Executive Summary. Comparing the downside risk (DR) framework with classic modern portfolio theory (MPT) is less straightforward than it may appear. Two recent studies have attempted to compare the two models in terms of portfolio risk. This study uses an empirical example to demonstrate the pitfalls of making such comparisons. Additionally, we suggest a means of making an appropriate comparison between DR and MPT.

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Introduction

There has been increasing interest in applying the downside risk (DR) approach to real estate portfolio management research. Two recent studies, Sivitanides (1998) and Sing and Ong (2000), have proposed the DR framework as a feasible alternative to classic modern portfolio theory (MPT). Although they differ in some respects, both studies present similar empirical comparisons between DR and MPT efficient frontiers, showing to different degrees that portfolios generated with a DR framework are more efficient in terms of risk-adjusted return than those generated with classic MPT. However, the comparisons these studies provide may be inappropriate and result in ill-advised adoption of the results of these two studies. Therefore, this study provides a note of caution. It employs an empirical example to demonstrate the logical pitfalls encountered when comparing DR and MPT.

The DR concept is well documented in the finance literature. See, for example, Mao (1970), Hogan and Warren (1974), Porter (1974), Fishburn (1977), Levy and Markowitz (1979), Scott and Horvath (1980), Harlow (1991) and Nawrocki (1991). The Sivitanides (1998) and Sing and Ong (2000) studies, which are the focus of this comment, also discuss relevant theoretical DR models and associated mathematical algorithms in some detail. Interested readers may refer to them for a basic understanding of the concept. Although multiple analyses are performed in the two referenced studies, the scope of this study is limited to the approach they take in comparing the portfolio efficiency of DR to classic MPT.
Comparing the efficiency of two portfolios is straightforward within a MPT framework. Through application of an optimization algorithm, such as quadratic programming, one can specify an expected portfolio return and minimize the variances of the two portfolios. The portfolio with the smallest minimized variance (or standard deviation) is said to be more efficient than the other portfolio. However, this approach is not appropriate for a MPT-minimized-portfolio vs. DR-minimized-portfolio comparison, because the MPT and DR models employ different risk measures. MPT uses a traditional variance (or standard deviation) risk measure, while the DR approach measures downside risk by means of the so-called lower partial moment (LPM).1 Directly comparing these two risk measures at any given return rate expectation is akin to the proverbial comparison of “apples with oranges.” One cannot logically conclude that a DR portfolio is more efficient than a MPT portfolio simply because the DR LPM is less than the MPT variance.

Recent Downside Risk Approach Studies

Sing and Ong (2000) argue that, at a given expected portfolio return, a DR portfolio is less risky than an MPT portfolio. Their conclusion is based on an empirical analysis indicating that, for a given expected portfolio return, DR portfolios have smaller standard deviations than MPT portfolios. The study uses the quarterly return data for three Singaporean assets—stocks, bonds and real estate—over the 1983:2 through 1997:2 period. Sing and Ong use the classic MPT approach and three different DR models to generate optimal portfolios under various expected portfolio returns as a means of comparing MPT and DR portfolio efficiency. Exhibit 1 is a partial reproduction of their comparison results. The exhibit presents each model’s optimal portfolio asset class weight, standard deviation and expected return. Sing and Ong claim, for any given portfolio return, that the three DR portfolios are more efficient than the MPT portfolio because they all have smaller standard deviations than the MPT portfolio. Apparently, they realized that standard deviation and downside risk are different and not directly comparable, since they chose to compare the portfolios on the basis of standard deviation.

Two consequent issues arise. First, does standard deviation appropriately measure DR portfolio risk? Standard deviation is not an appropriate DR risk measure because it is irrelevant for investors who are solely concerned with downside risk. In fact, at a given amount of downside risk, investors may prefer portfolios with larger standard deviations. Second, another look at Exhibit 1 reveals some peculiar results. Consider portfolios numbered 1 through 4 in the right-hand column, for example. Portfolio 1 is the MPT result with optimal asset weights of 5.04% real estate, 1.76% stocks and 93.2% bonds. By definition, this is the only portfolio composition having the minimum standard

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**Exhibit 1**

Comparison of MPT vs. DR by Sing and Ong

<table>
<thead>
<tr>
<th>Optimization Model</th>
<th>Portfolio Return</th>
<th>Portfolio Std. Dev. (%)</th>
<th>Real Estate Weight (%)</th>
<th>Stock Weight (%)</th>
<th>Bond Weight (%)</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical MPT</td>
<td>0.1</td>
<td>2.10</td>
<td>5.04</td>
<td>1.76</td>
<td>93.20</td>
<td>1</td>
</tr>
<tr>
<td>DR Model 1</td>
<td>0.1</td>
<td>1.50</td>
<td>2.97</td>
<td>5.11</td>
<td>91.93</td>
<td>2</td>
</tr>
<tr>
<td>DR Model 2</td>
<td>0.1</td>
<td>1.54</td>
<td>4.88</td>
<td>2.03</td>
<td>93.09</td>
<td>3</td>
</tr>
<tr>
<td>DR Model 3</td>
<td>0.1</td>
<td>1.49</td>
<td>6.14</td>
<td>0.00</td>
<td>93.86</td>
<td>4</td>
</tr>
<tr>
<td>Classical MPT</td>
<td>0.5</td>
<td>2.15</td>
<td>25.79</td>
<td>0.00</td>
<td>74.21</td>
<td>5</td>
</tr>
<tr>
<td>DR Model 1</td>
<td>0.5</td>
<td>1.29</td>
<td>20.80</td>
<td>8.01</td>
<td>71.18</td>
<td>6</td>
</tr>
<tr>
<td>DR Model 2</td>
<td>0.5</td>
<td>1.41</td>
<td>25.79</td>
<td>0.00</td>
<td>74.21</td>
<td>7</td>
</tr>
<tr>
<td>DR Model 3</td>
<td>0.5</td>
<td>1.29</td>
<td>25.79</td>
<td>0.00</td>
<td>74.21</td>
<td>8</td>
</tr>
</tbody>
</table>

Note: This table is a reproduction of part of the Exhibit 3 by Sing and Ong (2000). The three DR models differ slightly in their algorithms. For technical details, please refer to the original study. The last column is added by us for discussion purposes.
Exhibit 2
Sivitanides: Comparison of Downside Risk Profiles and Efficiency Ratios of DR and MPT Optimal Portfolios (Exhibit 5)

<table>
<thead>
<tr>
<th>Return (%)</th>
<th>Downsize Risk for MMR = 0%</th>
<th>Efficiency Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DR Portfolio (%)</td>
<td>MPT Portfolio (%)</td>
</tr>
<tr>
<td>9.40</td>
<td>0.876</td>
<td>0.967</td>
</tr>
<tr>
<td>9.45</td>
<td>0.916</td>
<td>1.035</td>
</tr>
<tr>
<td>9.50</td>
<td>0.960</td>
<td>1.102</td>
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<tr>
<td>9.55</td>
<td>1.019</td>
<td>1.170</td>
</tr>
<tr>
<td>9.60</td>
<td>1.124</td>
<td>1.237</td>
</tr>
<tr>
<td>9.65</td>
<td>1.236</td>
<td>1.304</td>
</tr>
<tr>
<td>9.70</td>
<td>1.354</td>
<td>1.373</td>
</tr>
</tbody>
</table>

Note: This table is a partial reproduction of the Exhibit 5 of Sivitanides (2000). The efficiency is defined as portfolio return in excess of risk-free rate divided by portfolio risk.

deviation of 2.1% given a 10% portfolio return. Logically then, any portfolio composition different from Portfolio 1—such as Portfolios 2, 3 and 4—must have a standard deviation greater than 2.1%. Even more peculiar, Portfolios 5, 7 and 8 consist of identical asset mixes, however Portfolios 7 and 8 have smaller standard deviations than Portfolio 5. This leads to the strange conclusion that a given portfolio can be more or less risky than another identical portfolio. Obviously, the standard deviations for the DR portfolios must have been miscalculated. Computed correctly, Sing and Ong would have concluded that the DR portfolios are more risky, in terms of standard deviation, and

Exhibit 3
Sivitanides: Return/Downside Risk Profile of MPT Portfolios vs. DR Portfolios (MRR = 0%)
thus inferior to the corresponding MPT portfolio by this measure. However, such a conclusion would still be incorrect because standard deviation is not an appropriate measure of DR portfolio risk.

Sivitanides (1998) presents a similar comparison between MPT and DR models. Unlike Sing and Ong (2000), his study compares the two models based on each model’s downside risk as measured by semi-deviation. Sivitanides selected four real estate asset classes from the NCREIF property return indices for the period 1979 through 1997—office, retail, R&D and warehouse. DR efficient frontiers are first derived under three different target returns, or minimum required return (MRR) as termed in the study (MRR = 0%, MRR = 5.56% and MRR = 8.55%). Then MPT efficient frontiers are calculated under the same three target return assumptions. Once this is done, downside risk is derived for each of the three MPT portfolios and DR and MPT downside risk are compared at returns ranging from 8.80% to 9.85% in 0.05% intervals for each of the three target return scenarios.

This result is not surprising. The MPT portfolios’ semideviations are not minimized by the optimization process used to derive the study’s three MPT efficient frontiers. On the other hand, the DR portfolio semideviations are minimized by the process used to derive the study’s three DR efficient frontiers. Hence, the results do not offer sufficient proof of downside risk superiority. In other words, the MPT portfolios appear to be more risky

Note: 1. All numbers are in percentage terms. The Downside risk is measured by semideviation with target return of 5.56%.

Exhibit 2 is a partial reproduction of Exhibit 5 in the study. Exhibit 3 is a reproduction of Sivitanides’ Exhibit 6, in which the DR efficient frontier is shown as being to the left of the MPT downside risk efficient frontier, implying that DR portfolios are more efficient than MPT portfolios. Although it is pointed out that the downside risk efficiency gain is small and occurs only within a narrow range along the efficient frontier, the results in Exhibit 3 clearly indicate that the downside risk of MPT portfolios are always greater than, or equal to, those of DR portfolios.
Exhibit 5
Which Portfolio is More Efficient Depends on How Risk is Measured

Panel A: When risk is measured with standard deviation

Panel B: When risk is measured with semideviation

because they are judged solely from a downside risk perspective. Had a comparison been made on a standard deviation basis, the DR portfolios would have appeared to be more risky than the MPT portfolios. The example in the next section demonstrates this point.

Empirical Example
For illustrative purposes, four asset classes in the United States were selected for analysis—common stocks, corporate bonds, treasury bills and real estate. Real estate returns are derived from the NCREIF annual total return index for all regions and all property types. Other asset class returns are based on the S&P 500, the Lehman Brother’s Long Bond Index and 1-year treasury bills. The time span of the illustration is from 1978 to 1999. For the purpose of this study, the real estate data has not been corrected for appraisal smoothing. However, the illustration would yield a similar result with unsmoothed data.

In order to show the futility of risk-based comparisons of MPT and DR efficient portfolios, the illustration follows the approaches taken by Sivitaniides (1998) and Sing and Ong (2000). First, the range of the expected portfolio returns is specified to be from 9% to 14% with 0.5% increments. For each expected portfolio return, one optimal MPT
portfolio and one optimal DR portfolio is generated. The optimal MPT portfolio weights are derived using classic quadratic programming. The optimal DR portfolio weights are derived by the method specified by Sing and Ong, which minimizes the $n$-degree, co-lower partial moment (CLPM$_n$). The analysis is confined to the second degree CLPM ($n = 2$) and measures optimal DR portfolio risk by semi-deviation. The target return for downside risk is arbitrarily assumed to be 5.56%, indicating the DR investor’s objective is to minimize the chance of portfolio returns falling below 5.56%.

The results are displayed in Exhibit 4. Eleven pairs of optimal DR and MPT portfolios are derived, and each optimal portfolio’s risk is measured in two ways—standard deviation and downside risk. By the standard deviation risk measure, all MPT portfolios are more efficient than their corresponding DR portfolios. In contrast, by the downside risk measure, all DR portfolios are more efficient than their corresponding MPT portfolios. Exhibit 5 plots these results. Clearly, the two frontiers take opposite dominant positions, depending on which risk measure is used for the horizontal axis. Portfolio efficiency dominance depends, therefore, on how risk is measured. The models produce differing optimal portfolios by minimizing different risk measures, and each model is viewed as inferior from the other model’s risk perspective. Hence, a fair comparison of DR and MPT results requires a common risk measure, which is not yet available.

**Conclusion**

Portfolio optimization using DR may be a viable alternative to MPT, but the argument cannot be based on DR producing less risky portfolios than MPT. In the absence of a common risk measure, the question of which model best reduces risk seems futile. A more appropriate goal might be to assess which model produces portfolios yielding higher returns over a given period. After all, given a certain investment horizon, it is total return that investors are concerned about. Nawrocki (1991) showed a brief comparison of the two models based on terminal wealth effect. However, he assumed that a MPT investor and a DR investor will select from the same set of asset classes and differ only in their allocations. Perhaps, given the variety of the investment universe, DR and MPT investors will be prone to construct portfolios from differing asset classes. MPT investors might prefer assets with small standard deviations, while DR investors might prefer assets with seemingly left-skewed (with a small left tail) return distributions. Hence, the reasonableness of investor return expectations may be enhanced by examining the extent to which DR and MPT produce ex post portfolios that yield better ex ante returns across a variety of investment horizons.

This said, one must consider some theoretical issues before drawing practical conclusions. For example, are real estate return distributions skewed? If asset returns are distributed symmetrically, DR and MPT models should produce similar optimal portfolios. Additionally, if real estate returns are skewed, as suggested by Myer and Webb (1994), are they stable over time? Young and Graff (1995) suggest that real estate return distributions vary from period to period. Low (1998) also provides evidence that the skewness of common stock returns may vary substantially over time. Non-stable return distribution is a problem in application of MPT and DR models, and may indicate that ex post findings are time-specific and should not be generalized to other time periods. Without an understanding of the characteristics of asset return distributions, conclusions regarding the superiority or MPT or DR is premature. Nonetheless, the theoretical appeal of downside risk and its potential applicability to portfolio diversification merits additional study. For this reason, we commend Sing and Ong (2000) and Sivitanides (1998) for introducing the DR concept to the field of real estate.

**Notes**

1. The second degree LPM is also known as semi-variance, and the square root of semi-variance is known as semi-deviation.
2. The Sivitanides study’s Exhibit 5 shows downside risk comparisons for all three target returns across the full 8.80% to 9.85% spectrum of portfolio returns. Exhibit 2 of this study only illustrates Sivitanides’ downside risk comparisons across the 9.40% to 9.70% range for MRR = 0%.
References
Low, C., The Bulls and Bears in the Cross-Section of Stock Returns: Exploring an Asymmetric Semivariance Risk Factor, Working paper, Yale School of Management.