left side zero, in which case our problem reduces to an eigenvector search problem [46]. Additionally, it is possible to expand the right hand side of Eq. (2) in powers of $1/Z$ in order to obtain the Drift vector field, that is, the so-called *gradient of selection* [11], $\mathbf{g}$, which provides information on the most likely direction of change of the population configuration with time. As shown in Appendix A, this is given by the difference between the probabilities of increasing and decreasing the number of individuals of a given strategy:

$$g_k(\mathbf{i}) = T_{\mathbf{i}}^{S_k+} - T_{\mathbf{i}}^{S_k-}. \tag{3}$$

The above transition probabilities depend on individual fitness, which defines how good a strategy is. At the mean-field level, each individual potentially interacts with any of the other $Z - 1$ players when assembling groups of size N. In each group interaction, individuals acquire a game payoff, such that individual fitness will be given by the average payoff resulting from all possible group compositions. This is readily computed employing hypergeometric sampling without replacement [11,33,47,48]. Thus, we may associate with each strategy, S_k, and for each configuration $\mathbf{i}$, a well-defined fitness, $f_i^{S_k}$, which results from the interactions with the other players (with given strategies). Let $\mathbf{j} = \{j_1, \dots, j_d\}$ be the configuration of all but one players in a group of size N ($\mathbf{j}$ is defined in the same way as $\mathbf{i}$ but replacing Z by $N - 1$, i.e., the configuration of the group interacting with a given player). Then, $f_{\mathbf{i}}^{S_k}$ is given (for an arbitrary number of strategies) by

$$f_{\mathbf{i}}^{S_k} = \binom{Z-1}{N-1}^{-1} \sum_{j_1+\dots+j_s \leqslant N-1} P_{\mathbf{j}}^{S_k} \binom{i_k - 1}{j_k} \prod_{l \neq k}^{s} \binom{i_l}{j_l} \tag{4}$$

where $P_{\mathbf{j}}^{S_k}$ is the payoff of an individual with strategy S_k in an N-person game with configuration $\mathbf{j}$, which must be such that the group of N contains at least one individual (the focal individual) with strategy S_k [11,12,27,33,48,49]. As a result, we write up the transition probability that an individual with a given strategy, S_l, changes into another specific strategy, S_k, $T_{S_l \to S_k} \equiv T_{\mathbf{i}}^{\mathbf{\Delta}=\{0_1,\dots,-1_l,\dots,+1_k,\dots,0_d\}}$ in the following way, making use of the Fermi update rule or pairwise comparison rule [25,50] with mutation:

$$T_{S_l \to S_k} = \frac{i_l}{Z} \left(\frac{i_k}{Z-1} \frac{1-\mu}{1+\exp(\beta \Delta_{S_l S_k})} + \frac{\mu}{d} \right) \tag{5}$$

where $\Delta_{S_l S_k} = f_i^{S_l} - f_i^{S_k}$, μ stands for the mutation probability and β, the inverse of the temperature, translates into noise associated with errors in the imitation process.

3. The effect of risk

We introduce the risk r as the probability of losing the benefit if the threshold is not met, inspired by the risk implementation adopted in behavioral experiments between **C**s and **D**s [13] on the same issue. Thus, the present model includes key factors present in any real setting, such as bounded rational individual behavior, peer-influence and the importance of risk assessment in meeting the goals defined.

In the context of evolutionary game theory, the evolution in time of the fraction x of **C**s (and $1 - x$ **D**s) in infinite populations can be conveniently described by means of a gradient of selection $g(x)$ associated to the so-called replicator equation $g(x) \equiv \dot{x} = x(1-x)(f_C(x) - f_D(x))$, which characterizes the rate of change of **C**s in the population.

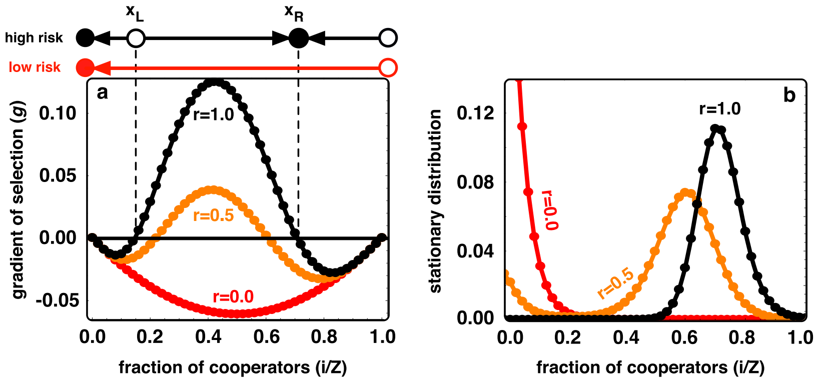

Fig. 1. Gradient of selection and prevalence of cooperation in finite populations. The right panel shows the stationary distribution corresponding to the prevalence of each fraction of Cs that emerges from the discrete gradient of selection $g(i)$ shown in the left panel. Each curve corresponds to a different value of risk, as indicated. Whenever risk is high, stochastic effects turn collective cooperation into a pervasive behavior, rendering cooperation viable and favoring the overcome of coordination barriers, irrespective of the initial configuration ($Z = 50$, $N = 6$, $n_{pg} = 3$, $c = 0.1$, $\mu = 0.005$, $\beta = 5.0$).

In this context and in the absence of risk ($r = 0$), it can be shown [33] that $g(x)$ is always negative, leading the population to full defection. Increasing risk, in turn, leads to the emergence of two mixed internal *equilibria* (x_L and x_R), rendering cooperation viable: for finite risk r, both **C**s (for ($x < x_L$) and **D**s (for ($x > x_R$) become disadvantageous when rare, turning co-existence between **C**s and **D**s stable at a fraction x_R which increases with r. Collective coordination becomes easier to achieve under high-risk and, once the coordination barrier (x_L) is overcome, high levels of cooperation will be reached. For fixed (and low) c/r, increasing n_{pg} will maximize cooperation (increase of x_R) at the expense of making it more difficult to emerge (increase of x_L) [33,47].

Without dismissing the usefulness of such analysis, real populations are finite and often rather small. This means that one has to be careful before inferring the overall behavior based on results obtained using infinite population approximations, which may be too simplistic [51]: Indeed, stochastic effects do play an important role, in particular, for the case of the world summits where group and population sizes are comparable and of the order of the hundreds. The finite population equivalent of the replicator equation corresponds to the discrete gradient of selection introduced in Eq. (3) – $g(i) = T_i^+ - T_i^-$, i.e., the difference between the probabilities to increase and decrease the number i of **C**s in the population by one. If we plot the gradient of selection of a finite population, $g(i)$, shown in the left panel of Fig. 1, we can describe the general behavior of the population in a manner which is qualitatively identical to that obtained from $g(x)$ in infinite populations [33,47] although only an analysis of the stationary distributions $p_\mathbf{i}$ – which provides information on the fraction of time the population spends in each possible configuration (specified by a given number of **C**s in the population) – allows us to assess the importance of the roots and magnitude of $g(i)$.

In the right panel of Fig. 1 we show the stationary distributions for different values of risk. They show that the population spends most of the time in configurations where **C**s prevail, irrespectively of the initial condition. This is a direct manifestation of the role of stochastic effects, which allow the "tunneling" through the coordination barrier associated with x_L, rendering such coordination barrier (x_L) irrelevant when extant and turning cooperation into the prevalent strategy. On the other hand, the existence of a stable root of $g(i)$ (probability attractor) leads to a maximum of $p_\mathbf{i}$ at this position.

4. Scale of agreements

Not only the total number of individuals taking part in the decision making process (Z), but also the group size (N) in which decisions are made can influence the dynamics. Both of these give us a notion of the scale at which the agreements should be attempted. Indeed, when one thinks on current attempts to achieve global agreements, we are lead to a portrait of a single global group of players, of the same size of the population Z. Sadly, when one increases the group size N, maintaining constant the population size Z, one observes a sharp reduction of the interval of *cost-to-risk ratios* in which a defection dominance dilemma is replaced by a combination of coordination and co-existence dilemmas; this implies a reduction of the overall levels of cooperation. In other words, the higher the

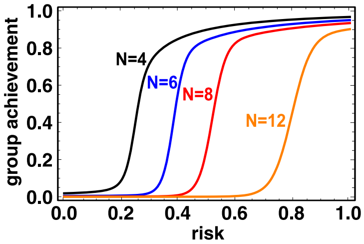

Fig. 2. Group size dependence for $n_{pg} = N/2$. Cooperation will be maximized in small groups, where the risk is high and goal achievement involves stringent requirements. Other parameters: $Z = 100$, $\mu = 1/Z$, $c = 0.1$, $b = 1$, $\beta = 1.0$.

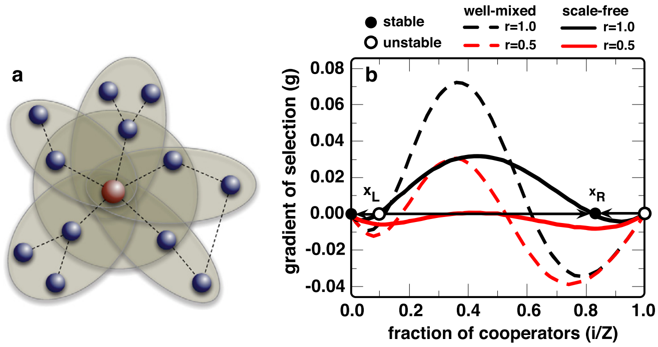

Fig. 3. a) Given an interaction network of size Z and average degree $\langle k\rangle$, where nodes represent individuals, and links exchanges or shared goals, N-person collective dilemmas may be setup in groups, each associated with a neighborhood in this network. As an example, the central individual participates in 6 groups, such that the individual fitness derives from the payoff accumulated from all games she/he participates. b) Gradients of selection $g(i)$ obtained analytically for homogeneous (well-mixed) setting and numerically for heterogeneous (scale-free) networks, for different values of risk ($n_{pg} = 3$, $Z = 500$, $\langle N\rangle = 7$, $\beta = 5.0$). In the heterogeneous case, we compute numerically the average probability that each $C(D)$ imitates a $D(C)$ randomly chosen from the population. $g(i) = T^{+}(i) - T^{-}(i)$ results from an average over 2×10^4 different distributions and 10 different *scale-free* interaction structures.

ratio N/Z the higher the perception of risk needed to achieve cooperation. In order to show such a striking dependence of cooperation on the scale at which agreements are tried, in Fig. 2 we resort to a useful quantity which can be used to compare our results directly with experimental observation, and that we can easily compute. To this end we start by computing the fraction of groups that succeed in overcoming the collective coordination problem for each behavioral configuration i, which we denote by $a_G(i)$. Since the stationary distribution $\bar{p}_i$ gives the pervasiveness in time of each possible behavioral composition of the population, the computation of the average fraction of groups that successfully produce (or maintain) the public good – a quantity we designate as "group achievement", η_G – can now be trivially determined from these two quantities:

$$\eta_G = \sum_i \bar{p}_i a_G(i). \tag{6}$$

The results shown in Fig. 2 indicate that Cooperation is better dealt with within small groups, contrary to world's most common attempts to solve the climate change problem. This trend remains valid both when the coordination threshold n_{pg} is constant and when it increases linearly with the group size [33].

5. Networks of overlapping agreements

In the previous section, we highlighted the importance of self-organized cooperation among multiple groups instead of aiming at a single successful agreement including the entire population. Yet, this result begs the question of how in fact these groups should be organized. One may easily anticipate that a few collective endeavors will involve a large number of participants, while many will involve just a few. Similarly, diversity in geographical positions, together with the complexity of political configurations, means that some *players* may face a larger number of collective action challenges than others. The overall number and size of the dilemmas faced by each player may be seen as a result of a complex interaction network, where nodes represent individuals, and links represent exchanges, collective investments or shared interests [52]. This idea is sketched in Fig. 3a, where each neighborhood of such a structure may represent a group of a size given by the connectivity of the focal individual (plus the focal individual). Given that, in reality, interaction structures are mostly heterogeneous, here we investigate the effect of such heterogeneity [52] in the problem at stake, using as a base reference the evolutionary dynamics in a homogeneous, well-mixed (WM) finite population, and the respective gradient of selection shown in the left panel of Fig. 1. For the heterogeneous case, we adopt the ubiquitous power law distribution of group sizes, resulting from a *scale-free* interaction network of contacts, assembled via growth and preferential attachment [53].

In Fig. 3b, we show the result of the gradient of selection $g(i)$ for both homogeneous and heterogeneous structures, showing that the existence of a heterogeneous distribution of group sizes further increases the chances of success in coordinating towards a collective good [33], by significantly raising (at high risk) the fraction of **C**s at which co-existence occurs. This favorable effect of diversity is widespread, extending to other cooperation dilemmas [52, 54]. In this case, the benefit results from the different nature of the games played in different sites. Because for fixed n_{pg} coordination is better achieved in large groups, highly connected players (hubs) at the group centers will acquire a larger fitness. Whenever hubs happen to be occupied by **C**s [52], they will influence the participants of small groups (the majority) to cooperate, hence enabling small groups to overcome their more stringent coordination requirements. This positive feedback vanishes, however, for lower levels of collective risk, indicating once again how the risk of collective failure constitutes one of most important variables whenever the emergence of cooperation is at stake in public goods games.

6. The role and scale of sanctioning institutions

Another issue often associated with the limited success of existing attempts to reach global cooperation is the lack of sanctioning institutions and mechanisms to deal with those who do not contribute to the welfare of the planet. In this section, we investigate the impact of two distinct types of institutions in deterring non-cooperative behaviors, in particular when those institutions may be most needed, i.e., when the overall perception of risk is low.

As shown in the previous section, to let the entire population form a single group engaging in the threshold **PGG** is detrimental to cooperation [33] and, hence, it is much better to establish smaller groups. Thus, it is in a scenario of variable (and preferentially small) group size that one should assess the role of sanctioning institutions in the presence of risk [27]. Individual decisions, which evolve in time according to the standard model, should also evolve regarding the behavior towards the existence and maintenance of sanctioning institution(s).

To this end, we introduce a new behavioral strategy – the Punishers (**P**). Like **C**s, **P**s contribute to the public good; unlike **C**s, **P**s also contribute with a *punishment tax* (π_{tax}) to an institution which, whenever endowed with enough funding ($n_p\pi_{tax}$) will effectively punish **D**s by fining them by an amount π_{fine}. Hence, establishing a sanctioning institution stands as a new, "second-order" public good [19,55], which is only achieved when a certain *threshold number of contributors* n_p [11] is reached. The payoff of **P**s and the modified Payoffs of **C**s and **D**s are now given by

$$\begin{aligned}
P^C(j_C, j_P) &= -cb + b\Theta(j_C + j_P - n_{pg}) + (1-r)b\big(1 - \Theta(j_C + j_P - n_{pg})\big),\\
P^P(j_C, j_P) &= P^C(j_C, j_P) - \pi_t,\\
P^D(j_C, j_P) &= P^C(j_C, j_P) + cb - \Delta_{scale},
\end{aligned} \tag{7}$$

where Δ_{scale} is given by $\Delta_{scale}^{local} = \pi_{fine}\Theta(j_P - n_p)$ whenever local institutions are being considered, and by $\Delta_{scale}^{global} = \pi_{fine}\Theta(i_P - n_p)$ whenever global institutions are at stake.

The fact that both public goods dilemmas contain thresholds implies that contributors may pay a cost in vain, thus increasing the realism (and the inherent complexity) of the decision process modeled here. Also, the institution need not be global (such as the United Nations) – supported by all **P**s – that overviews all group-interactions in the population. Institutions may also be local – created by **P**s within each group – to enforce cooperation in that group. In what follows we shall consider both cases computing, in all of them, the "group achievement", η_G (see Eq. (6)).

Empirical results (in the absence of any sanctioning) show that group achievement increases with the value of risk [13], correlating nicely with the dependence shown in Fig. 4 with a black dotted line. Indeed, Fig. 4 shows the behavior of η_G as a function of *risk* in the absence of any institutions (black dotted line), under one *global* institution (red dashed line) and under *local* institutions (blue solid line). Comparison between the three curves shows that global

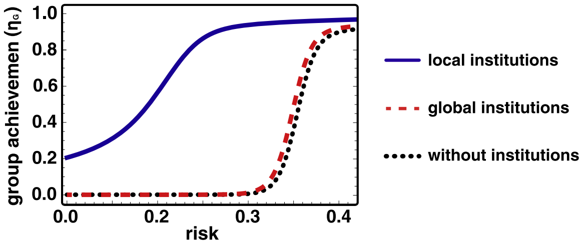

Fig. 4. Group achievement. Group achievement $-\eta_G$ (average fraction of groups that are able to attain the public good) is shown as a function of *risk* (r). The red dashed line corresponds to sanctions that are enacted by a *global*, population-wide institution, responsible for fining all those who do not contribute to the pubic goods game. The blue line shows results obtained from *local institutions*, responsible for fining those (in the group) who do not contribute. The black dotted line can be used as reference, corresponding to the results obtained in absence of any kind of institution. Global institutions provide at best marginal improvements of overall cooperation. The coordination threshold (n_{pg}) is set to 75% of the group size, whereas local (global) institutions are created whenever 25% of the group (population) contributes to its establishment (n_p). Punishment tax is $\pi_{tax} = 0.03$, whereas the punishment fine for defecting is $\pi_{fine} = 0.3$. Other parameters: $Z = 100$, $N = 4$, $c/b = 0.1$, $\mu = 1/Z = 0.01$ in (a) and (c), and $r = 0.3$ in (b) and (d). See detailed definitions in main text. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

institutions may provide a marginal improvement compared to no institutions at all, a scenario akin to most climate agreements attempts [16,37,47]. On the contrary, under *local*, group-wide, *sanctioning institutions*, group achievement is substantially enhanced, in particular for low values of the perception of risk and whenever individuals face stringent requirements (high n_{pg}) to avoid a collective disaster [27], a feature that we predict will remain valid for all values of risk and group sizes.

The success of local institutions is closely connected with their resilience. Local institutions prevail for longer periods than a (single) global one, promoting systematically more widespread cooperation than global ones. The efficiency of both kinds of institutions is also enhanced in those situations in which participants change their decisions more frequently [27]. This scenario may be relevant, given the multitude of (often conflicting) factors that contribute to the process of decision-making [17,23,27]. Nonetheless, we find that neither local nor global institutions are robust to free riding, a result which has been recently confirmed experimentally [56]. Finally, behavioral mutations enhance the incidence of population configurations in which a diversity of strategies coexist, which in turn increases the chances of having enough **P**s to establish institutions and cooperation. Thus, whenever perception of risk of collective disaster, alone, is not enough to ensure global cooperation, better conditions both for cooperation to thrive and for ensuring the maintenance of such institutions can be achieved by a decentralized, polycentric, bottom-up approach [15], involving multiple institutions instead of a single global one.

7. Collective action under threshold uncertainty

A potentially unavoidable issue in climate negotiations is the role played by uncertainties associated with incomplete information regarding what targets must be met to tame the planet's climate [7,57]. Such *threshold uncertainties*, which are not directly related to the overall risk perception dealt with already, have been shown in recent experiments to play a very important role [7]. Indeed, for big enough uncertainties, and in the language of the standard model introduced here, the game can change from a N-person coordination game into a N-person Prisoner's Dilemma.

In what follows, we introduce threshold uncertainty by replacing the sharp threshold n_{pg} defined previously by a "fuzzy" threshold which, at any time, is drawn from a uniform probability distribution over $[n_{pg} - \delta, n_{pg} + \delta]$. This is the simplest possible assumption, although the results for other possible profiles of the probability distribution will not change the qualitative nature of the conclusions that will be drawn here. The larger is δ, the larger the uncertainty associated with the threshold. Fig. 5a shows how this uncertainty induces the regime shift described above. This shift leads, in turn, to a radical change in the profile of the stationary distribution, shown in Fig. 5b, with corresponding impact in the likelihood of group achievement $\eta_G(r)$, shown in Fig. 5c. As stated above, the precise shape of the threshold uncertainty profile is of little importance in what concerns the main message stemming from its effect: Threshold uncertainty may lead to a sudden regime shift, as observed experimentally [7].

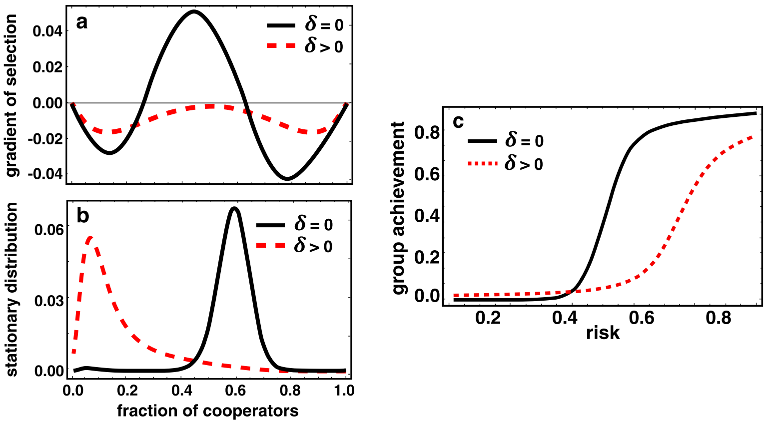

Fig. 5. Threshold uncertainty effect. Panel (a) shows the gradient of selection, while (b) shows the stationary distribution, that is, the fraction of time the population spends in each population composition specified by the fraction of cooperators. Panel (c) shows the fraction of groups that are successful in overcoming the threshold as a function of the risk r. The black lines provide results for no threshold uncertainty ($\delta = 0$) whereas the red dashed lines show results for $\delta = 2.75$. Other parameters are: $Z = 200$, $N = 8$, $M = 4$, $c = 0.1$, $b = 1.0$, $\beta = 6.0$, $\mu = 1/Z$ and $r = 0.6$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

8. Conclusions

When modeling decision-making processes associated with environmental sustainability one cannot overlook the uncertainty associated with the process of attempted collective action. This uncertainty can be understood both in terms of the risk of failure to comply with the challenge at stake, or in terms of the uncertainty associated with incomplete information on what targets must be met to tame the Planet's Climate. Here we propose a simple way of modeling these two types of uncertainty, which not only agrees with the limited data compiled in recent experiments [13,33], but also largely extends the range of conditions in which to search for solutions to the problem, allowing us to make several concrete predictions. We do so in the framework of N-person evolutionary game theory, a dynamical systems framework of modeling (e.g., political) decision-making, combined with tools from many body stochastic processes of non-equilibrium statistical mechanics. We use a N-person game where the risk of collective failure is explicitly introduced on top of a "*simple*" collective coordination dilemma. Moreover, instead of resorting to complex and rational planning or rules, combined with inductive reasoning, individuals revise their behavior by peer-influence, creating a complex dynamics akin to many evolutionary and ecological systems. This framework allowed us to address the impact of risk and different types of sanctioning in several configurations, from large to small groups, from deterministic to stochastic behavioral dynamics and from well-mixed to networked populations. The same framework was further extended to investigate the role of threshold uncertainty in risky public goods games.

Overall, we have shown how the emerging dynamics depends on the perception of risk. The larger the overall perception of risk, the easier it is to successfully coordinate for the global good. Large threshold uncertainties, in turn, can overshadow the somewhat bright message stemming from a pure risky world without them. Clearly, in the framework adopted here, agreements to be sought are necessarily short-term, giving individuals the opportunity to revise their strategy frequently. This, is turn, may contribute to minimize threshold uncertainty, since short-term goals are easier to define than long-term ones.

Our model clearly predicts that a decentralized, polycentric, bottom-up approach [15], involving multiple institutions instead of a single, global one, provides better conditions both for cooperation to thrive and for ensuring the maintenance of such supervising institutions.

Altogether, these results call for a reassessment of policies towards the promotion of public endeavors. Global institutions, such as the UN, do not increase the odds of overcoming the climate change problem. On the other hand, decentralized agreements between smaller groups (small N), possibly focused on region-specific issues, where risk is high and goal achievement involves tough requirements (large relative n_{pg}), are prone to significantly raise the probability of success in coordinating to tame the planet's climate. Thus, collective cooperation is easier to achieve in a distributed way, eventually involving regions, cities [58], *NGO*s and, ultimately, all groups of citizens. Moreover, by promoting regional or sectorial agreements, we are opening the door to the diversity of economic and political structure of all parties, which, as shown, can be beneficial to cooperation.

Present day local initiatives, such as the **WCI** (Western Climate Initiative [7]), have started with a small group of US-states. With time, the **WCI** group-size has grown to include additional Canadian and Mexican provinces. Recently, the emergence of similar initiatives involving Central and East-Coast US-states (and, again, Canadian provinces) has fostered the appearance of the America 2050 initiative, providing an example of a hierarchical bottom-up approach in dealing with Climate Change issues. Such a hierarchical aggregation, already envisioned theoretically [15] may be able to overcome the fact that larger groups are clearly more difficult to coordinate (at once) into widespread cooperation (see Sections 4 and 6).

We believe that the insights provided by our *standard model* of climate change provide important clues, whose causes are easy to trace, in what regards devising agreements towards a sustainable planet. We believe that the motto "*think globally, act locally*" nicely encapsulates the conclusions in this work.

Naturally, we are aware of the many limitations of a bare model such as ours, in which the complexity of Human interactions has been overlooked. From higher levels of information [59] and reactive behaviors [60], to non-binary investments, additional layers of realism can be added to our model. We believe the introduction of wealth inequality [59,61] is an important ingredient that is not included in this work (see also Ref. [62]). These add-ons, in turn, may render analytical treatments such as those employed here, impossible, leading one to resort to agent-based computational models [63–65]. The present results, however, settle the conditions under which successful negotiations may arise in the absence of all these refinements. Clearly, the simplicity of the dilemma introduced here constitutes both the strength and the weakness of the present model. For instance, Humans have devised ingenious ways of cooperating and defecting which extend beyond the unconditional strategies modeled here [55]. We believe that the solution of the threshold uncertainty problem (circumventing also the difficulties that humans generally face, at present, to deal with uncertainty) may precisely lie in the possibility of employing more complex strategies, such as conditional cooperation [60]. Work along these lines is in progress. Notwithstanding, and when combined with the existing arsenal of tools that Humans have successfully developed throughout their history to coordinate into cooperation, our decentralized, bottom-up approach, provides reasons of hope to win the "game that concerns all of us, and we cannot afford to lose" [28]. Last but not least, similar dynamical situations, in which cooperation nucleating in a small group expands into a larger and larger group, provide a plethora of problems to which tools such as those here unfolded may apply. In this sense, the impact of these results may go well beyond decision-making towards global warming.

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Appendix A. KM coefficients and gradient of selection

A stochastic, Markov, birth-death process $\mathbf{i}(t)$ follows the Master-Equation, Eq. (2). Here we show how to obtain the *gradient of selection*, $\mathbf{g}$ (Drift vector field), by performing a Kramers–Moyal (KM) expansion of the right side of Eq. (2) in powers of $1/Z$ (we shall make use of definitions introduced in Section 2.2).

We start by rewriting in terms of the fraction of individuals in the population, $\mathbf{x} = \mathbf{i}/Z$, all transition probabilities and statistical properties defined in the main text as functions of the number of individuals using different strategies, $\mathbf{i}$. Thus, the discrete stochastic process $\mathbf{x}$ will become a continuous process as $Z \to +\infty$:

$$\begin{aligned} & x_k \equiv i_k/Z, \quad \boldsymbol{\delta} \equiv \boldsymbol{\Delta}/Z, \\ & p_{\mathbf{i}}(t) \to p(\mathbf{x}, t) \\ & T_{\mathbf{i}}^{\boldsymbol{\Delta}} \to T^{\boldsymbol{\delta}}(\mathbf{x}) \\ & \rho(\mathbf{x}, t) \equiv Z^d p(\mathbf{x}, t). \end{aligned} \tag{i}$$

$T^{\boldsymbol{\delta}}(\mathbf{x})$ now represents a transition from configuration $\mathbf{x}$ to configuration $\mathbf{x}' = \mathbf{x} + \boldsymbol{\delta}$, in the direction $\boldsymbol{\delta}$. Rewriting the Master-Equation we get

$$\rho(\mathbf{x}, t+\tau) - \rho(\mathbf{x}, t) = \sum_{\boldsymbol{\delta}\neq 0}\big(T^{-\boldsymbol{\delta}}(\mathbf{x}+\boldsymbol{\delta})\rho(\mathbf{x}+\boldsymbol{\delta}, t) - T^{\boldsymbol{\delta}}(\mathbf{x})\rho(\mathbf{x}, t)\big). \tag{ii}$$

We expand the terms $T^{-\boldsymbol{\delta}}(\mathbf{x}+\boldsymbol{\delta})\rho(\mathbf{x}+\boldsymbol{\delta}, t)$ on right hand side in powers of $\boldsymbol{\delta}$ (which scales with $1/Z$, see Eq. (i)). We obtain:

$$T^{-\boldsymbol{\delta}}(\mathbf{x}+\boldsymbol{\delta})\rho(\mathbf{x}+\boldsymbol{\delta}, t) = T^{-\boldsymbol{\delta}}(\mathbf{x})\rho(\mathbf{x}, t) + \sum_{n=1}^{+\infty}\frac{1}{n!}\sum_{k_1,\ldots,k_n}^{d}\left[\prod_{m=1}^{n}\delta_{k_m}\frac{\partial}{\partial x_{k_m}}\right]T^{-\boldsymbol{\delta}}(\mathbf{x})\rho(\mathbf{x}, t). \tag{iii}$$

Summing all terms without derivatives leads to

$$\sum_{\boldsymbol{\delta}\neq 0}\big(T^{-\boldsymbol{\delta}}(\mathbf{x})\rho(\mathbf{x}, t) - T^{\boldsymbol{\delta}}(\mathbf{x})\rho(\mathbf{x}, t)\big) = \sum_{\boldsymbol{\delta}\neq 0}\big(T^{\boldsymbol{\delta}}(\mathbf{x}) - T^{\boldsymbol{\delta}}(\mathbf{x})\big)\rho(\mathbf{x}, t) = 0. \tag{iv}$$

The KM coefficients can be obtained by reorganizing the remaining terms:

$$\begin{aligned}
&\sum_{\boldsymbol{\delta}\neq 0}\sum_{n=1}^{+\infty}\frac{1}{n!}\sum_{k_1,\ldots,k_n}^{d}\left[\prod_{m=1}^{n}\delta_{k_m}\frac{\partial}{\partial x_{k_m}}\right]T^{-\boldsymbol{\delta}}(\mathbf{x})\rho(\mathbf{x}, t)\\
&\quad=\sum_{n=1}^{+\infty}(-1)^n\sum_{k_1,\ldots,k_n}^{d}\sum_{\boldsymbol{\delta}\neq 0}\left[\prod_{m=1}^{n}\delta_{k_m}\prod_{l=1}^{n}\frac{\partial}{\partial x_{k_l}}\right]\frac{(-1)^n}{n!}T^{-\boldsymbol{\delta}}(\mathbf{x})\rho(\mathbf{x}, t)\\
&\quad=\frac{1}{Z}\sum_{n=1}^{+\infty}(-1)^n\sum_{k_1,\ldots,k_n}^{d}\prod_{l=1}^{n}\frac{\partial}{\partial x_{k_l}}\left(Z\frac{(-1)^n}{n!}\sum_{\boldsymbol{\delta}\neq 0}\left[\prod_{m=1}^{n}\delta_{k_m}\right]T^{-\boldsymbol{\delta}}(\mathbf{x})\right)\rho(\mathbf{x}, t).
\end{aligned} \tag{v}$$

Formally, they can be written

$$D^{(n)}_{k_1,\ldots,k_n} = Z\frac{(-1)^n}{n!}\sum_{\boldsymbol{\delta}\neq 0}\left[\prod_{m=1}^{n}\delta_{k_m}\right]T^{-\boldsymbol{\delta}}(\mathbf{x}). \tag{vi}$$

The gradient of selection is simply $g_k(\mathbf{x}) = D^{(1)}_k(\mathbf{x})$, whereas the diffusion term is given by $D^{(2)}_{kl}(\mathbf{x})$. Notice that the n-th KM coefficient contains a factor which scales roughly as $Z|\boldsymbol{\delta}|^n$, which shows that higher order terms in this KM expansion are increasingly small. Moreover, it shows that the dynamics of infinite populations becomes deterministic, since all coefficients but the first tend to zero and, therefore, its Langevin equation is no longer a stochastic differential equation (see Eq. (x) and Eq. (xi)).

For the birth-death process with Fermi update we consider in the main text, we obtain for the gradient of selection:

$$\begin{aligned}
g_k = D^{(1)}_k &= -Z\sum_{\boldsymbol{\delta}\neq 0}\delta_k T^{-\boldsymbol{\delta}}(\mathbf{x}) = -Z\sum_{\boldsymbol{\delta}:\ \delta_k\neq 0}\delta_k T^{-\boldsymbol{\delta}}(\mathbf{x})\\
&= -\frac{Z}{Z}\left(\sum_{\boldsymbol{\delta}:\ \delta_k\neq 1/Z}T^{-\boldsymbol{\delta}}(\mathbf{x}) - \sum_{\boldsymbol{\delta}:\ \delta_k\neq -1/Z}T^{-\boldsymbol{\delta}}(\mathbf{x})\right)\\
&= -\left(\sum_{\boldsymbol{\delta}:\ \delta_k\neq 1/Z}T^{-\boldsymbol{\delta}}(\mathbf{x}) - \sum_{\boldsymbol{\delta}:\ \delta_k\neq 1/Z}T^{\boldsymbol{\delta}}(\mathbf{x})\right)\\
&= T^{S_k+} - T^{S_k-}
\end{aligned} \tag{vii}$$

with

$$T^{S_k\pm} = \sum_{\boldsymbol{\delta}:\ \delta_k\neq 1/Z}T^{\pm\boldsymbol{\delta}}(\mathbf{x}). \tag{viii}$$

Using the symmetry $\Delta_{S_l S_k} = -\Delta_{S_k S_l}$ and the identity

$$\big(1+\exp(x)\big)^{-1}-\big(1+\exp(-x)\big)^{-1}=\tanh(-x/2)$$

we can write

$$\begin{aligned}T^{S_k+}-T^{S_k-}&=\sum_{l\neq k}^{s}\left[\frac{i_l}{Z}\left(\frac{i_k}{Z-1}\frac{1-\mu}{1+\exp(\beta\Delta_{S_lS_k})}+\frac{\mu}{d}\right)-\frac{i_k}{Z}\left(\frac{i_l}{Z-1}\frac{1-\mu}{1+\exp(\beta\Delta_{S_kS_l})}+\frac{\mu}{d}\right)\right]\\&=\frac{1-\mu}{Z(Z-1)}i_k\sum_{l\neq k}^{s}\big[i_l\tanh(\beta\Delta_{S_kS_l}/2)\big]+\frac{\mu}{dZ}(N-si_k).\end{aligned}\tag{ix}$$

Taking into account the first two terms in this expansion, one is led to a Fokker–Planck equation which can be transformed into the Langevin equation below

$$\frac{d(\mathbf{i}/Z)}{dt}=\mathbf{g}(\mathbf{i})+\sqrt{\mathbf{D}^{(2)}(\mathbf{i})}\boldsymbol{\Gamma}(t)\tag{x}$$

where, in the Itô–Langevin interpretation, $\mathbf{g}(\mathbf{i})=\mathbf{D}^{(1)}(\mathbf{i})$, with $\sqrt{\mathbf{D}^{(2)}(\mathbf{i})}$ a symbolic notation for a matrix that satisfies $\sqrt{\mathbf{D}^{(2)}}\sqrt{\mathbf{D}^{(2)}}^{T}=\mathbf{D}^{(2)}$. $\boldsymbol{\Gamma}(t)$, in turn, is a delta-correlated random variable with zero mean [46,66,67]. In one dimension, i.e., considering only two strategies ($d=s-1=1$), this equation reduces to

$$\frac{d(i/Z)}{dt}=T_i^{+}-T_i^{-}+\frac{1}{\sqrt{Z}}\sqrt{\frac{T_i^{+}+T_i^{-}}{2}}\Gamma(t).\tag{xi}$$

For large populations, the fluctuations in the fraction of individuals of a given strategy become increasingly small compared to their actual value. Therefore, neglecting the stochastic term in Eq. (x), one finds a system of ordinary differential equations, closely related to the famous *replicator equation* [68] of population dynamics – in fact, Eq. (xi) leads to the *replicator equation* whenever an expansion to first order in β is carried out, which corresponds to the weak-selection approximation, often combined with the infinite population approximation in many theoretical studies of population dynamics.

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