Comment on “A close examination of the motion of an adiabatic piston,” by Eric A. Gislason [Am. J. Phys. 78 (10), 995–1001 (2010)]

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A recent paper by Gislason treats the adiabatic piston, a system of two ideal gases in a horizontal cylinder and separated by an insulating piston that moves without friction. The analysis in this paper is comprehensive and useful as a teaching tool, but is somewhat misleading if not understood in the appropriate context. The evolution to equilibrium involves two mechanisms, a faster one leading to the equalization of pressures, and a slower one bringing the system to identical temperatures. Gislason addressed only the first mechanism. We note that the eventual final state is described by thermodynamics. Therefore, a discussion of the adiabatic piston can be enriched to promote a proper and general view of thermodynamics.

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I. INTRODUCTION

In a recent paper, Gislason analyzed the motion of the “adiabatic piston,” which consists of two subsystems of the same ideal gas contained in a horizontal cylinder with insulating walls.[1] Gislason made several important points and elaborates on the first mechanism that brings the piston to rest when the pressures of the two gases become equal. Significant insight given by Gislason concerns the damping of the piston motion as a result of the dynamic pressure on the piston “because the pressure is greater when the piston is moving toward the gas than when the piston is moving away from the gas.”[1] Gislason cites several papers that point out that “temperature and pressure fluctuations in the two gases will slowly act to bring the two temperatures to equality.”[1] He correctly states that the “time scale for this slow mechanism is much longer than the time scale for the piston to come to rest,”[1] and cautions that this slower mechanism is not discussed in the paper. Gislason asserts that “thermodynamics cannot predict what the final temperatures will be,”[1] which is correct only in the context of the analysis of the first mechanism. He adds that “to achieve complete equilibrium the piston must be able to conduct energy, which cannot occur for an adiabatic piston.”[1] As we will discus, this statement is not valid if we keep in mind the second mechanism as well. It is interesting to analyze the first process as done in Ref. 1, but readers should be aware of the approximations involved and the conceptual problems it hides. The purpose of this comment is to clarify this issue by using the formalism of thermodynamics to extend the investigation to the second mechanism.

An intuitive and beautiful discussion of the second mechanism was made by Feynman,[2] and a quantitative molecular dynamics simulation, establishing beyond doubt the state of equal pressures and temperatures as the final equilibrium state, was published by Kestemont and co-workers.[3] A careful use of thermodynamics must give the same final results as molecular dynamics, because the latter is a microscopic interpretation of the former.

The remainder of this comment is structured as follows. The way in which thermodynamics may handle the “adiabatic piston” problem is shown in Sec. II. A short discussion and an identification of the origin of some common misunderstandings are given in Sec. III. Finally, Sec. IV summarizes our main conclusions.

II. THERMODYNAMIC APPROACH

The equality of pressures is a necessary condition for mechanical equilibrium, corresponding to the first mechanism. It is not sufficient for thermodynamic equilibrium, which also requires the second, slower process and the establishment of thermal equilibrium.

The two subsystems together must satisfy the conditions of constant total volume and total energy. The collisions between the gas particles and the piston make the position of the piston fluctuate, allowing an exchange of energy between both gases. This energy exchange will take place even if the piston is not a thermal conductor, because they are a result of the momentum transfer in the collisions.[2] As a consequence, the system will pass through the different available configurations toward greater entropy. Therefore, we cannot impose the condition $dS=0$ once the pressures are equal,[4] although this constraint is sometimes confused with the “adiabatic” condition (see Sec. III). Moreover, the assertion that “to achieve complete equilibrium, the piston must be able to conduct energy, which cannot occur for an adiabatic piston”[1] does not hold.

If we take into account these considerations, the system is described by the set of equations,[4]

$$dU_1 = -P_1 dV_1 + T_1 dS_1, \tag{1}$$

$$dU_2 = -P_2 dV_2 + T_2 dS_2. \tag{2}$$

We have the condition

$$dS = dS_1 + dS_2 \geq 0. \tag{3}$$

Equations (1) and (2) can be written in the form

$$dS_1 = \frac{dU_1}{T_1} + \frac{P_1}{T_1} dV_1, \tag{4}$$

$$dS_2 = \frac{dU_2}{T_2} + \frac{P_2}{T_2} dV_2. \tag{5}$$

As long as the system reaches mechanical equilibrium, we have

$$dE_k = -dU_1 - dU_2 = 0, \tag{6}$$

where E_k is the kinetic energy of the piston. Furthermore,

$$dV = dV_1 + dV_2 = 0. \tag{7}$$

Hence, $dU_2 = -dU_1$ and $dV_2 = -dV_1$. If we substitute Eqs. (4) and (5) into the equilibrium condition $dS = 0$, we obtain

$$dS = \left(\frac{1}{T_1} - \frac{1}{T_2}\right)dU_1 + \left(\frac{P_1}{T_1} - \frac{P_2}{T_2}\right)dV_1 = 0. \tag{8}$$

Therefore, the solution is $P_1 = P_2$ and $T_1 = T_2$, and both mechanical and thermodynamical equilibria are obtained. Thermodynamics can predict that the final variables are equal.

III. DISCUSSION

We have shown that thermodynamics correctly predicts that the system will evolve to a final state of equal pressures and equal temperatures. The reason a different and inaccurate statement is repeated by many authors is related to a problem of language and misconceived notions associated with the meaning of adiabatic. If the piston is adiabatic, an additional condition is often imposed, based on faulty physical intuition, specifically,

$$dU_i = -P_i dV_i \quad (i = 1,2). \tag{9}$$

The argument is that, because the piston is adiabatic, $dQ = 0$. If this were the case, we would have, substituting Eq. (9) into Eq. (8),

$$dS = -\left(\frac{1}{T_1} - \frac{1}{T_2}\right)P_1 dV_1 + \left(\frac{P_1}{T_1} - \frac{P_2}{T_2}\right)dV_1 = 0. \tag{10}$$

Equation (10) would be valid if mechanical equilibrium $P_1 = P_2$ holds, without the need for the equality of the temperatures. If we let $P_2 = P_1$ in Eq. (10),

$$dS = -\left(\frac{1}{T_1} - \frac{1}{T_2}\right)P_1 dV_1 + \left(\frac{1}{T_1} - \frac{1}{T_2}\right)P_1 dV_1, \tag{11}$$

we find $dS = 0$, regardless of the values of T_1 and T_2.

The term adiabatic piston means a piston with zero heat conductivity. If the piston is held in place, there is no energy transfer from one subsystem to another. However, if the piston is released, both systems are coupled, and can interact and exchange energy. We can say that a piston, which is adiabatic when it is fixed, is not adiabatic when it can move freely. The condition $dQ = 0$ cannot be imposed.

It is not difficult to show that Eq. (9) does not hold in general and cannot be demonstrated.[4] Conservation of energy is expressed by the first part of Eq. (6), $dE_k + dU_1 + dU_2 = 0$. In contrast, the work done on the piston is

$$dW = dE_k = (\tilde{P}_1 - \tilde{P}_2)dV_1, \tag{12}$$

where $\tilde{P}_1$ and $\tilde{P}_2$ are dynamic pressures (they are denoted by P_1 and P_2 in Ref. 1), that is, the pressures the gases exert on the moving piston. Therefore,

$$dU_1 + dU_2 = -(\tilde{P}_1 - \tilde{P}_2)dV_1. \tag{13}$$

Equation (13) does not imply that Eq. (9) is generally valid, although it can be a good approximation during the fast process. Hence, even after the first process, when the pressures are equal but the temperatures are still different, we have

$$dU_i = -P_i dV_i + T_i dS_i \neq -P_i dV_i, \tag{14}$$

and Eq. (9) is incorrect.

After the attainment of mechanical equilibrium, the piston has no kinetic energy and the evolution to the final equilibrium continues with $dU_1 = -dU_2$, or

$$-P_1 dV_1 + T_1 dS_1 = +P_2 dV_2 - T_2 dS_2. \tag{15}$$

Because $P_1 = P_2$ and $dV_1 = -dV_2$, we have

$$T_1 dS_1 = -T_2 dS_2. \tag{16}$$

If $T_1 > T_2$ initially, and we take into account Eq. (3), $dS_2 > 0$ and $dS_1 < 0$, and the global change of entropy is positive.[4] Accordingly, the temperature T_2 will slowly increase and T_1 will decrease until both temperatures become equal and thermodynamic equilibrium is achieved.

IV. CONCLUSION

A recent paper raises several interesting points on thermodynamics using the example of the adiabatic piston.[1] As asserted in Ref. 1, its results must be used only to describe the first process leading to mechanical equilibrium. We have shown that the slow evolution to thermodynamic equilibrium is well described within classical thermodynamics and complete thermodynamic equilibrium is achieved, even if the piston is not a thermal conductor. Our discussion can help to promote a general and proper view of thermodynamics. In addition, it may provide a link to the microscopic interpretation of entropy. Additional insight of the problem, including the analysis of the first process and the damped oscillations of the piston, can be found in Refs. 5–7.

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#1 Au: Please update Ref. 4 if possible.
#2 Au: Please update Ref. 7 if possible.