Modelling stock return sensitivities to economic factors
with the Kalman filter and Neural networks

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Abstract
Sensitivity analysis of asset returns to various economic variables provides investors with a useful tool to build portfolios and manage their risk. However, there are strong reasons to believe that stock exposures evolve through time and that factor models involving them are only pertinent if they use reliable estimates of future sensitivities. Both Kalman filtering and Neural Networks may be used to provide such estimates. While the Kalman filter is good at modelling the time structure of sensitivities, Neural network are capable of relating them to exogeneous variables in a non-linear way. Furthermore, because the two approaches perform complementary tasks of sensitivity forecasting, they may be combined to achieve better performances. These procedures are evaluated in a controlled simulation experiment and in a real stock exposure analysis. Stock sensitivities to interest and exchange rates are forecasted for 90 French shares and portfolios are built accordingly.

Introduction
Factor models for equity investment have enjoyed a great popularity over the past two decades in both academic and practitioners communities. There are two main reasons for this popularity. Firstly, the emergence of factor models such as the CAPM (Sharpe64, Merton73) or the APT (Ross 76) has been useful in explaining how assets are priced in the financial markets by relating their performance to their risk, viewed in terms of asset exposure to market return or economic factors. Secondly, although these factor models do not provide any prediction of future asset returns, they do provide an important information for risk management, i.e. the sensitivity of stock returns to various economic variables. Indeed, knowing stock exposures enables fund managers to build portfolios that are immune against changes in those variables or, alternatively, portfolios that are exposed to them, if they anticipate a change in the economic environment.

While the concept behind factor models is theoretically appealing, its effective application remains problematic, mainly because the relationship between asset returns and the various economic factors is not fixed but tends to vary through time. It is generally believed that these changes are the results of micro-economic factors, such as changes in the operational structure of the corporation, or macro factors, such as general business conditions or expectations about relevant future events. The implication of this non-stationarity in the underlying relationships is that it prevents fund managers from using estimates of present sensitivities, since these estimates will be different from future ones.

This paper describes and compares two techniques, designed to estimate asset future sensitivities to economic factors. The first technique is a Kalman filter (Kalman60), which is increasingly used by social scientists interested in analysing relationships that vary over time. The Kalman filter is used to model the underlying dynamics of the sensitivity coefficients. The second technique is a multi-layer neural network trained with the standard backpropagation algorithm. Because Neural Networks are able to model conditional factor, they constitute a powerful tool to model the apparent non-stationarity in the relationships that is related to a bad model specification rather than real changing rules.

Finally, we propose to combine both techniques in a single framework in order to model the time structure as well as conditional influences to the underlying sensitivities. The performances of the different models will be compared using both an artificial dataset and a non-trivial application in exposure analysis of stock returns to multiple economic factors.
1. Sensitivity analysis with the Kalman Filter

In order to model non-stationary systems, in which the underlying relationships evolve through time, an extension of the linear regression is needed, for which the coefficients are no longer fixed but stochastic. A convenient way to handle such stochastic parameter models is to use a state space representation and to use Kalman filtering, which provides optimal estimates of the non-observable sensitivity structure (or "states") of the system.

Two equations define the process. The observation equation describes the linear relationship between the observed variables $X$ (e.g. asset returns) and the explanatory factors $F$:

$$X_t = F_t^T S_t + n_t$$

with $S_t$ the non-observable states and $n_t$ the observation error

The transition equation describes how the states evolve over time:

$$S_t = G_t S_{t-1} + w_t$$

with $G_t$ the transition matrix and $w_t$ a deviation vector

This representation is fairly general and allows various time structures for sensitivities. For instance, if we assume that the coefficient of the regression model follows a random walk, we can re-write equations (2) in the form: $S_t = S_{t-1} + w_t$ with sensitivity vector $S_t^T = [\alpha, \beta_1, ..., \beta_n]$, and factor vector $F_t^T = [1, f_1, ..., f_n]$. In the particular case where $w_t = 0$, the $S_t$ are constant and we are back to the familiar linear regression model with constant coefficients.

The Kalman filter provides then a method to update the state vector $S_t$ when a new observation becomes available. Kalman filtering has three major advantages. Firstly, it adapts fairly quickly to any change in the underlying states, without requiring a fixed size rolling window for the estimation of the parameters (unlike regression models). Secondly, it can easily deal with the time structure of the sensitivity coefficients. Thirdly, the calculations are recursive, so that, although the current estimates are based on the whole past history of measurement, there is no need for an ever-expanding memory.

To illustrate the ability of the Kalman filter to adapt quickly to changes in sensitivity, let us consider a synthetic example for which we know the actual sensitivity. In this example the dependant variable is generated by equation (3):

$$Y_t = A + a_t \times X_t + e_t$$

with $A$ being a constant

$a_t = 0$ in some periods and 0.5 in others, $a_t$ is the actual sensitivity of $Y$ to $X$

$X_t$ being an observed random variable (i.e. factors)

$e_t$ and random variable (noise)

Figure 1 represents the sensitivities of $Y$ to $X$ calculated by a standard linear regression model on a rolling window, and by a Kalman filter, with a random walk model for the underlying sensitivity.

Figure 1: Sensitivities calculated by an OLS regression model, and a Kalman filter.
As shown in this figure, the stochastic coefficient regression model has a clear advantage over the fixed coefficient model as far as reaction time is concerned, because it is not limited by the choice of a fixed size window to estimate the various coefficients.

However, changing sensitivities as measured by the Kalman filter are not always related to the non-stationarity in the underlying dynamics. It can also be related to a bad specification of the linear model, in particular when the underlying influences contains interactions effects between variables. In these cases a Neural Network approach can be a useful tool. This issue will now be discussed.

2. Neural Network approach to conditional sensitivity modelling

Non-stationarity in the relationships between asset returns and economic factors may be explained by a change in the micro- and macro-economic environment of the company. Sometimes these changes can be related to other indicators, and therefore may be partly predicted. In other cases, we do not know which indicators can be used to model these changes in sensitivity and can only apply a univariate model.

The stochastic coefficient regression model will not make any difference between the two cases, because it is not able to model conditional sensitivities, but only the sensitivity time structure in a linear way. Neural networks are known for their ability to model, without any a priori assumption about the underlying dynamics, non-linear relationships, in particular interaction effects, i.e. conditional sensitivities.

We show in the following example how Neural networks can be used to measure and predict these conditional sensitivities. Let us consider the artificial dataset used previously and assume that the $a_i$ are not a smooth function of time but a function of another observable indicator that is independent from time.

The sensitivity of the network output to an input is calculated by the partial derivative $\left( \frac{\partial o_i}{\partial i} \right)$, at each point in time. The result is shown in figure 2.

![Figure 2: Sensitivities calculated by a Kalman filter and a neural network.](image)

As shown by this figure, the Kalman Filter is only effective when the underlying sensitivities evolve gradually. If the sensitivity can be related to other indicators, then a Neural Network is more appropriate.

In most cases however, apparent changing sensitivities are the results of both non-stationarity in the underlying relationships and interaction effects between variables. Therefore, because Kalman filtering and Neural Networks deal with different aspects of the problem, they should be combined in a single framework to achieve better descriptive and predictive performance when applied to real market data.
3. Combining Kalman filtering and Neural Networks to better predict sensitivities

In order to take account of both the sensitivities time structure and their relationships with financial indicators, Neural Networks and Kalman filtering are combined in a single framework.

The relevance of such an approach is evaluated in the context of investment management by applying the procedure to a real sensitivity forecasting problem. The aim of this application is to model the sensitivity of stock relative returns to changes in interest and exchange rates. The dataset utilised contains weekly prices of 90 French stocks, as well as financial indicators and Earning per Share estimations collected between December 1987 and January 1995.

As an example, figure 3 represents the relative sensitivity of Elf Aquitaine to changes in long term interest rates, measured by a Neural Network and a Kalman Filter.

![Figure 3: Sensitivity of Elf Aquitaine's relative return to changes in long term interest rates](image)

In the full paper, we use an investment strategy based on sensitivity forecasts to compare the performance of the different prediction methods.

Conclusions and future work

We described and compared three procedures for predicting stock sensitivity to economic factors. While the strength of the Kalman Filter is in modelling the sensitivity time structure, Neural Networks are more appropriate to relate these sensitivities to exogeneous variables. Both methods are therefore complementary and should be used together to achieve better predictive performances.

In the light of this analysis, we will proceed with the study by investigating sector exposures to economic factors, stock clustering according to their exposures, as well as risk analysis based on scenario simulation.

References


