

How Fast is a Fast Equilibrium? A New View of Reversible Reactions

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Reversible reactions are described in terms of kinetic cycles. From this description, two useful parameters arise: 1) average cycle duration and 2) average number of cycles. The latter applies to cycles with at least one unstable species. These parameters allow answering the question "How fast is a fast equilibrium?" in abso-

lute and in relative terms, respectively. The general interest of the approach is demonstrated by application to reactions drawn from several areas: Acid–base equilibria, enzyme kinetics, excited-state proton transfer, thermally activated delayed fluorescence, monomer–excimer kinetics and homo-FRET.

1. Introduction

According to the principle of microscopic reversibility, all elementary chemical reactions are reversible to some extent.^[1,2] In many cases the degree of reversibility is small, and the corresponding reverse reaction can be neglected. Effectively reversible reactions are nevertheless very common in both ground- and excited-state mechanisms. Chemical examples comprise acid–base and enzyme reactions, whereas photochemical processes include excited-state proton transfer and homo-FRET.

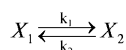
Although the kinetic treatment of these relatively simple mechanisms is known,^[3] the consequences of reversibility are not fully disclosed by the available results that mainly consist of special mathematical approximations, namely those applying to pre-equilibrium conditions^[4] or to relaxation kinetics.^[5]

Herein, two-state reversible reactions are viewed according to a new approach, based on the concept of cycle. Two useful parameters are introduced: 1) average cycle duration and 2) average number of cycles, with the latter applying to cycles with at least one unstable species. These parameters allow answering the question "How fast is a fast equilibrium?" in absolute and relative terms, respectively. The usefulness of the approach is demonstrated by its application to several systems.

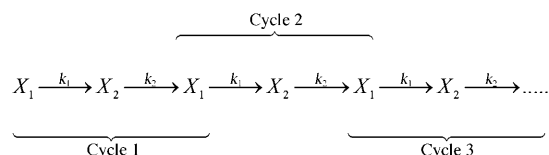
2. Results and Discussion

2.1. The Simplest Case

Consider an elementary unimolecular (or pseudo-unimolecular) equilibrium, shown in Scheme 1. For a molecule that starts as the X_1 species, the equilibrium can be viewed as an infinite sequence of cycles of the type $X_1 \rightarrow X_2 \rightarrow X_1$ (Scheme 2). The duration of each cycle is a random variable $t = t_1 + t_2$, where t_1 and t_2 are the survival times of X_1 and X_2 , respectively. The cycle duration density distribution is given by Equation (1):



Scheme 1. Elementary unimolecular equilibrium.



Scheme 2. Elementary unimolecular equilibrium viewed as a sequence of cycles.

$$f_c(t) = \frac{k_1 k_2}{k_1 - k_2} (e^{-k_2 t} - e^{-k_1 t}) \quad (1)$$

as shown in the Appendix. The average duration of a cycle, τ_c , is shown in Equation (2):

$$\tau_c = \frac{1}{k_1} + \frac{1}{k_2} \quad (2)$$

This equation answers the question "How fast is a fast equilibrium?" in absolute terms.

It is of interest to contrast the average duration of a cycle for a given system with the relaxation time of the same system, τ_r [Eq. (3)]:

$$\tau_r = \frac{1}{k_1 + k_2} \quad (3)$$

When the forward and reverse reactions have very different unimolecular (or pseudo-unimolecular) rate constants, τ_r is de-

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terminated by the fast constant, whereas τ_c is controlled by the slow constant.

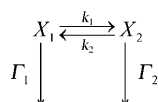
The distribution of the duration of n cycles is obtained as Equation (4):

$$f_n(t) = \underbrace{f_c(t) \otimes \dots \otimes f_c(t)}_{n \times} \quad (4)$$

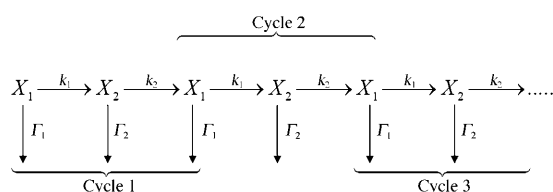
where \otimes denotes the convolution between two functions (see the Appendix). For a large number of cycles, this distribution becomes a narrow Gaussian with mean $n\tau_c$ and standard deviation $\sqrt{n}\tau_c$.

2.2. Cycles with at Least One Unstable Species

Now, consider Scheme 3, where X_1 and X_2 are unstable species and namely excited states. Again, assuming that a molecule starts as X_1 , this scheme can be redrawn as a sequence of cycles, as shown in Scheme 4. In this case it makes sense to ask how many $X_1 \rightarrow X_2 \rightarrow X_1$ cycles take place before decay (via the Γ_1 or Γ_2 channels) occurs. Clearly, the number of cycles n is



Scheme 3. Unimolecular equilibrium for unstable species.



Scheme 4. Unimolecular equilibrium for unstable species viewed as a sequence of cycles.

a random variable. We first compute the probability of decay via each of the two channels. The probability of decay via Γ_1 and after n cycles is given by Equation (5):

$$p_1(n) = (1 - \Phi_2)(\Phi_1\Phi_2)^n \quad (5)$$

where Equation (6):

$$\Phi_1 = \frac{k_2}{k_2 + \Gamma_2} \quad (6)$$

is the probability that X_2 yields X_1 when it reacts, and Equation (7):

$$\Phi_2 = \frac{k_1}{k_1 + \Gamma_1} \quad (7)$$

is likewise the probability that X_1 yields X_2 upon reaction. The

probability of decay via Γ_2 and after n cycles is given by Equation (8):

$$p_2(n) = (1 - \Phi_1)\Phi_2(\Phi_1\Phi_2)^n \quad (8)$$

According to Equation (9), the probability of decay after n cycles is thus:

$$p(n) = p_1(n) + p_2(n) = (1 - \Phi_1\Phi_2)(\Phi_1\Phi_2)^n \quad (9)$$

and the average number of cycles is finally obtained as Equation (10):

$$\bar{n} = \sum_{n=0}^{\infty} np(n) = \left(\frac{1}{\Phi_1\Phi_2} - 1 \right)^{-1} \quad (10)$$

or, in terms of the rate constants of the elementary processes, as Equation (11):

$$\bar{n} = \left[\left(1 + \frac{\Gamma_1}{k_1} \right) \left(1 + \frac{\Gamma_2}{k_2} \right) - 1 \right]^{-1} \quad (11)$$

Equation (11) answers the question "How fast is a fast equilibrium?" in relative terms, by comparing the equilibrium rate constants with those of the competing decay processes.

For large \bar{n} there is a rapid equilibrium (in relative terms) between X_1 and X_2 , and Equation (11) becomes Equation (12):

$$\bar{n} = \left(\frac{\Gamma_1}{k_1} + \frac{\Gamma_2}{k_2} \right)^{-1} \quad (12)$$

This situation is in fact a pre-equilibrium, as X_1 and X_2 eventually die out. It has been shown^[4] that for a pre-equilibrium between m species that disappear via m unimolecular channels Γ_i ($i=1, 2, \dots, m$), there is a long-time common decay rate given by Equation (13):

$$k = \frac{1}{\tau} = \sum_{i=1}^m x_i \Gamma_i \quad (13)$$

where the x_i are the fractions of each species. In the case of Scheme 3, Equation (13) reduces to Equation (14):

$$\frac{1}{\tau} = x_1 \Gamma_1 + x_2 \Gamma_2 \quad (14)$$

and, as quasi-equilibrium holds, one also has Equation (15):

$$\frac{x_2}{x_1} = \frac{k_1}{k_2} \quad (15)$$

Equation (14) yields Equation (16):

$$\tau = \frac{k_1 + k_2}{k_2 \Gamma_1 + k_1 \Gamma_2} \quad (16)$$

Now, rearranging and using Equation (2), Equation (17) is obtained:

$$\frac{\tau}{\tau_c} = \left(\frac{\Gamma_1}{k_1} + \frac{\Gamma_2}{k_2} \right)^{-1} \quad (17)$$

This corresponds exactly to Equation (12), confirming that the average number of cycles in the case of a pre-equilibrium is given by Equation (18):

$$\bar{n} = \frac{\tau}{\tau_c} \quad (18)$$

In monomer–excimer intermolecular kinetics, as well as in thermally activated delayed fluorescence, k_2 markedly increases with temperature, whereas the remaining rate constants have a much weaker temperature dependence. In this way, for sufficiently high temperatures, the average number of cycles attains its maximum value, given by Equation (19):

$$\bar{n}_\infty = \left(\frac{1}{\Phi_1^\infty \Phi_2^\infty - 1} \right)^{-1} \quad (19)$$

which reduces to Equation (20):

$$\bar{n}_\infty = \left(\frac{1}{\Phi_1^\infty} - 1 \right)^{-1} \quad (20)$$

when $\Phi_2^\infty \approx 1$.

The importance of reversibility for a given reaction under specific conditions can be ascertained by computing the fraction of molecules undergoing at least one full cycle. This can be obtained from Equation (9) as Equation (21):

$$p(n \geq 1) = \frac{\bar{n}}{1 + \bar{n}} \quad (21)$$

Average numbers of cycles of 9, 19 and 99 respectively correspond to 90%, 95% and 99% of all molecules undergoing at least one full cycle.

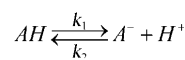
When X_1 is continuously generated (either by its continuous feed to a stirred reactor, or by continuous irradiation of a precursor ground state species), a steady state is attained after some time, and the concentration of X_1 obeys Equation (22):

$$\frac{[X_1]}{[X_1]^0} = 1 + \bar{n} \quad (22)$$

where $[X_1]^0$ is the concentration that X_1 would have in the absence of reversibility, that is, for $\Phi_1 = 0$ (which includes $k_2 = 0$ and $\Gamma_2 \rightarrow \infty$ cases), all other conditions being kept constant.

2.3. Acid–Base Equilibrium

Of the many possible kinds of reactions that are described by Scheme 1, the acid–base (proton transfer) equilibrium as shown in Scheme 5 is probably the most common situation. The average duration of an acid–base cycle is given by Equation (23):



Scheme 5. Acid–base equilibrium.

$$\tau_c = \frac{1}{k_1} + \frac{1}{k_2[H^+]} \quad (23)$$

For acetic acid in water at room temperature,^[6] for instance, $k_1 = 9 \times 10^5 \text{ s}^{-1}$ and $k_2 = 5 \times 10^{10} \text{ M}^{-1} \text{ s}^{-1}$, hence the average cycle duration is 1 μs for $\text{pH} < 4$, but becomes 33 min for $\text{pH} 14$ (see Figure 1), owing to the extreme slowness of the reverse reac-

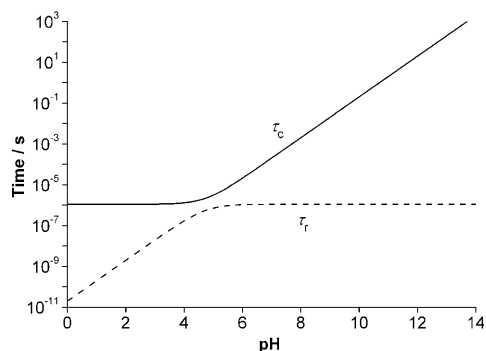
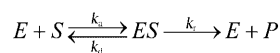


Figure 1. Average cycle duration and relaxation time for acetic acid in water as a function of pH.

tion under these conditions. The infinite dilution relaxation time ($[H^+] \gg [A^-]$), on the other hand, is of the order of 200 ps for $\text{pH} 1$, and increases for higher pH values, stabilizing at 1 μs for $\text{pH} > 6$ (see Figure 1).

2.4. Enzyme Kinetics

As a further application of Scheme 1, consider the simplest description of enzyme kinetics, the Henri–Michaelis–Menten mechanism shown in Scheme 6. Here, the substrate S is in



Scheme 6. Henri–Michaelis–Menten mechanism.

large excess. A central quantity is the turnover rate r_t given by Equation (24):

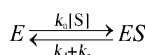
$$r_t = \frac{1}{[E]_T} \frac{d[P]}{dt} \quad (24)$$

where $[E]_T$ is the total enzyme concentration. Equation (24) can be rewritten as Equation (25) in terms of the rate constants of the elementary steps and of the substrate concentration^[7]:

$$r_t = \frac{k_a k_r [S]}{k_d + k_r + k_a [S]} = \frac{k_{cat} [S]}{K_m + [S]} \quad (25)$$

where Equation (25) corresponds to the usual form and notation, with $k_{cat} = k_r$ and $K_m = (k_d + k_r)/k_a$. Considering the cycle description of the same mechanism, where Scheme 6 is replaced by Scheme (7), the average cycle duration is now as shown by Equation (26):

$$\tau_c = \frac{1}{k_a [S]} + \frac{1}{k_d + k_r} = \frac{1}{k_a} \left(\frac{1}{[S]} + \frac{1}{K_m} \right) \quad (26)$$



Scheme 7. Cyclic form of the Henri-Michaelis-Menten mechanism.

On the other hand, one can define a yield per cycle for the formation of product, as in Equation (27):

$$\Phi_p = \frac{k_r}{k_d + k_r} = \frac{k_{cat}}{k_a K_m} \quad (27)$$

This is the probability that the enzyme-substrate complex will result in the formation of the product.

The turnover rate can now be written as the product of the average number of cycles per unit time and the yield per cycle [Eq. (28)]:

$$r_t = \frac{\Phi_p}{\tau_c} \quad (28)$$

This equation has a clear dynamical interpretation, namely that the turnover rate is the number of successful $E \rightarrow ES \rightarrow E$ cycles per unit time. Increasing the substrate concentration does not change the yield of product formation per cycle, but it can reduce the average cycle duration [Eq. (26)]. For very efficient enzymes, for which the whole process is determined by diffusion only, Φ_p is close to unity and the average cycle duration is $\tau_c = (k_a [S])^{-1}$, where k_a corresponds to diffusion control.

Using representative values for β -galactosidase and RGP (resorufin- β -D-galactopyranoside) substrate,^[8] namely $k_a = 5 \times 10^7 \text{ M}^{-1} \text{ s}^{-1}$, $k_d = 18000 \text{ s}^{-1}$ and $k_r = 900 \text{ s}^{-1}$, $\Phi_p = 5\%$ is obtained. For a substrate concentration of $50 \mu\text{M}$ the average cycle duration is $450 \mu\text{s}$, implying a turnover rate of 110 s^{-1} , see Figure 2. The minimum cycle duration, attained for substrate concentrations higher than about 20 mM , is $50 \mu\text{s}$, for which the turnover rate attains its maximum value, $k_r = 900 \text{ s}^{-1}$.

2.5. Excited-State Proton Transfer

Excited-state proton transfer kinetics, to which Scheme 3 applies, was studied in detail for the 7-hydroxyquinolinium ion.^[9] In 4.0 M perchloric acid, the rate constants have the following values:^[9] $k_1 = 1.6 \times 10^{10} \text{ s}^{-1}$, $k_2 = 2.7 \times 10^{10} \text{ s}^{-1}$, $\Gamma_1 = 9.3 \times 10^7 \text{ s}^{-1}$, and $\Gamma_2 = 3.7 \times 10^8 \text{ s}^{-1}$. In this way, the average cycle duration is

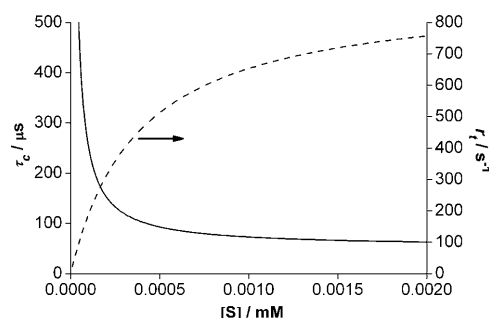


Figure 2. Average cycle duration and turnover number for β -galactosidase as a function of substrate (RGP) concentration.

100 ps , and the average number of cycles, computed from Equation (11), is 51, which shows that a large number of proton-transfer cycles are effected in the excited state before decay to the ground state occurs. The distribution function of the number of cycles is shown in Figure 3.

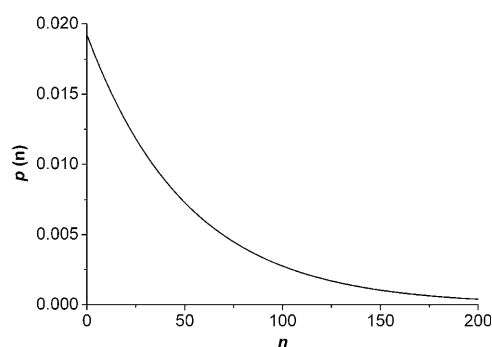


Figure 3. Probability of decay of the 7-hydroxyquinolinium ion in 4.0 M perchloric acid after n excited-state cycles. The average number of cycles is 51.

The effect of the analytical concentration of perchloric acid on the average cycle duration and on the average number of cycles is displayed in Figure 4. The existence of a minimum for

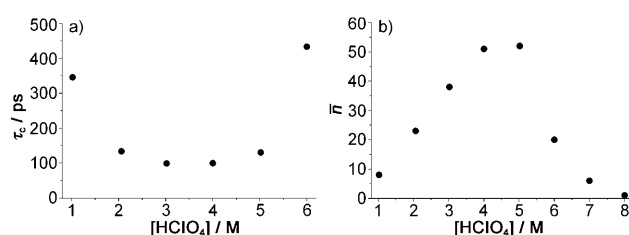
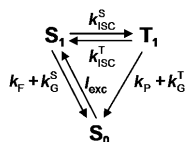


Figure 4. a) Average excited-state cycle duration for the 7-hydroxyquinolinium ion in perchloric acid. b) Average number of excited-state cycles for the 7-hydroxyquinolinium ion in perchloric acid.

the average cycle duration, and of a maximum for the average number of cycles at intermediate $[\text{HClO}_4]$ (but not for the same values) is a consequence of the dependence of k_1 and k_2 on $[\text{HClO}_4]$.^[9]

2.6. Thermally Activated Delayed Fluorescence

The accepted kinetic model for thermally activated delayed fluorescence (TADF) in the condensed phases is a three-state system (thus assuming fast decoherence) that can be represented by Scheme 8. Here $I_{\text{exc}}(t)$ is the excitation intensity, k_F



Scheme 8. Kinetic Scheme for TADF.

and k_P are the radiative rate constants for fluorescence and phosphorescence, respectively, k_G^S and k_G^T are the nonradiative rate constants for deactivation to the ground state (internal conversion from S_1 and intersystem crossing from T_1 , respectively), and k_{ISC}^S and k_{ISC}^T are the intersystem crossing (ISC) rate constants for singlet-to-triplet and triplet-to-singlet conversion, respectively. For delta excitation, this scheme reduces to Scheme 3. The simplest form for the triplet-to-singlet ISC rate constant (k_{ISC}^T) is the Arrhenius equation,^[10,11] given by Equation (29):

$$k_{\text{ISC}}^T(T) = A \exp\left(-\frac{\Delta E_{\text{ST}}}{k_B T}\right) \quad (29)$$

where ΔE_{ST} is the S_1 – T_1 energy gap. Owing to the relative energies of S_1 and T_1 , the triplet-to-singlet ISC is always an activated process that is strongly temperature-dependent.

The average number of cycles \bar{n} can in this case be written as Equation (30):

$$\bar{n} = \left[\frac{1}{\Phi_T} \left(1 + \frac{1}{k_{\text{ISC}}^T \tau_P^0} \right) - 1 \right]^{-1} \quad (30)$$

where Φ_T is the quantum yield of triplet formation, $\Phi_T = k_{\text{ISC}}^S / (k_F + k_G^S + k_{\text{ISC}}^S)$ and $\tau_P^0 = 1 / (k_P + k_G^T)$. The maximum average number of cycles value, \bar{n}_{max} , attained for sufficiently high temperatures, is thus [Eq. (31)]:

$$\bar{n}_{\text{max}} = \left(\frac{1}{\Phi_T} - 1 \right)^{-1} \quad (31)$$

As the fluorescence intensity I is proportional to the concentration of S_1 , Equation (32) follows from Equation (22):

$$\frac{\Phi_F}{\Phi_{\text{PF}}} = \frac{I_F}{I_{\text{PF}}} = 1 + \bar{n} \quad (32)$$

where Φ_F and Φ_{DF} respectively are the total and delayed fluorescence quantum yields.^[11] In this way, the increase in fluorescence intensity owing to TADF is a direct measure of the average number of $S_1 \rightarrow T_1 \rightarrow S_1$ cycles performed. This result is rea-

sonable, as each return from T_1 to S_1 brings a new opportunity for fluorescence emission.

The combination of several remarkable photophysical properties of fullerene C_{70} , specifically the Φ_T very close to one, the small ΔE_{ST} gap, and the long intrinsic phosphorescence lifetime, lead to an exceptionally strong TADF in this molecule.^[10–12] Using the following set of data, obtained for C_{70} in polystyrene:^[11,12] $\Phi_T = 0.99$, $\tau_F = 630$ ps, $\tau_P^0 = 28$ ms, $A = 8 \times 10^7 \text{ s}^{-1}$, $\Delta E_{\text{ST}} = 29 \text{ kJ mol}^{-1}$, the maximum average number of cycles is estimated to be 99, and the maximum fluorescence intensification factor to be 100. The computed average number of cycles as a function of temperature is displayed in Figure 5a. It is seen that a large number of excited state cycles are already effected at moderate temperatures, as experimentally observed for C_{70} and for a C_{70} monoadduct,^[11] shown in Figure 5b.

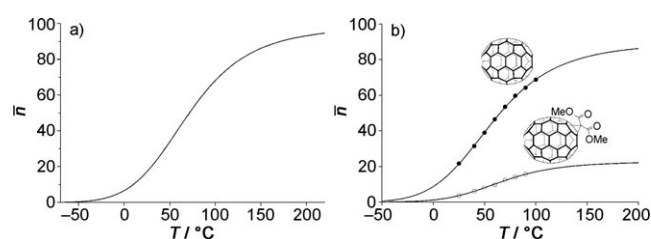


Figure 5. a) Computed average number of $S_1 \rightarrow T_1 \rightarrow S_1$ cycles as a function of temperature for C_{70} in polystyrene. b) Average number of $S_1 \rightarrow T_1 \rightarrow S_1$ cycles as a function of temperature for both C_{70} and a C_{70} -derivative in polystyrene.

2.7. Monomer–Excimer Kinetics

Scheme 3 also applies to monomer–excimer kinetics in fluid media^[13,14] if X_1 is identified with the monomer (intrinsic decay rate Γ_1) and X_2 with the excimer (intrinsic decay rate Γ_2). For the intermolecular case, the forward rate constant k_1 is pseudo-unimolecular, as it is the product of the diffusion-controlled bimolecular rate constant by the monomer concentration. The backward rate constant k_2 is thermally activated, and increases markedly with temperature. At the high-temperature limit of monomer–excimer kinetics, a fast equilibrium is attained in the excited state in many cases. It is thus of interest to estimate the average number of cycles, which allows a more precise characterization of such an equilibrium.

Two different excimer-forming molecules are considered here: pyrene^[13] and toluene.^[15] One of the most stable, the pyrene excimer has a binding energy of 40 kJ mol^{-1} , and the system is only weakly reversible at room temperature. The opposite is the case with toluene, whose excimer has a small binding energy (16 kJ mol^{-1}).

The rate coefficients for pyrene in degassed cyclohexane at 30°C are:^[13] $k_1 = 6.7 \times 10^9 [M] \text{ s}^{-1}$, where $[M]$ is the monomer concentration in mol dm^{-3} , $k_2 = 6.5 \times 10^6 \text{ s}^{-1}$, $\Gamma_1 = 2.3 \times 10^6 \text{ s}^{-1}$, and $\Gamma_2 = 1.6 \times 10^7 \text{ s}^{-1}$. For $[M] = 10^{-2} \text{ M}$, which is near the solubility limit of pyrene in cyclohexane, the average cycle duration is 170 ns, below the monomer lifetime (440 ns), but significantly above the excimer lifetime (63 ns), and the average number

of cycles is only 0.4. Therefore $p(n \geq 1) = 0.29$, that is, only about a third of the excited monomers are reformed from an excimer. It is interesting to note that a hypothetical increase in the concentration of monomer above 10^{-2} M does not markedly affect either the average cycle duration or the average number of cycles, as both parameters are controlled by the rates of excimer dissociation and decay in the high-concentration range.

Quite a different picture is obtained for toluene in degassed cyclohexane, for which the rate coefficients at 20°C are:^[15] $k_1 = 5 \times 10^9 \text{ [M] s}^{-1}$, where $[M]$ is the monomer concentration in mol dm^{-3} , $k_2 = 3.1 \times 10^{10} \text{ s}^{-1}$, $\Gamma_1 = 4.2 \times 10^7 \text{ s}^{-1}$, and $\Gamma_2 = 6.3 \times 10^7 \text{ s}^{-1}$. For $[M] = 10^{-2}$ M, the average cycle duration is 20 ns, close to the monomer lifetime (24 ns) and to the excimer lifetime (16 ns), and the average number of cycles is 1.2. The estimated dependence of the average cycle duration and of the average number of cycles on the concentration of toluene is displayed in Figure 6. Contrary to the pyrene case, both

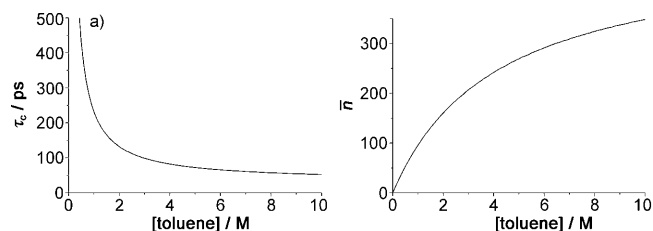


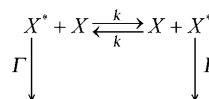
Figure 6. a) Average excited-state cycle duration for toluene in cyclohexane. b) Average number of excited-state cycles for toluene in cyclohexane.

parameters change significantly with the monomer concentration, for example, for 1 M toluene, the average cycle duration is 230 ps and the average number of cycles is 96. The average number of cycles increases with the monomer concentration, and is predicted to reach a value of 350 for 10 M toluene, whereas the computed average cycle duration attains 52 ps for the same concentration.

It is important to remark that in reactions involving bimolecular steps the actual molecules participating in the cycling process may change with time. In the monomer–excimer situation the initially excited monomer is part of the first excimer, but as this one dissociates, it may or may not become the new excited monomer again. Also, the ground state partner that joins the excited one to yield the new excimer may even be a third molecule. What cycles is the excitation energy.

2.8. Reversible Excitation Energy Transfer (Homo-FRET)

Fully reversible excitation energy transfer (homotransfer), as depicted in Scheme 9, is also an example of a cyclic process in the excited state. We restrict the analysis herein to a molecular pair at a fixed distance and with fixed relative orientation. Assuming a dipolar mechanism (Förster resonance energy transfer, FRET),^[16] k is given by Equation (33):



Scheme 9. Homotransfer of electronic excitation energy.

$$k = \Gamma \left(\frac{R_0}{r} \right)^6 \quad (33)$$

where R_0 is the effective critical radius. According to Equation (34), the average number of cycles is thus:

$$\bar{n} = \frac{\left(\frac{R_0}{r} \right)^{12}}{1 + 2 \left(\frac{R_0}{r} \right)^6} \quad (34)$$

and is independent of the lifetime. For $R_0 \ll r$, Equation (34) reduces to $\bar{n} = (R_0/r)^{12}$ whereas for $R_0 \gg r$ it becomes $\bar{n} = \frac{1}{2} (R_0/r)^6$. Homo-FRET can lead to a very high number of excited-state cycles (Figure 7). Considering typical values $R_0 =$

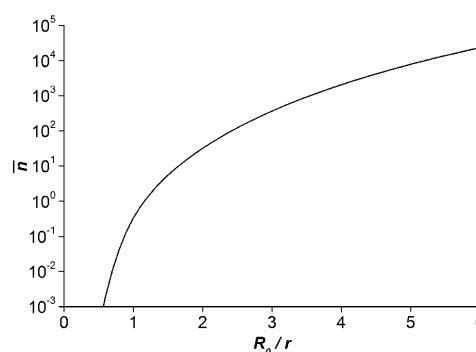


Figure 7. Average number of excited-state cycles vs R_0/r for a molecular pair undergoing homo-FRET.

40 Å and $r = 10$ Å, the average number of cycles is of the order of 2000. Assuming a lifetime of 5 ns, the computed average cycle duration is 2 ps. Note that the above picture is valid only in case of thermalization of X^* prior to each transfer.

3. Conclusions

Two-state reversible reactions were viewed according to a new approach, based on the concept of cycle. Two parameters were introduced, namely the average cycle duration [Eq. (2)] and the average number of cycles [Eq. (11)], the last one applying to cycles with at least one unstable species. These parameters allow answering the question “How fast is a fast equilibrium?” in absolute and relative terms, respectively. In this way, what is generally simply termed a fast equilibrium can be better characterized, and a distinction made in terms of the defined parameters.

The usefulness of the approach was demonstrated by application to selected ground-state mechanisms (acid–base equilibrium, enzyme kinetics) and excited-state processes (proton

transfer, thermally activated delayed fluorescence, monomer–excimer kinetics, homo-FRET).

It was shown that for reversible excited-state processes the average number of cycles can range from a few tens (proton transfer, thermally activated delayed fluorescence) to many thousands (homo-FRET).

Appendix

According to Scheme 1, once formed, both X_1 and X_2 decay unimolecularly with rate constants k_1 and k_2 , respectively, hence the respective survival probability follows an exponential distribution, Equation (A1):

$$f_i(t) = k_i \exp(-k_i t) \quad (i = 1, 2). \quad (\text{A1})$$

The average duration or lifetime (also called transit time^[17]) is given by Equation A2:

$$\bar{t}_i = \int_0^\infty t f_i(t) dt = \frac{1}{k_i} \quad (i = 1, 2). \quad (\text{A2})$$

For the assumed mechanism the survival probabilities of X_1 and X_2 are uncorrelated.

The cycle duration is a random variable that is the sum of two independently and exponentially distributed variables—the survival probabilities of X_1 and X_2 . In this way, the probability density function for the cycle duration is given by the convolution of the two PDFs in the form of Equation (A3)^[18]:

$$f_c(t) = f_1(t) \otimes f_2(t) = \int_0^t f_1(u) f_2(t-u) du \quad (\text{A3})$$

which leads to Equation (A4)

$$f_c(t) = \frac{k_1 k_2}{k_1 - k_2} (e^{-k_2 t} - e^{-k_1 t}). \quad (\text{A4})$$

The average cycle duration is given by Equation (A5)

$$\tau_c = \bar{t}_1 + \bar{t}_2 = \int_0^\infty t f_c(t) dt = \frac{1}{k_1} + \frac{1}{k_2} \quad (\text{A5})$$

The PDF for the duration of n cycles is again obtained by taking into account that it is a random variable resulting from the sum of n independent variables, all distributed according to Equation (A4), hence Equation (A6)

$$f_n(t) = \underbrace{f_c(t) \otimes \dots \otimes f_c(t)}_{n \times} \quad (\text{A6})$$

It follows from the central limit theorem^[19] that for large n this PDF approaches a Gaussian PDF with mean $n\tau_c$ and standard deviation $\sqrt{n}\tau_c$.

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