Addendum: Distribution of neighbors other than the nearest

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The distribution of the nearest neighbor was recently discussed in this Journal[1] and elsewhere.[2] However, the distribution of other neighbors has not been covered in those discussions. It may be of interest to know that all the distributions (including the nearest-neighbor distribution) can be derived in a simple manner for the case of independently distributed particles.

Consider $N+1$ particles distributed at random. A certain particle will have N "neighbors." If the ith neighbor is at a distance from the particle smaller than r, then i or more particles must lie within that range. Now, since the particles are independently distributed, the probability that exactly k particles have distances smaller than r, $P_k(r)$, is given by the binomial law

$$P_k(r) = \binom{N}{k} [F(r)]^k [1 - F(r)]^{N-k}, \tag{1}$$

where $F(r)$ is the probability for a particle to have a distance smaller than r. Therefore, the probability that the ith neighbor has a distance smaller than r, $F_i(r)$, is

$$F_i(r) = \sum_{k=i}^{N} \binom{N}{k} [F(r)]^k [1 - F(r)]^{N-k}. \tag{2}$$

The distance distribution function $w_i(r) = dF_i/dr$ is then obtained as

$$w_i(r) = i\binom{N}{i} w(r) [F(r)]^{i-1} [1 - F(r)]^{N-i}, \tag{3}$$

where $w(r) = dF/dr$. Equation (3) is a generalization to the ith neighbor of Eq. (18) in Ref. 1, valid for the nearest neighbor. The asymptotic form ($N \to \infty$) of Eq. (3)

$$w_i(r) = \frac{N^i}{(i-1)!} w(r) [F(r)]^{i-1} \exp[-NF(r)], \tag{4}$$

is of special relevance, since usually one has $N \gg 1$. If the particles are uniformly distributed in a (essentially infinite) volume V, then $w(r) = 4\pi r^2/V$, $F(r) = 4\pi r^3/3V$, and Eq. (4) becomes

$$w_i(r) = \frac{3}{(i-1)!} \left(\frac{4\pi n}{3}\right)^i r^{3i-1} \exp\left(-\frac{4\pi r^3 n}{3}\right), \tag{5}$$

where $n = N/V$ is the number density. For $i = 1$ we get the classical nearest-neighbor distribution.[1-3] The average distance from a particle to its ith neighbor, D_i, computed from Eq. (5), is given by

$$D_i = \int_0^\infty r w_i(r) dr = \frac{\Gamma(i + 1/3)}{(i-1)!} \left(\frac{3}{4\pi}\right)^{1/3} n^{-1/3}. \tag{6}$$

It is perhaps remarkable how slowly D_i increases with i: this distance is $2D_1$ only for the sixth neighbor. Note that for large i the average distance reduces to $D_i = (3i/4\pi n)^{1/3}$.

[1] M. Berberan Santos, Am. J. Phys. **54**, 1139 (1986).

[2] A. M. Stoneham, J. Phys. C **16**, 285 (1983).

[3] S. Chandrasekhar, Rev. Mod. Phys. **15**, 1 (1943).

Erratum: "Gravitational fields and the cosmological constant in multidimensional Newtonian universes" [Am. J. Phys. 54, 726 (1986)]

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In Eq. (8), replace $O(b^2)$ with $O(b^4)$.

In the line following Eq. (12b), and in Eqs. (14), (17), and (18), replace ∂_{n2} by δ_{n2}.

At both of its appearances in Sec. IV, Ref. 15 should be changed to Ref. 6.

Erratum: "Slinky whistlers" [Am. J. Phys. 55, 130 (1987)]

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The following corrections should be made in the article cited above.

Equation (3) should read $t = L/v_g = 1.14/(fd)^{1/2}$. (The d was omitted.)

Also, on p. 131, column two, in the second line of the second full paragraph $f = 55$ kHz should be replaced by $f = 5.5$ kHz.