An introduction to
Plasma Tomography

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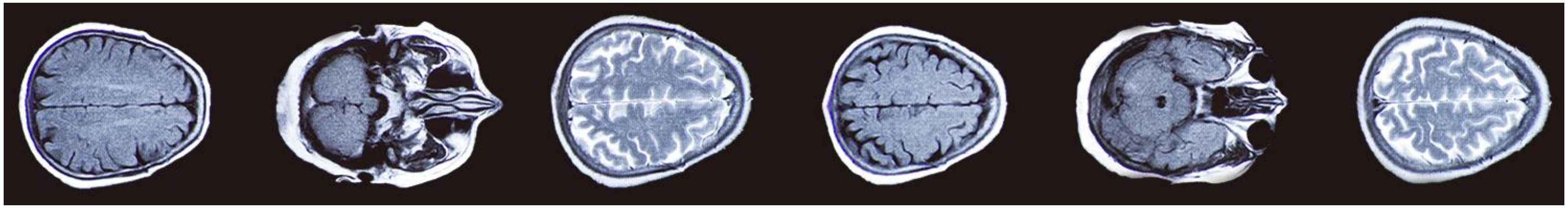
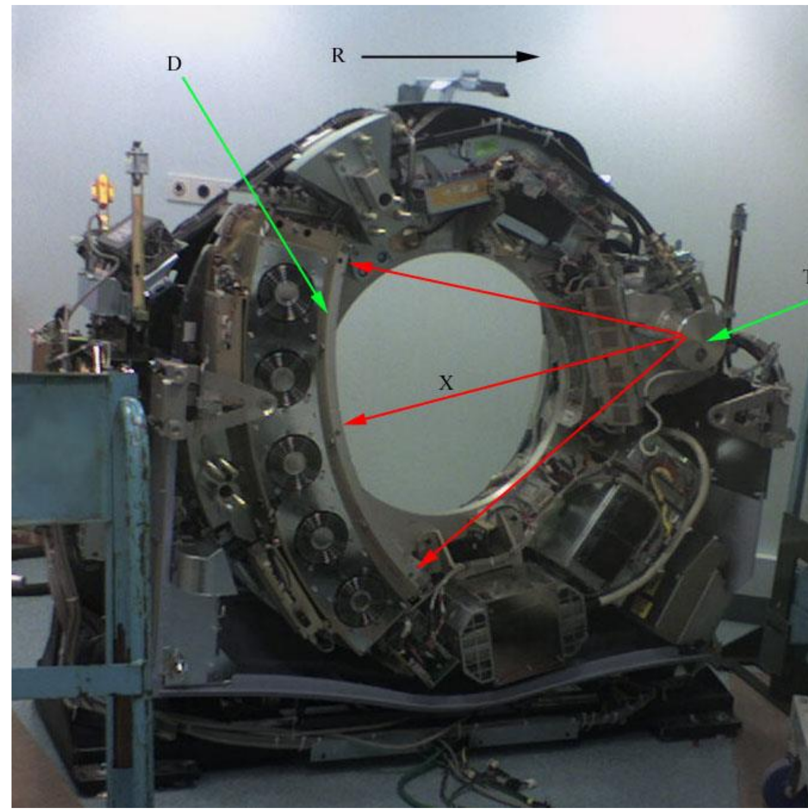
Computed Tomography

- Medical applications



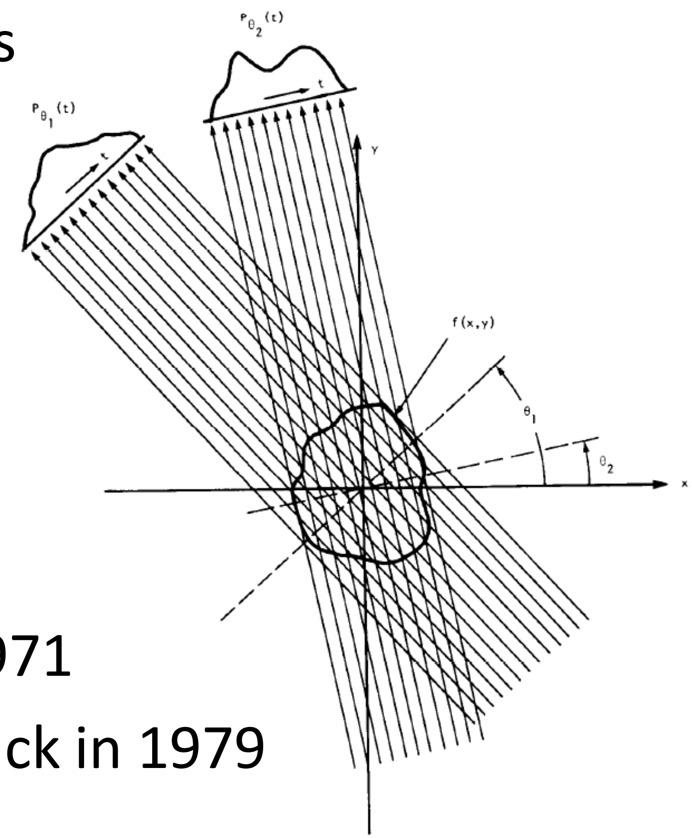
Computed Tomography

- CT scanner internals



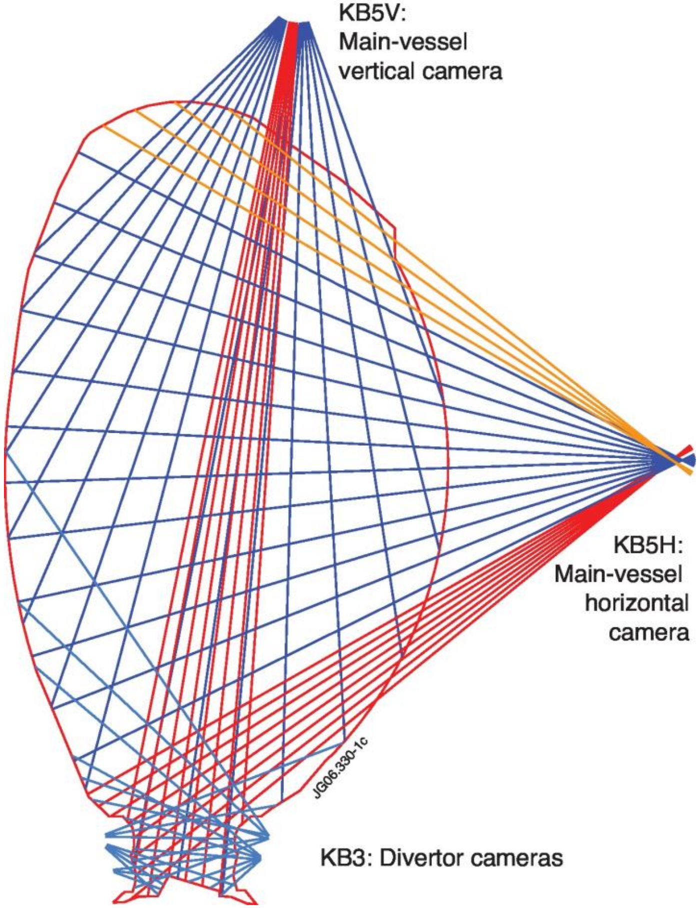
Computed Tomography

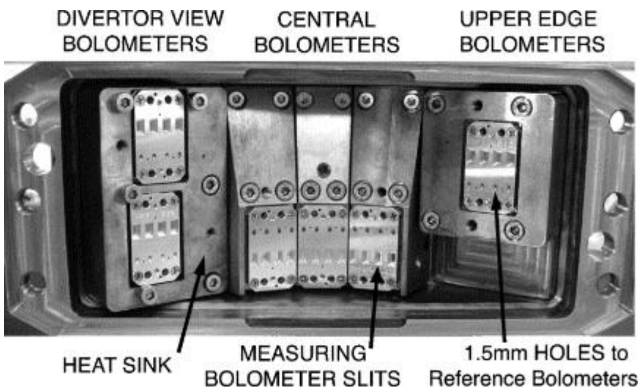
- Tomography problem
  - reconstruct image from its projections
    - each projection at a different angle
    - integral of the image at that angle
  - paper by J. Radon in 1917
    - Radon transform
    - inverse Radon transform
  - algorithm by A. Cormack in 1963-64
  - first CT scanner by G. Hounsfield in 1971
  - Nobel prize for Hounsfield and Cormack in 1979

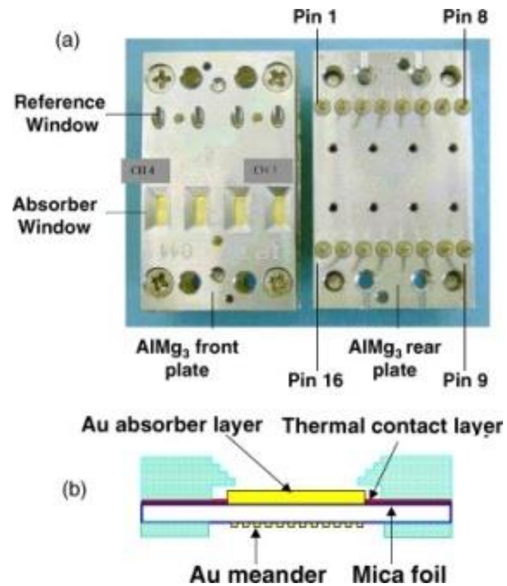


Plasma Tomography

- Tomography at the Joint European Torus (JET)







Plasma Tomography

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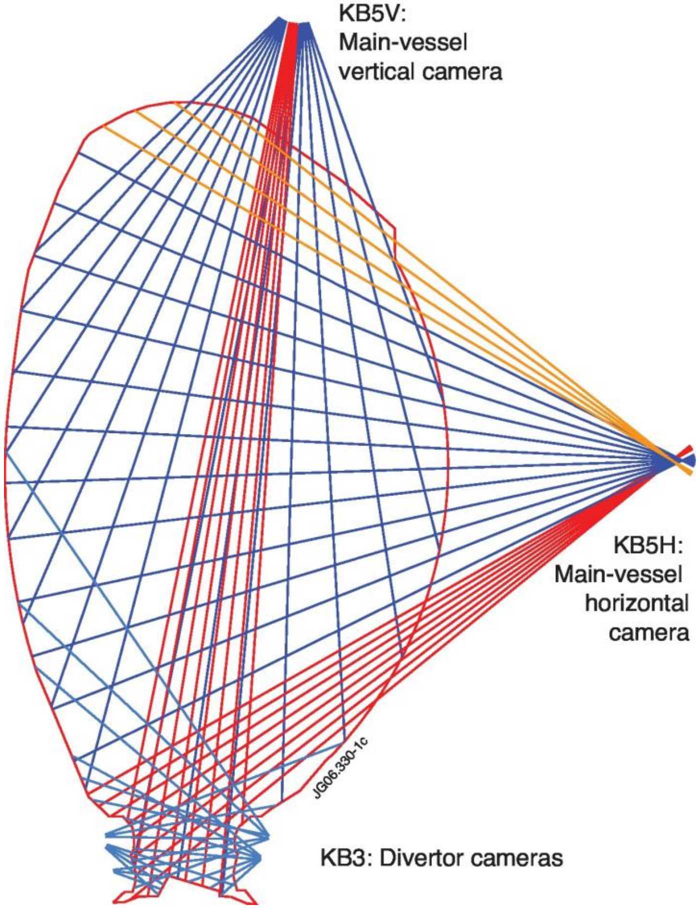

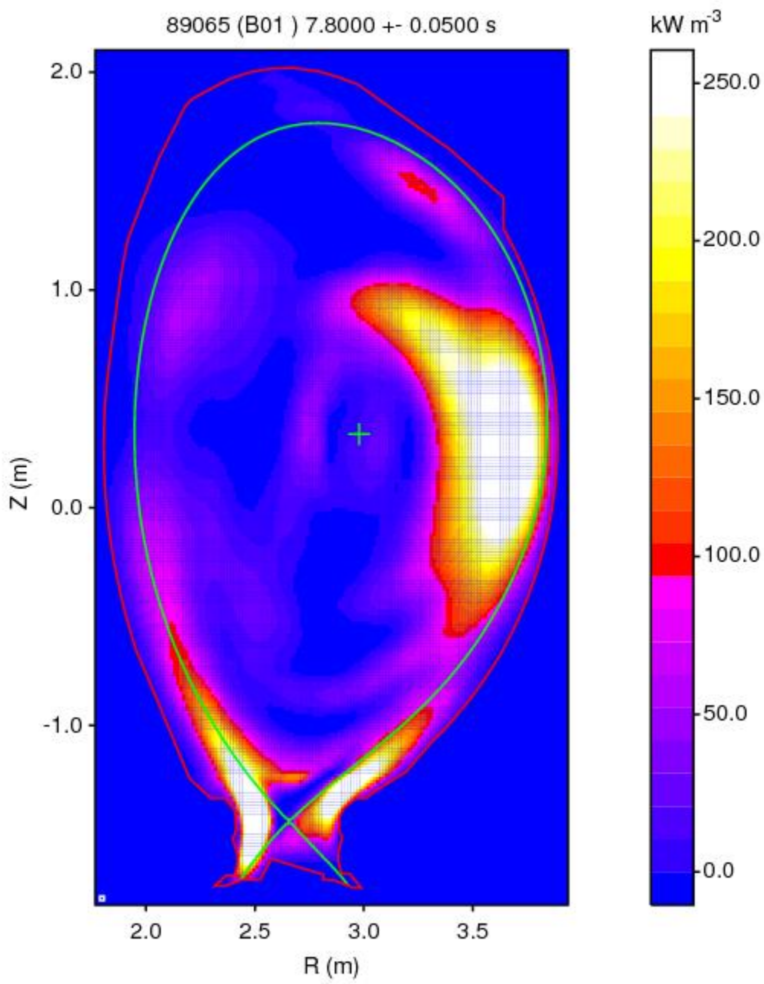

Plasma Tomography

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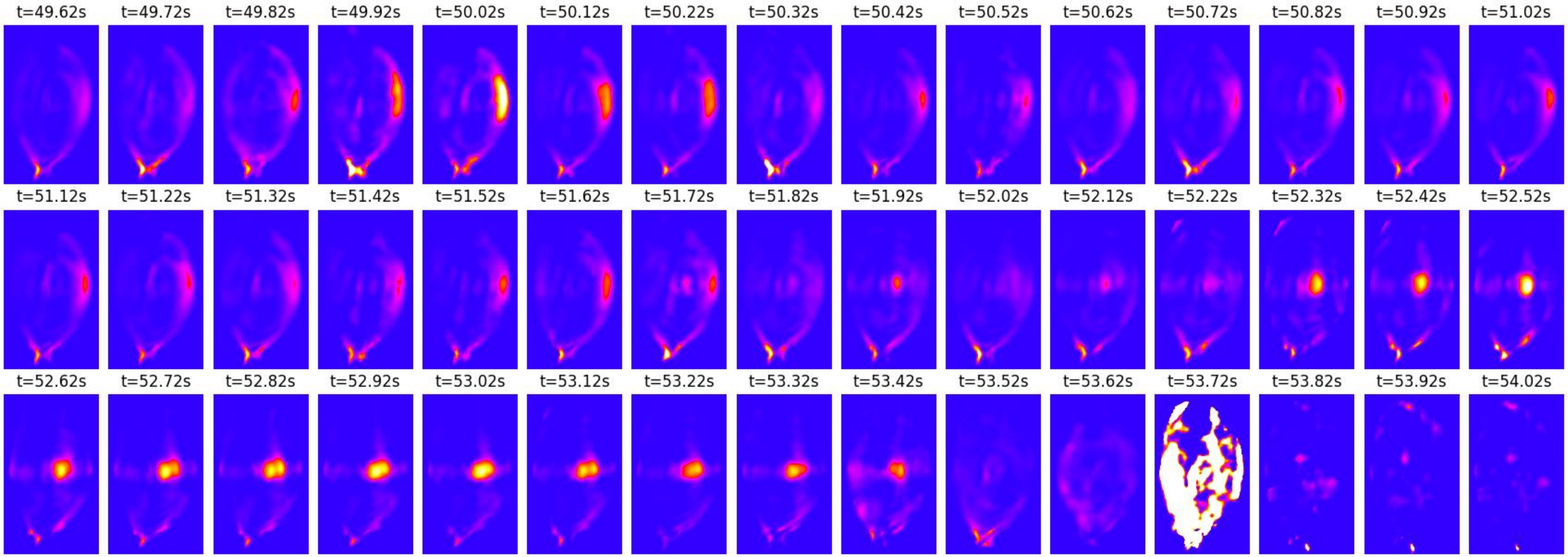
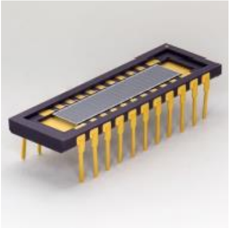
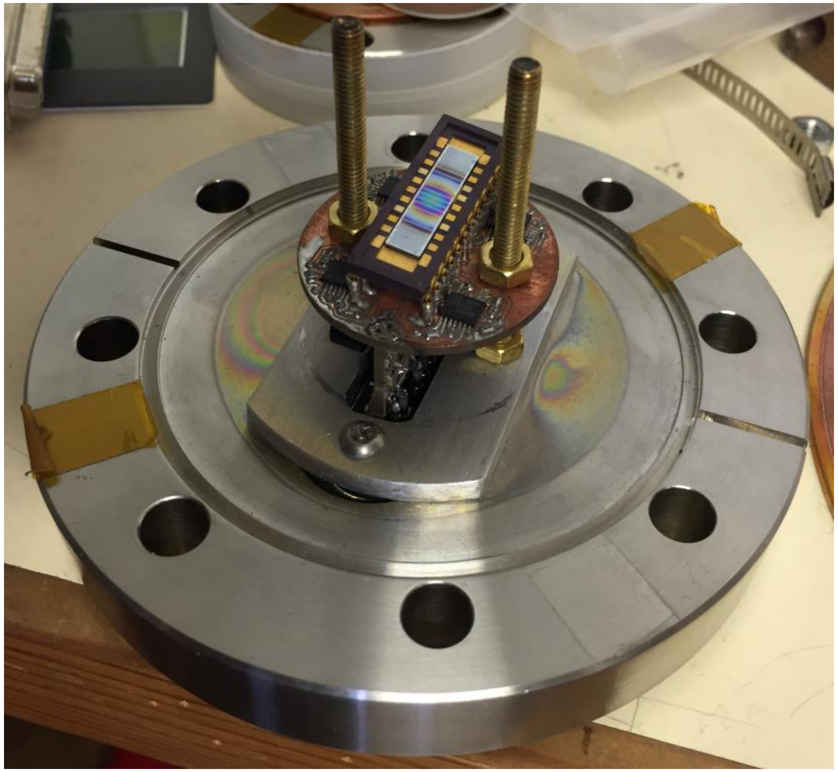
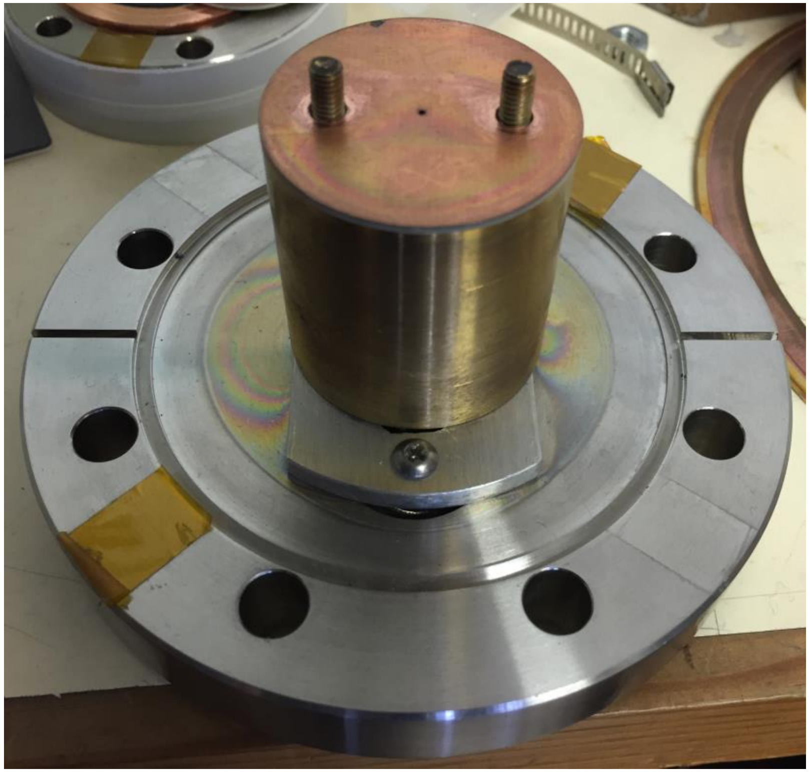


Plasma Tomography

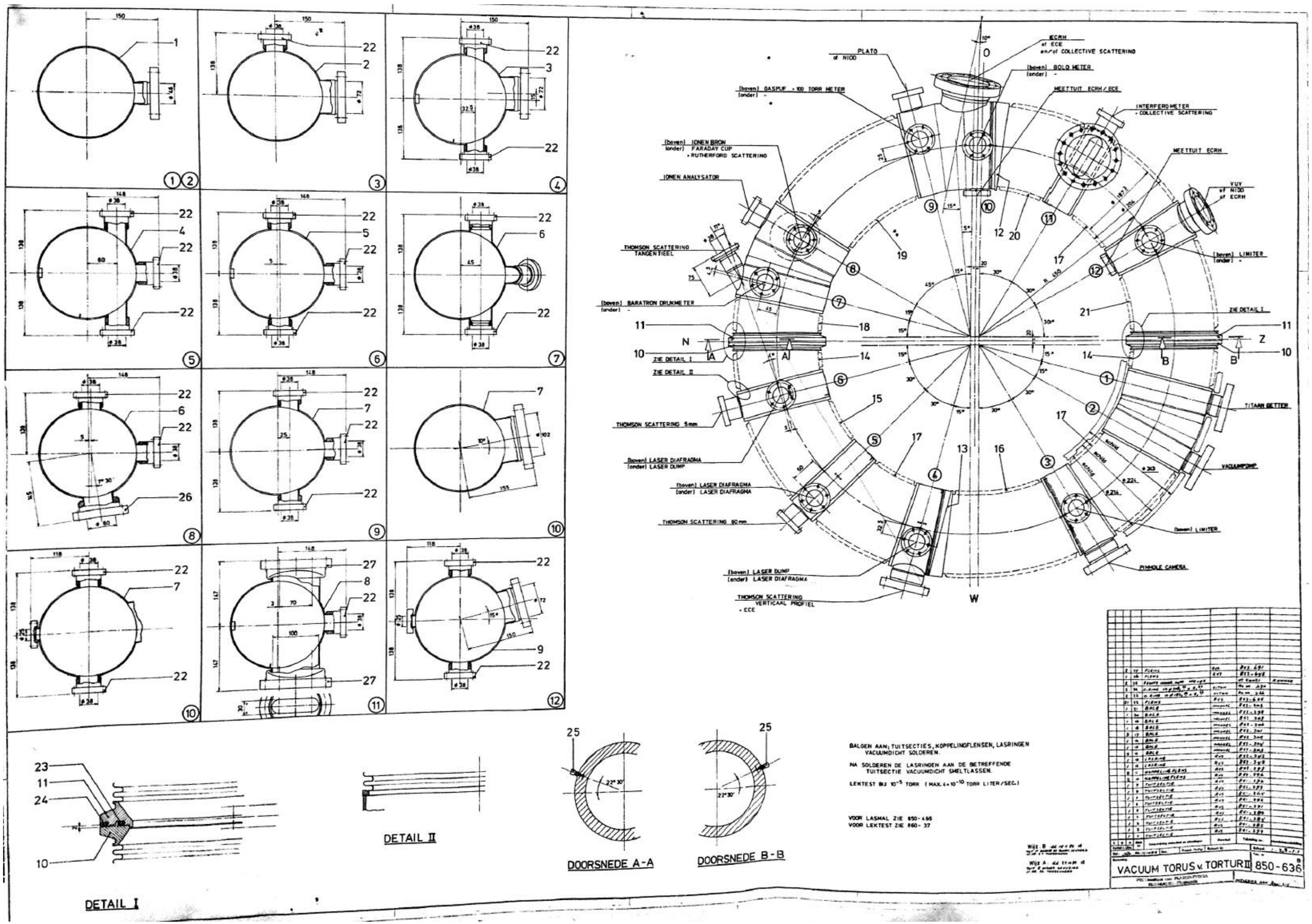
- Tomography at ISTTOK
  - cameras based on photodiode array + pinhole





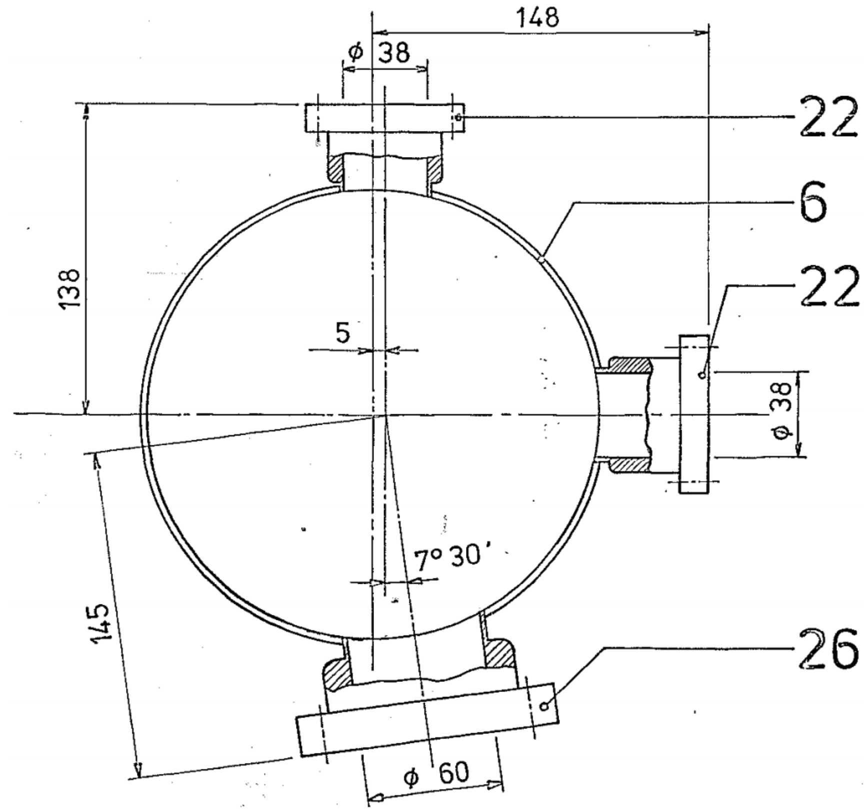


Plasma Tomography



Plasma Tomography

- ISTTOK setup (2019)
  - 2 cameras
    - vertical, horizontal
  - 16 detectors per camera
    - in fact 20 detectors, but 4 are not used
  - lines of sight can be derived from detector and pinhole positions

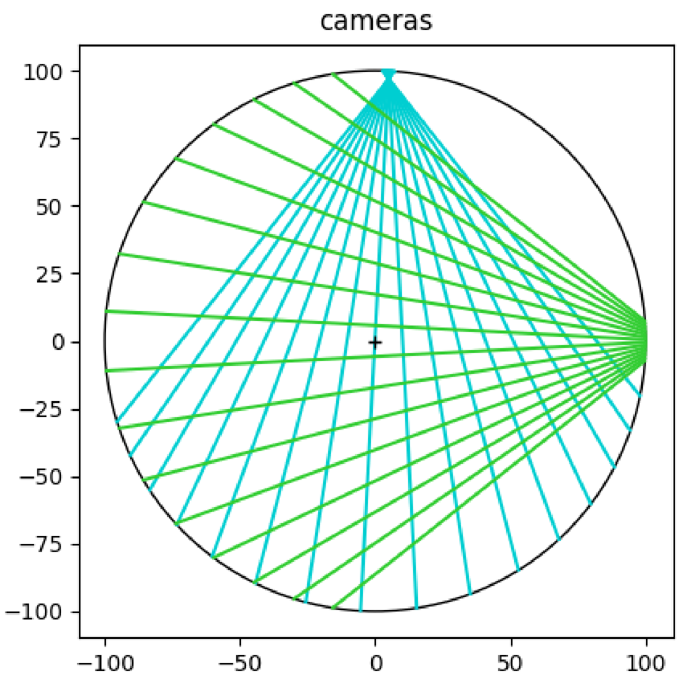


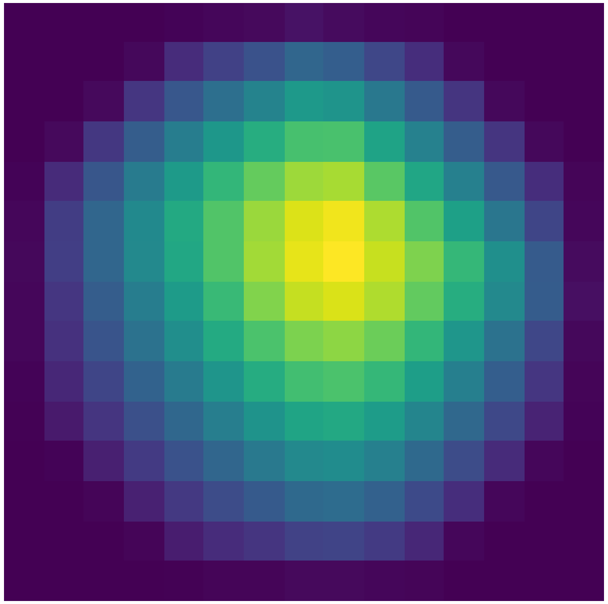
Plasma Tomography

- Tomography methods
  - analytical methods (Fourier-based)
    - Fourier slice theorem
    - filtered backprojection (FBP)
    - Cormack’s approach with basis functions
  - algebraic methods (pixel-based)
    - system of linear equations
    - iterative reconstruction techniques such as ART
    - solutions using regularization

Plasma Tomography

- Inverse problem
  - from detector measurements to plasma profile

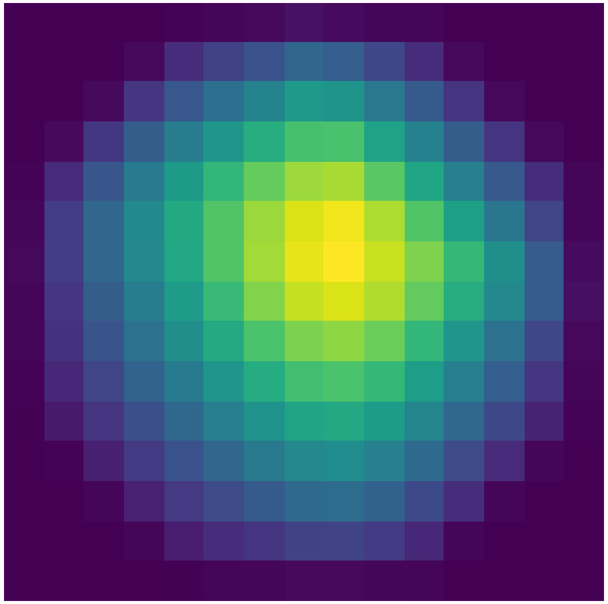




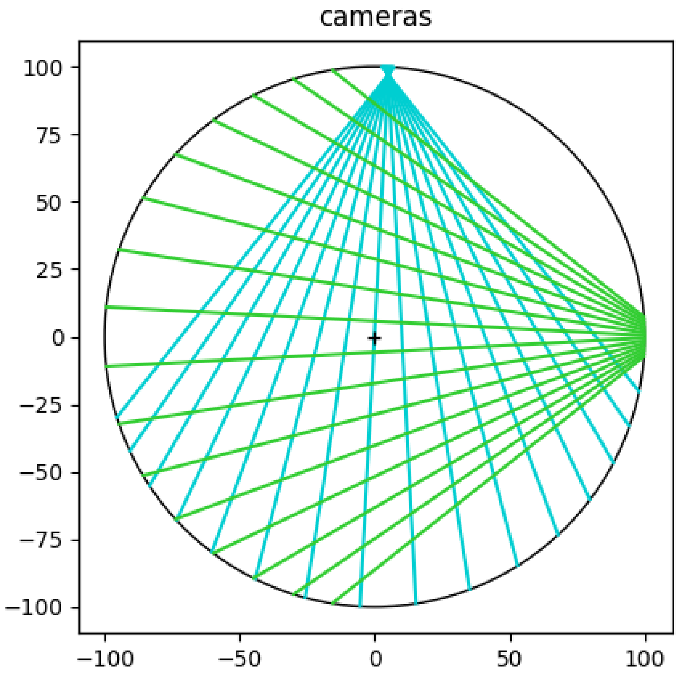
15x15 resolution

Plasma Tomography

- Forward problem
  - from plasma profile to detector measurements

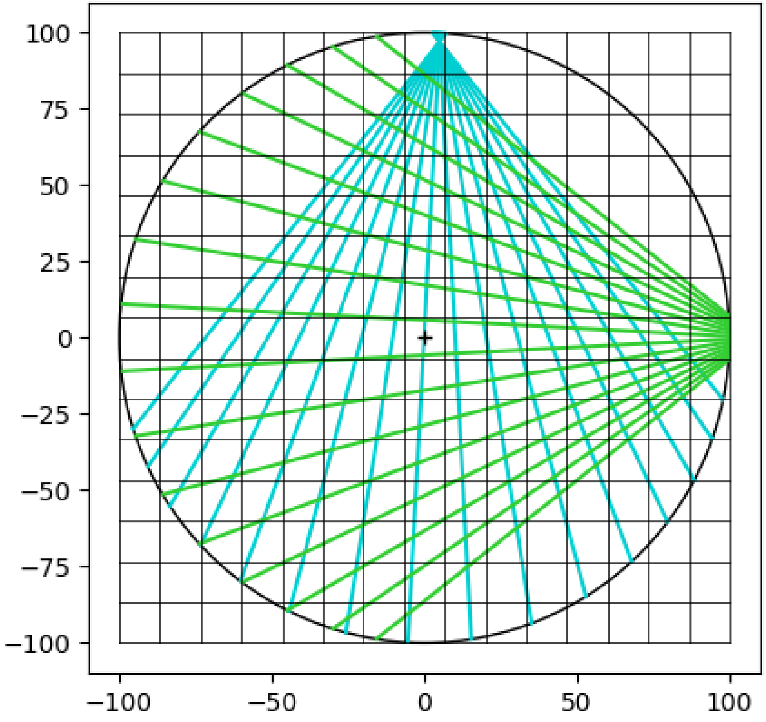


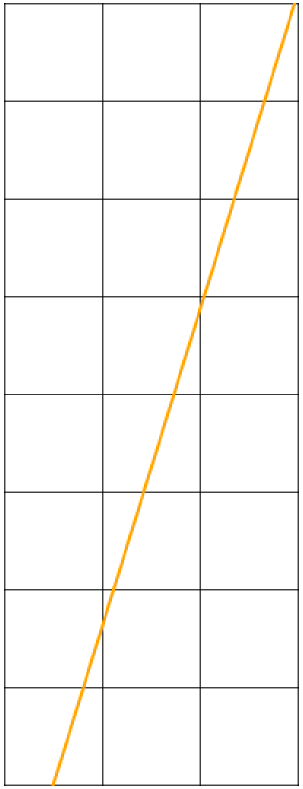
15x15 resolution



Plasma Tomography

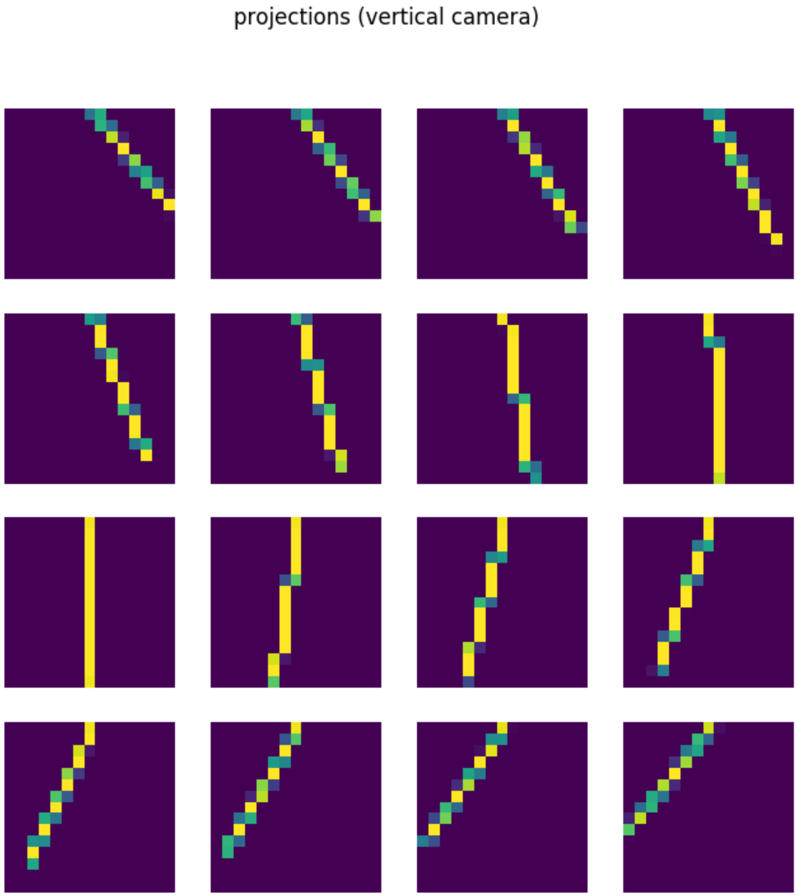
- Geometry of the problem
  - find the contribution of each pixel for each line of sight

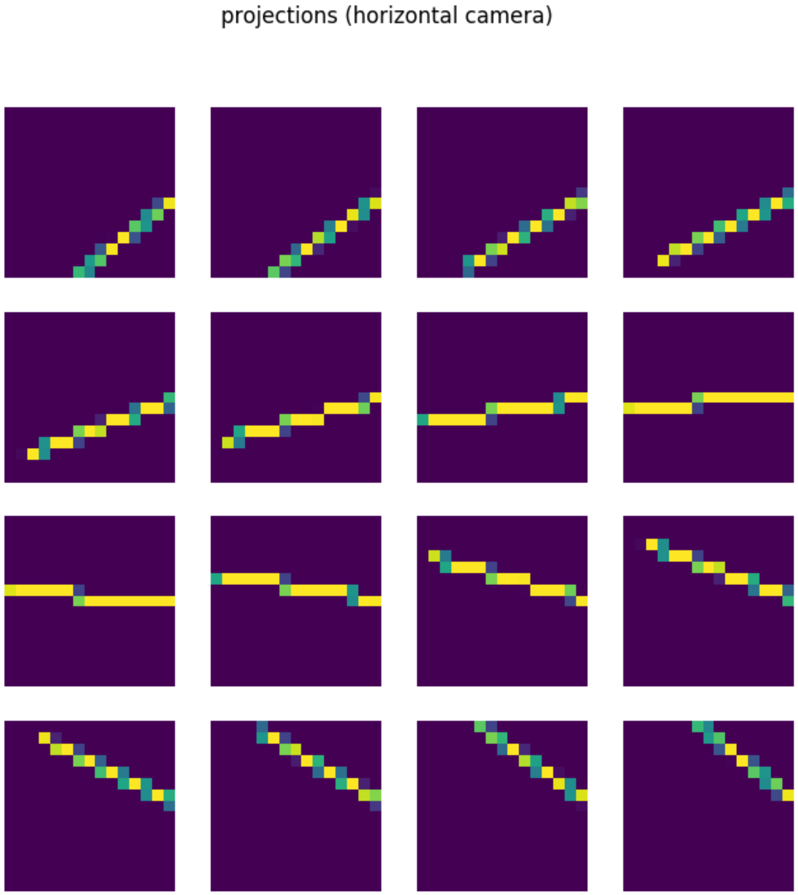




Plasma Tomography

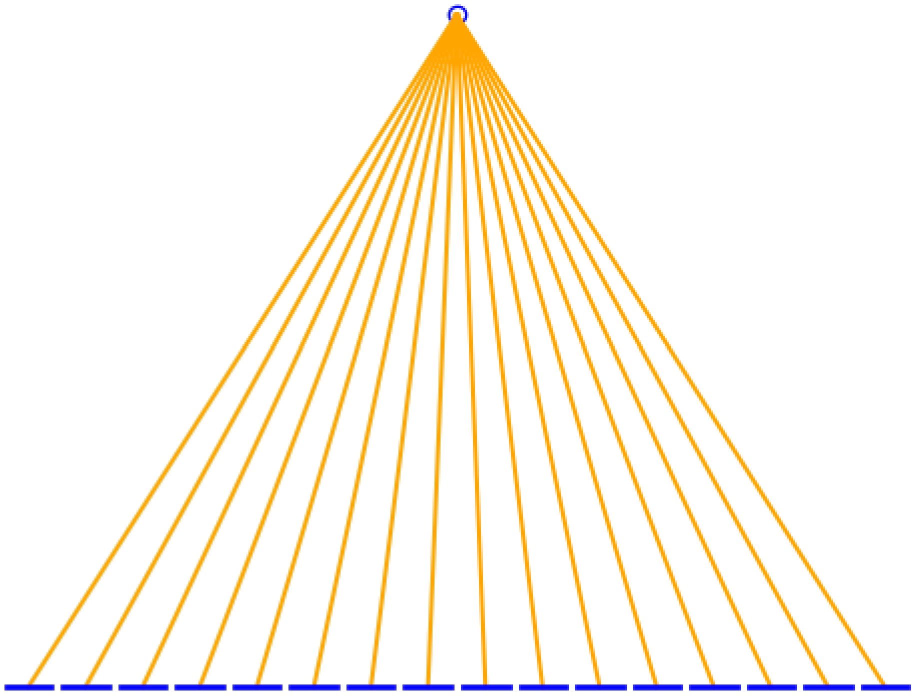
- Contribution of each pixel to each line of sight

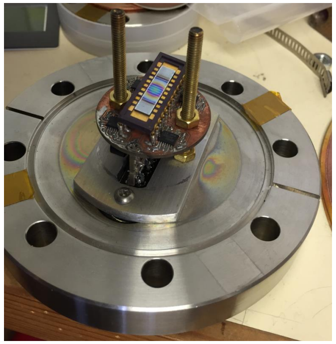


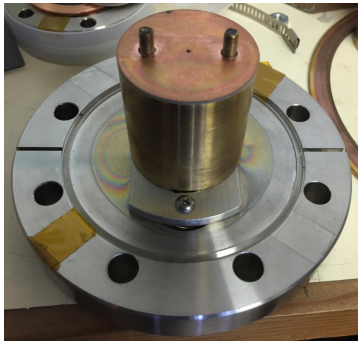


Plasma Tomography

- Calibration factors (*étendue*)
  - angle of incidence on the detector
  - angle through the pinhole (and thickness)



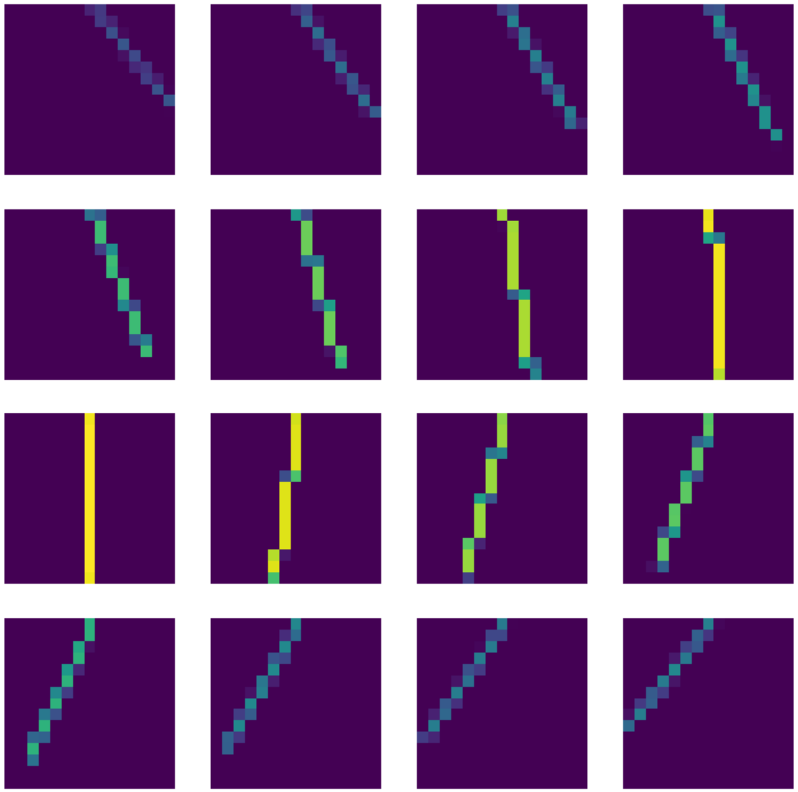




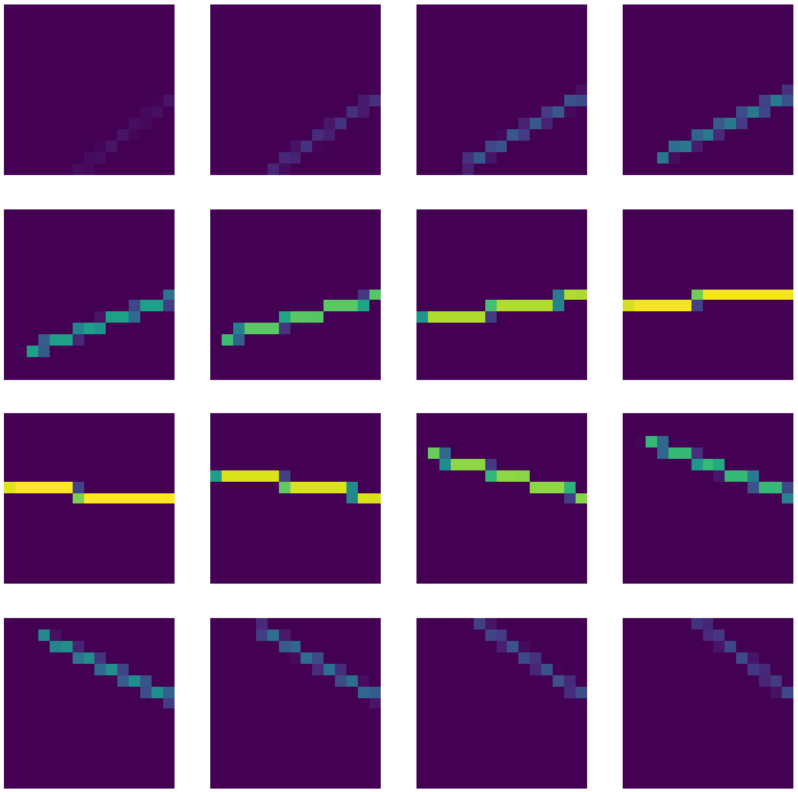
Plasma Tomography

- Contribution of each pixel to each line of sight

projections (vertical camera)



projections (horizontal camera)



Plasma Tomography

- In matrix form:

projection matrix

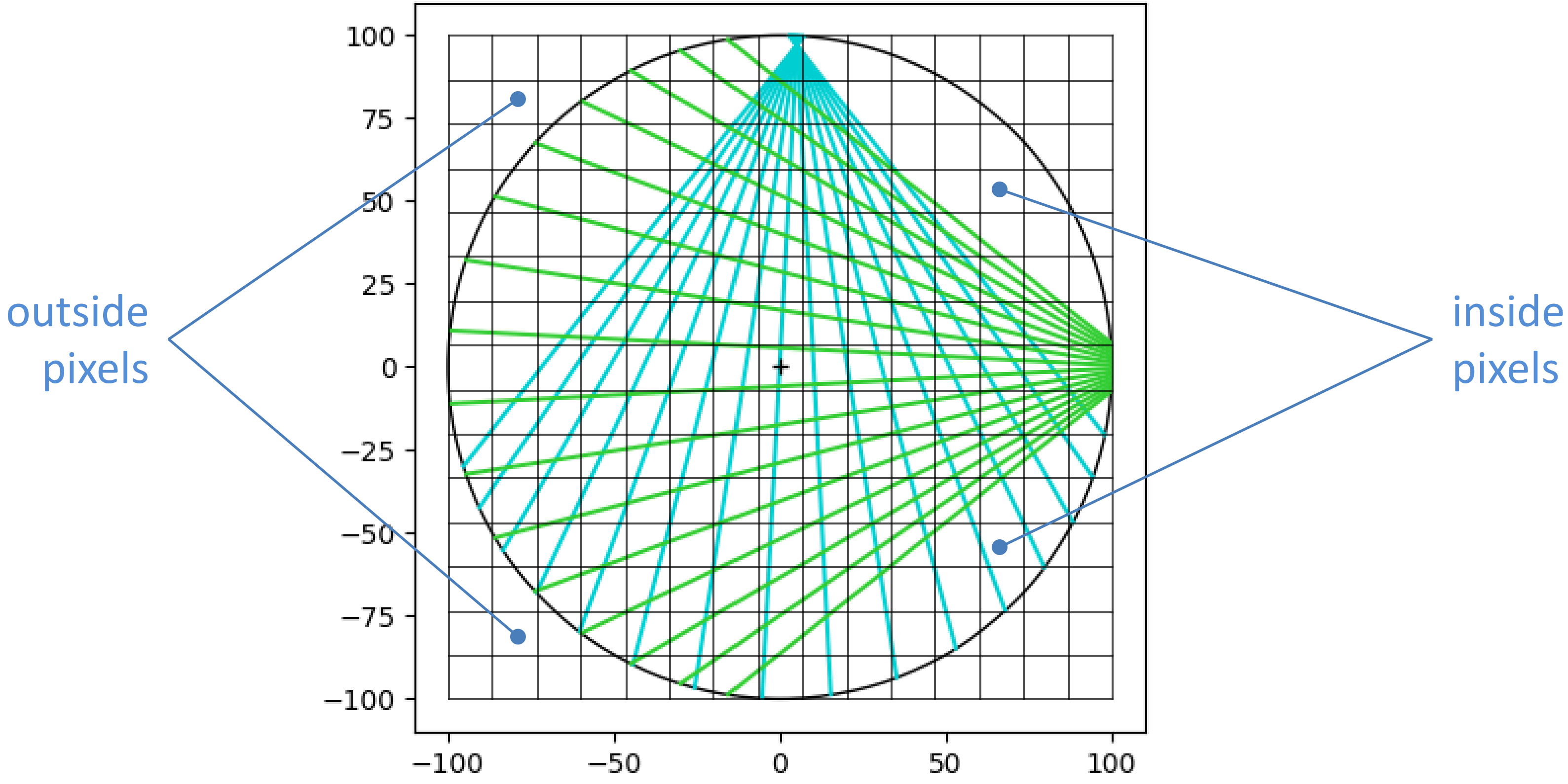
detector measurements → $$\mathbf{f} = \mathbf{P} \cdot \mathbf{g}$$ ← reconstruction (as column vector)

32x1 32x225 225x1

underdetermined system
(32 equations for 225 unknowns)

Plasma Tomography

- Underdetermined system



Plasma Tomography

- Regularization (general)
  - minimize:

$$\phi = \|\mathbf{f} - \mathbf{P}\mathbf{g}\|^2 + \alpha\|\mathbf{R}\mathbf{g}\|^2$$

$$\frac{\partial\phi}{\partial\mathbf{g}} = 0 \;\Rightarrow\; \mathbf{g} = \left(\mathbf{P}^{\mathrm{T}}\mathbf{P} + \alpha\mathbf{R}^{\mathrm{T}}\mathbf{R}\right)^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{f}$$

$$\mathbf{g} = \left(\mathbf{P}^{\mathrm{T}}\mathbf{P} + \alpha_1\mathbf{R}_1^{\mathrm{T}}\mathbf{R}_1 + \alpha_2\mathbf{R}_2^{\mathrm{T}}\mathbf{R}_2 + \cdots\right)^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{f}$$

Plasma Tomography

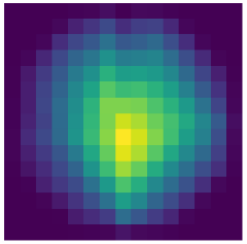
- Regularization (simple approach)
  - for every pixel
    - minimize the horizontal and vertical differences to neighbors
  - for outside pixels
    - minimize their norm

$$\phi = \|\mathbf{f} - \mathbf{P}\mathbf{g}\|^2 + \alpha_1 \|\mathbf{D}_\mathrm{h}\mathbf{g}\|^2 + \alpha_2 \|\mathbf{D}_\mathrm{v}\mathbf{g}\|^2 + \alpha_3 \|\mathbf{I}_\mathrm{o}\mathbf{g}\|^2$$

$$\mathbf{g} = (\mathbf{P}^\mathrm{T}\mathbf{P} + \alpha_1 \mathbf{D}_\mathrm{h}^\mathrm{T}\mathbf{D}_\mathrm{h} + \alpha_2 \mathbf{D}_\mathrm{v}^\mathrm{T}\mathbf{D}_\mathrm{v} + \alpha_3 \mathbf{I}_\mathrm{o}^\mathrm{T}\mathbf{I}_\mathrm{o})^{-1}\mathbf{P}^\mathrm{T}\mathbf{f}$$

Plasma Tomography

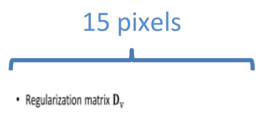
- Regularization matrix $\mathbf{D}_{\mathrm{h}}$



225x225

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & & 0 & 0 \\ \vdots & & & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & 0 & & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

Plasma Tomography



- Regularization matrix $\mathbf{D_v}$

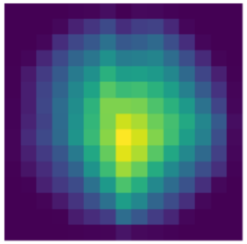
15 pixels

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & & 0 \\
\vdots & & & & & & & & & & & & & & & & & & \ddots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & & 1
\end{bmatrix}$$

225x225

Plasma Tomography

- Regularization matrix $\mathbf{I_o}$



225x225

$$
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 1 & 0 & & 0 & 0 & 0 & & 0 & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & & 0 & 0 & 0 & & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\
0 & 0 & 0 & & 0 & 0 & 0 & & 0 & 1 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 1
\end{bmatrix}
$$

Plasma Tomography

- Tomographic inversion
  - one reconstruction

$$\mathbf{g} = (\mathbf{P}^{\mathrm{T}}\mathbf{P} + \alpha_1 \mathbf{D}_{\mathrm{h}}^{\mathrm{T}}\mathbf{D}_{\mathrm{h}} + \alpha_2 \mathbf{D}_{\mathrm{v}}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}} + \alpha_3 \mathbf{I}_{\mathrm{o}}^{\mathrm{T}}\mathbf{I}_{\mathrm{o}})^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{f}$$

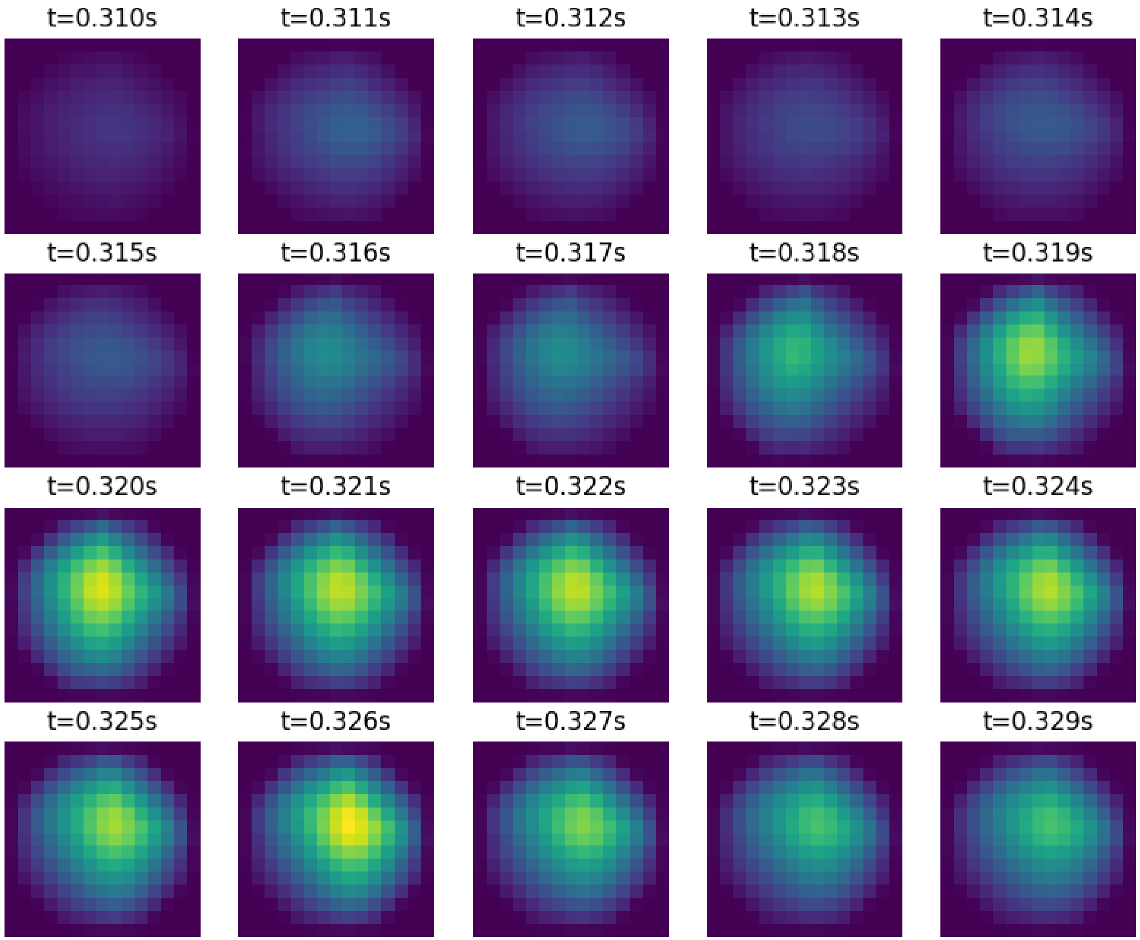
  - multiple reconstructions

$$\mathbf{M} = (\mathbf{P}^{\mathrm{T}}\mathbf{P} + \alpha_1 \mathbf{D}_{\mathrm{h}}^{\mathrm{T}}\mathbf{D}_{\mathrm{h}} + \alpha_2 \mathbf{D}_{\mathrm{v}}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}} + \alpha_3 \mathbf{I}_{\mathrm{o}}^{\mathrm{T}}\mathbf{I}_{\mathrm{o}})^{-1}\mathbf{P}^{\mathrm{T}}$$

$$\mathbf{g} = \mathbf{M} \cdot \mathbf{f}$$

Plasma Tomography

- Tomographic reconstructions for shot 47238



Plasma Tomography

- Source code
  - available at: https://github.com/diogoff/isttok-tomography
  - cameras.py
    - finds the lines of sight for a given geometry
  - projections.py
    - finds the projection matrix for a given pixel resolution
  - signals.py
    - reads the camera signals for a given shot number
  - reconstructions.py
    - calculates the reconstructions at given times

Plasma Tomography

- Other forms of regularization
  - generic
    - e.g. minimum Fisher information (MFI)
  - specific
    - e.g. smoothness along magnetic flux surfaces

Plasma Tomography

- Minimum Fisher information (MFI)

$$I_F = \int \frac{g'(x)^2}{g(x)} dx$$

  - inspired by the concept of Fisher information
  - differences should be small, but they are allowed to be larger where **g** itself is large

$$\mathbf{g} = (\mathbf{P}^{\mathrm{T}}\mathbf{P} + \alpha_1 \mathbf{D}_{\mathrm{h}}^{\mathrm{T}}\mathbf{D}_{\mathrm{h}} + \alpha_2 \mathbf{D}_{\mathrm{v}}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}} + \alpha_3 \mathbf{I}_{\mathrm{o}}^{\mathrm{T}}\mathbf{I}_{\mathrm{o}})^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{f}$$

$$\mathbf{D}_{\mathrm{h}}^{\mathrm{T}}\mathbf{D}_{\mathrm{h}} \rightarrow \mathbf{D}_{\mathrm{h}}^{\mathrm{T}}\mathbf{W}\mathbf{D}_{\mathrm{h}}$$

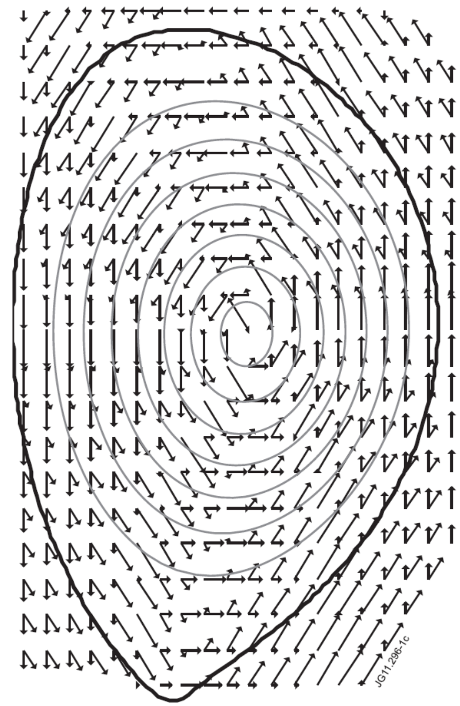
$$\mathbf{D}_{\mathrm{v}}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}} \rightarrow \mathbf{D}_{\mathrm{v}}^{\mathrm{T}}\mathbf{W}\mathbf{D}_{\mathrm{v}}$$

$$\mathbf{W} = diag\left(\frac{1}{\mathbf{g}}\right)$$

  - system becomes non-linear; solve iteratively for **g**

Plasma Tomography

- Smoothness along magnetic flux surfaces
  - differences are taken along the direction of magnetic flux surfaces
  - plasma equilibrium (e.g. by EFIT) must be provided beforehand
  - system remains linear but now depends on data from other diagnostics



$$\mathbf{g} = (\mathbf{P}^{\mathrm{T}}\mathbf{P} + \alpha_1 \mathbf{D}_{\mathrm{h}}^{\mathrm{T}}\mathbf{D}_{\mathrm{h}} + \alpha_2 \mathbf{D}_{\mathrm{v}}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}} + \alpha_3 \mathbf{I}_{\mathrm{o}}^{\mathrm{T}}\mathbf{I}_{\mathrm{o}})^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{f}$$

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