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Exercises for the practical classes of Knowledge Representation and Reasoning

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20010/2011

Conteúdo

1 First Order Logic

Ch 2, Ex 1
For each of the following sentences, give a logical interpretation that makes that sentence false and the other two sentences true:

1. $\forall x \forall y \forall z[(P(x, y) \wedge P(y, z)) \supset P(x, z)]$
2. $\forall x \forall y[(P(x, y) \wedge P(y, x)) \supset (x = y)]$
3. $\forall x \forall y[P(a, y) \supset P(x, b)]$

Ch 2, Ex 4
In a certain town, there are the following regulations concerning the town barber:

- Anyone who does not shave himself must be shaved by the barber
- Whomever the barber shaves, must not shave himself.

Show that no barber can fulfill these requirements. That is, formulate the requirements as sentences of FOL and show that in any interpretation where the first regulation is true, the second one must be false. (This is called the barber's paradox and was formulated by Bertrand Russel.)

Hint: introduce a constant $barber$ for the *unique* barber and a binary predicate $Shaves(x, y)$ meaning that x shaves y.

2 First Order Logic

new - 2.1.
Consider the following set of sentences:

S1 Andrew is the father of Bob.

S2 Bob is the father of Chris.

S3 Every grandfather is someone´s father.

S4 Andrew is a grandfather of Chris.

1. Translate these sentences into first-order logic, using binary predicates $Father$ and $Grandfather$ and constants a, b, c for Andrew, Bob and Chris.
2. Show semantically (by reasoning about interpretations) that **S1**-**S3** do not logically entail **S4**.
3. Write in first-order logic an additional sentence that defines a general property of grandfathers, and show that **S1**-**S3** together with this new sentence entail **S4**.

Ch 3, Ex 1
Consider the following piece of knowledge:

> Tony, Mike and John belong to the Alpine Club. Every member of the Alpine Club who is not a skier is a mountain climber. Mountain climbers do not like rain, and anyone who does not like snow is not a skier. Mike dislikes whatever Tony likes, and likes whatever Tony dislikes. Tony likes rain and snow.

1. Prove that the given sentences logically entail that there is a member of the Alpine Club who is a mountain climber but not a skier.
2. Suppose we had been told that Mike likes whatever Tony dislikes, but we had not been told that Mike dislikes whatever Tony likes. Prove that the resulting set of sentences no longer logically entails that there is a member of the Alpine Club who is a mountain climber but not a skier.

3 Resolution

Ch 4, Ex 1
Determine whether the following sentence is valid using resolution:

$$\exists x \forall y \forall z ((P(y) \supset Q(z)) \supset (P(x) \supset Q(x)))$$

Ch 4, Ex 2, follow-up of Ch 3, Ex 1
Use resolution to prove that there exists a member of the Alpine club who is a climber but not a skier.

new - 3.3.
Consider the following set of sentences.
Using resolution, prove that $Mammal(winnie)$.

S1 $\forall x[(Animal(x) \wedge HasHair(x)) \rightarrow Mammal(x)]$

S2 $\forall x[Bear(x) \rightarrow (Animal(x) \wedge HasHair(x))]$

S3 $\forall x[Rabbit(x) \rightarrow Mammal(x)]$

S4 $Bear(winnie)$

S5 $Rabbit(bugsbunny)$

S6 $Animal(sylvester) \wedge HasHair(sylvester)$

4 Horn Clauses

new - 3.1.
Assume given a set of facts of the form `father(name1,name2)` (`name1` is the father of `name2`).

1. Define a predicate `brother(X,Y)` which holds iff `X` and `Y` are brothers.
2. Define a predicate `cousin(X,Y)` which holds iff `X` and `Y` are cousins.
3. Define a predicate `grandson(X,Y)` which holds iff `X` is a grandson of `Y`.
4. Define a predicate `descendant(X,Y)` which holds iff `X` is a descendant of `Y`.
5. Consider the following genealogical tree:

```
father(a,b).   % 1
father(a,c).   % 2
father(b,d).   % 3
father(b,e).   % 4
father(c,f).   % 5
```

   whose graphical representation is:

```
     a
    / \
   b   c
  / \  |
 d   e f
```

   Say which answers, and in which order, are generated by your definitions for the queries, assuming that you ask for all the solutions (using `;`).

```
?- brother(X,Y).
?- cousin(X,Y).
?- grandson(X,Y).
?- descendant(X,Y).
```

new - 3.2.
Define a predicate `mylength(L,N)` which holds iff `N` is the length of the list `L`.

new - 3.3.
Define a predicate `occurrences(X,L,N)` which holds iff the element `X` occurs `N` times in the list `L`.

new - 3.4.
Define a predicate `occurs(L,N,X)` which holds iff `X` is the element occurring in position `N` of the list `L`.

new - 3.5.
Define a predicate `sumlist(L,N)` which, given a list of integers `L`, returns the sum `N` of all the elements of `L`.

new - 3.6.
Define a predicate `mymerge(L,K,M)` which, given two ordered lists of integers `L` and `K`, returns an ordered list `M` containing all the elements of `L` and `K`.

new - 3.7.
Explain why the following program for `minimum3` does not produce the expected results.

```
/*
  minimum3(X,Y,Min) :- Min is the minimum of numbers X and Y.
*/
minimum3(X,Y,X) :- X =< Y, !.
minimum3(X,Y,Y).
```

new - 3.8.
Explain the problem with Prolog´s cut in the following program:

```
/*
  member3(X,L) :- X e um member de L.
*/
member3(X,[X|_]) :- !.
member3(X,[_|Ys]) :- member3(X,Ys).
```

5 Production Systems; Frames

new - 4.1.
Explain the notions of rule matching, rule instance, conflict set, and conflict resolution strategy in rule-based systems. Give two examples of common conflict resolution strategies. Illustrate your answers on the following example of rules and working memory elements. State what the conflict set is for the current state of the working memory and which rules will be fired first under each conflict resolution strategy. You can also refer to the conflict set at the next cycle, after the selected rules are fired.

F1 $animal(tiger)$
F2 $animal(cat)$
F3 $large(tiger)$
F4 $eatsMeat(tiger)$
F5 $eatsMeat(cat)$
R1 $\forall x[(animal(x) \wedge large(x) \wedge eatsMeat(x)) \supset dangerous(x)]$
R2 $\forall x[animal(x) \supset breathesOxygen(x)]$
R3 $\forall x[dangerous(x) \supset runAwayNow]$

new - 4.2.
Explain how decision tables can be used for knowledge elicitation and designing an expert system.

new - 4.3.
Suppose that all you have to work with in designing an expert system for recognising spam email is the following set of correctly classified messages. Produce a decision table based on this set of examples. Do not include irrelevant checks in the rules.

Message1 Properties: has an attachment, does not contain images, sender is in the receiver's address book, subject line contains "Prize". Decision: spam.
Message2 Properties: no attachments, contains images, sender is not in the receiver's address book, subject line contains "Goods". Decision: spam.
Message3 Properties: has an attachment, contains images, sender is in the receiver's address book, subject line contains "Prize". Decision: spam.
Message4 Properties: no attachments, does not contain images, sender is not in the receiver's address book, subject line does not contain "Prize"or "Goods". Decision: not spam.
Message5 Properties: has an attachment, does not contain images, sender is not in the receiver's address book, subject line contains "Prize". Decision: spam.
Message6 Properties: has no attachments, contains images, sender is in the receiver's address book, subject line contains "Goods". Decision: not spam.
Message7 Properties: has no attachments, does not contain images, sender is not in the receiver's address book, subject line contains "Goods". Decision: spam.
Message8 Properties: has no attachments, contains images, sender is not in the receiver's address book, subject line does not contain "Prize"or "Goods". Decision: not spam.

Ch 8, Ex 1
Consider a possible frame-based application for a classroom scheduler.

We want to build a program that helps schedule rooms for classes of various sizes at a university, using the sort of frame technology (frames slots and attached procedures) discussed in the text. Slots of frames might be used to record when and where a class is to be held, the capacity of a room, and so on, and **if-added** and other procedures might be used to encode constraints as well as to fill in implied values when the KB is updated.

In this problem, we want to consider updating the KB in several ways: (1) asserting that a class of a given size is to be held in a given room at a given time; the system would either go ahead and add this to its schedule or alert the user that it was not possible to do so; (2) asserting that a class of a given size is to be held at a given time, with the system providing a suitable room (if one is available) when queried; (3) asserting that a class of a given size is desired, with the system providing a time and a place when queried.

- Design a set of frames and slots to represent the schedule and any ancillary information needed by the assistant.
- For all slots of all frames, write in English pseudo-code the **if-added** or **if-needed** procedures that would appear there. Annotate these procedures with comments explaining why they are there (e.g., what constraints they are enforcing).
- Briefly explain how your system would work (what procedures would fire and why they do) on concrete examples of your choosing, illustrating each of the three situations mentioned in the description of the application.

6 Description Logics

new - 6.1.
Assume that you have an atomic concept $Woman$, roles $Child$ and $Employer$, and a constant uon for the University of Nottingham. Define the following concepts:

1. Attendee (of some work-life balance workshop) is a working mother employed by the University of Nottingham.
2. Someone all of whose children only have female children themselves (that is a person who only has granddaughters, if he or she has any grandchildren).
3. Someone who has children, and all of whose children have children.

new - 6.2.
Answer the following questions:

1. Do $d1 \sqsubseteq d2$ and $d2 \sqsubseteq d3$ entail $d1 \sqsubseteq d3$?
2. Do $c \rightarrow d1$ and $d2 \sqsubseteq d1$ entail $c \rightarrow d2$?
3. Do $c \rightarrow d1$ and $d1 \sqsubseteq d2$ entail $c \rightarrow d2$?

new - 6.3.
Consider a description logic with the following definition of a concept (note that it is slightly different from the one in the textbook, namely the first concept constructor is new and the forth concept constructor is different from `[EXISTS` n r`]`):

- $\top$ is a special atomic concept which describes any object (it is a property which is trivially true for everything)
- an atomic concept is a concept
- if r is a role and b is a concept, then `[ALL` r b`]` is a concept (describing objects all of whose r-successors are described by b)
- if r is a role and b is a concept, then `[EXISTS` r b`]` is a concept (describing objects which have at least one r-successor which is described by b)
- if r is a role and c is a constant, then `[FILLS` r c`]` is a concept (describing objects which have an r-successor denoted by c)
- if $b_1, \ldots, b_n$ are concepts, `[AND` b_1 $\ldots b_n$`]` is a concept (describing objects which are described by all of $b_1, \ldots, b_n$)

and the following definition of a sentence:

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s)
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)

- if c is a constant and b a concept then $c \to b$ is a sentence (the individual denoted by c satisfies the description expressed by b)

1. Define interpretations (D, I) for this description logic and give an inductive definition of the meaning of concepts (extend the interpretation mapping I to complex concepts). Give conditions for the truth of sentences in an interpretation.
2. Given the atomic concepts $Female$, $Male$, roles $Child$, $Sibling$ and constant $alice$, define in the description logic above the following concepts:
   (a) "Mother of Alice" (someone female whose child is Alice)
   (b) "Parent" (someone who has a child) [Hint: use $\top$ to describe the child]
   (c) "Uncle" (someone male who has a sibling who has a child)
3. Using the same atomic concepts as in part (b), translate the following sentences in description logic:
   (a) Every grandparent is a parent
   (b) Alice is a grandmother

new - 6.4.
Recall the description logic DL given in the textbook:

*****************definition of DL******************************

Concepts:

- an atomic concept is a concept
- if r is a role and b is a concept, then `[ALL` r b`]` is a concept (e.g. `[ALL` $:Child$ $Girl$`]` describes someone all of whose children are girls).
- if r is a role and n is a positive integer, then `[EXISTS` n r`]` is a concept (e.g. `[EXISTS 2` $:Child$`]` describes someone who has at least 2 children)
- if r is a role and c is a constant, then `[FILLS` r c`]` is a concept (e.g. `[FILLS` $:Child$ $john$`]` describes someone whose child is John).
- if $b_1, \ldots, b_n$ are concepts, `[AND` b_1 `...`b_n`]` is a concept.

Sentences:

- if b_1 and b_2 are concepts then $b_1 \sqsubseteq b_2$ is a sentence (all b_1s are b_2s)
- if b_1 and b_2 are concepts then $b_1 \doteq b_2$ is a sentence (b_1 is equivalent to b_2)
- if c is a constant and b a concept then $c \to b$ is a sentence (the individual denoted by c satisfies the description expressed by b)

*****************end definition of DL*************************

1. Express the following concepts and sentences in DL using constants $john$, $g51prg$, roles $Module$ and $Supervision$ and atomic concepts $Academic$, $Lecturer$, $Compulsory$:

   C1 concept of an academic who has some project students (supervises the students)

   C2 concept of an academic who teaches at least two modules

   C3 concept of an academic who teaches only compulsory modules

   C4 concept of someone who teaches G51PRG

   S1 a lecturer is an academic who has at least 8 project students and teaches at least 2 modules

   S2 John teaches at least 3 modules and they are all compulsory

2. At the moment the logic does not contain concept negation `NOT`. It also cannot say that there exists some individual connected by a role which is in a concept b (namely, we have `[ALL` r b`]` but no `[EXISTS` r b`]`). If we add concept negation `NOT`, with the obvious meaning that `[NOT` b`]` is a concept containing all individuals which are not in b, explain how we can then define `[EXISTS` r b`]`.

new - 6.5.
Express the following concepts and sentences in DL using the constants, roles and atomic concepts that you find the most useful.

> Computers have at least one input device and one output device, which are input devices and output devices, respectively. Keyboards and mice are different types of input devices. Screens and columns are different types of output devices. C1 is a computer whose keyboard is K1.

What can be inferred about K1?

new - 6.6.
Express the following concepts and sentences in DL using the constants, roles and atomic concepts that you find the most useful.

> There are several types of drinks: water, alcoholic drinks and fruit drinks. Drinks are described by their ingredients, which are edible stuff. Alcoholic drinks are also described by their alcohol contents, which is an integer. W1 is a wine whose alcohol contents is 12. S2 is a fruit drink containing water, pineapple juice and coconut juice.

What can be inferred about S2?

7 Inheritance

Ch 10, Ex 1
Consider the following collection of assertions:

George is a Marine.

George is a chaplain.

A Marine is typically a beer drinker.

A chaplain is typically not a beer drinker.

A beer drinker is typically overweight.

A Marine is typically not overweight.

1. Represent the assertions in an inheritance network.
2. What are the credulous extensions of the network?
3. Which of them are preferred extensions?
4. Give a conclusion that a credulous reasoner might make but that a skeptical reasoner would not.
5. Are there conclusions where a skeptical reasoner and an ideally skeptical reasoner would disagree given this network?

new - 7.2.
Consider the following collection of assertions:

a is a B.

a is a C.

Bs are typically Es.

Cs are typically not Es.

Cs are typically Ds.

Ds are typically Es.

1. Represent the assertions in an inheritance network.
2. What are the credulous extensions of the network?
3. Which of them are preferred extensions?
4. Give a conclusion that a credulous reasoner might make but that a skeptical reasoner would not.

8 Default Reasoning

Ch 11, Ex 1
Although the inheritance networks of Chapter 10 are in a sense much weaker than the other formalisms considered in this chapter for default reasoning, they use default assertions more fully. Consider the following assertions:

> Canadians are typically not francophones.
>
> All Quebecois are Canadians.
>
> Quebecois are typically francophones.
>
> Robert is a Quebecois.

Here is a case where it seems plausible to conclude by default that Robert is a francophone.

1. Represent these assertions in an inheritance network (treating the second one as defeasible), and argue that it unambiguously supports the conclusion that Robert is a francophone.
2. Represent them in first-order logic using two abnormality predicates, one for Canadians and one for Quebecois, and argue that, as it stands, minimizing abnormality would not be sufficient to conclude that Robert is a francophone.
3. Show that minimizing abnormality would work if we add the assertion
   All Quebecois are abnormal Canadians,
   but will not work if we only add
   Quebecois are typically abnormal Canadians.
4. Repeat the exercise in default logic: Represent the assertions as two facts and two normal default rules, and argue that the result has two extensions. Eliminate the ambiguity using a non-normal default rule. You may use a variable-free version of the problem where the letters q, c and f stand for the propositions that Robert is Quebecois, Canadian, and francophone respectively, and where defaults are considered only with respect to Robert.
5. Write a variable-free version of the assertions in autoepistemic logic, and show that the procedure described in the text generates two stable expansions. How can the unwanted expansion be eliminated?

9 Default Reasoning

new - 9.1.
Consider the following knowledge base:

$$
\begin{aligned}
KB = \{&NorthOf(coimbra, faro),\\
&NorthOf(chaves, porto),\\
&NorthOf(coimbra, lisboa),\\
&NorthOf(chaves, coimbra),\\
&\forall x \forall y \forall z[(NorthOf(x, y) \wedge NorthOf(y, z)) \supset NorthOf(x, z)]\}
\end{aligned}
$$

1. Does it hold that $KB \models_{CWA} NorthOf(chaves, faro)$? Explain why.
2. Does it hold that $KB \models_{CWA} \neg NorthOf(chaves, faro)$? Explain why.
3. Does it hold that $KB \models_{CWA} NorthOf(porto, faro)$? Explain why.
4. Does it hold that $KB \models_{CWA} \neg NorthOf(porto, faro)$? Explain why.

new - 9.2.
For the following KB:

$$
\begin{aligned}
KB = \{&SouthOf(milan, paris),\\
&SouthOf(milan, london),\\
&SouthOf(milan, moscow),\\
&paris \neq london,\\
&london \neq moscow,\\
&paris \neq moscow,\\
&\neg WarmerThan(milan, paris) \vee \neg WarmerThan(milan, london),\\
&\forall x[(SouthOf(milan, x) \wedge \neg Ab(x)) \supset WarmerThan(milan, x)]\}
\end{aligned}
$$

state whether the sentence $WarmerThan(milan, moscow)$ is minimally entailed, and explain why.

new - 9.3.
Consider the following knowledge base:

S1 Cats usually don't attack people

S2 Wild cats are cats

S3 Wild cats when threatened attack people

S4 a is a cat

S5 b is a wild cat and is different from a

S6 b is threatened

1. Translate this knowledge base into first-order logic, using the circumscription approach to translate the default rule **S1**. Translate **S2** and **S3** as normal first order implications, which are true without exceptions. Use unary predicates C for cat, W for wild cat, A for attack people, T for being threatened.

2. Does this knowledge base minimally entail $\neg A(a)$ (a does not attack people)?
3. Does this knowledge base minimally entail $\neg A(b)$ (b does not attack people)?
4. Translate this knowledge base into default logic.
5. What can you conclude about a and b using default logic?
6. Translate this knowledge base into autoepistemic logic.
7. What can you conclude about a and b using autoepistemic logic?

10 Probabilistic Reasoning

Ch 12, Ex 2
Consider the following example:

> Metastatic cancer is a possible cause of a brain tumor and is also an explanation for an increased total serum calcium. In turn, either of these could cause a patient to fall into occasional coma. Severe headache could also be explained by a brain tumor.

1. Represent these causal links in a belief network. Let a stand for 'metastatic cancer', b for 'increased total serum calcium', c for 'brain tumor', d for 'occasional coma', and e for 'severe headaches'.
2. Give an example of an independence assumption that is implicit in this network.
3. Suppose the following probabilities are given:

   $Pr(a) = 0.2$
   $Pr(b|a) = 0.8$
   $Pr(b|\neg a) = 0.2$
   $Pr(c|a) = 0.2$
   $Pr(c|\neg a) = 0.05$
   $Pr(e|c) = 0.8$
   $Pr(e|\neg c) = 0.6$
   $Pr(d|b \wedge c) = 0.8$
   $Pr(d|b \wedge \neg c) = 0.8$
   $Pr(d|\neg b \wedge c) = 0.8$
   $Pr(d|\neg b \wedge \neg c) = 0.05$

   and assume that it is also given that some patient is suffering from severe headaches but has not fallen into a coma. Calculate joint probabilities for the eight remaining possibilities (that is, according to whether a, b, and c are true or false).
4. According to the numbers given, the a priori probability that the patient has metastatic cancer is 0.2. Given that the patient is suffering from severe headaches but has not fallen into a coma, are we now more or less inclined to believe that the patient has cancer? Explain.

new - 10.1.
Given the following belief network:

```
        a
       / \
      /   \
     /     \
    V       V
    b       c
```

And the following probabilities:

$Pr(a) = 1/5$

$Pr(b|a) = 2/3$

$Pr(b|\neg a) = 1/6$

$Pr(c|a) = 1/6$

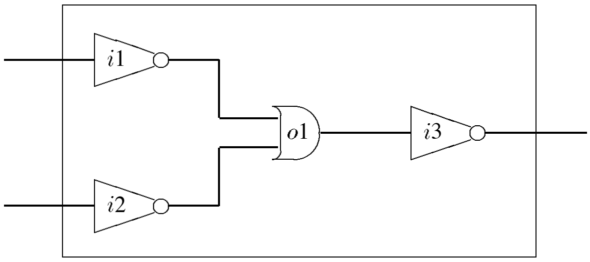
$Pr(c|\neg a) = 2/3$

1. Give an example of an independence assumption that is implicit in this network.
2. What is the probability that a, b and c are all true?
3. What is the probability that a, b and c are all false?
4. What is the probability of $b \wedge c$?
5. Is $b \wedge c$ more probable when a is true or when a is false?

11 Explanation and Diagnosis

Ch 13, Ex 3

Consider the binary circuit for logical AND depicted in this figure, where $i1$, $i2$ and $i3$ are logical inverters, and $o1$ is an OR gate.



1. Write sentences describing this circuit: its components, connectivity, and normal behaviour.
2. Write a sentence for a fault model saying that a faulty inverter has its output the same as its input.
3. Assuming the above fault model and that the output is 1 given inputs of 0 and 1, generate the three abductive explanations for this behaviour.
4. Generate the three consistency-based diagnoses for this circuit under the same conditions.
5. Compare the abductive and consistency-based diagnoses and explain informally why they are different.

12 Actions and Plans

Ch 14, Ex 1
In the exercises below, and in the follow-up exercises of Chapter 15, we consider three application domains where we would like to be able to reason about action and change:

Pots of water: Consider a world with pots that may contain water. There is a single fluent $Contains$, where $Contains(p, w, s)$ is intended to say that a pot p contains w litres of water in situation s. There are only two possible actions, which can always be executed: $empty(p)$ which discards all the water contained in the pot p, and $transfer(p, p')$, which pours as much water as possible without spilling from pot p to p', with no change when $p = p'$. To simplify the formalization, we assume that the usual arithmetic constants, functions and predicates are also available. (You may assume that axioms for these have already been provided or built-in.)

1. Write the precondition axioms for the actions.
2. Write the effect axioms for the actions.
3. Show how successor state axioms for the fluents would be derived from these effect axioms. Argue that the successor state axioms are not logically entailed by the effect axioms, by briefly describing an interpretation where the effect axioms are satisfied but the successor state ones are not.
4. Show how frame axioms are logically entailed by the successor state axioms.

Ch 15, Ex 1
These exercises are continuations of the exercises from Chapter 14. For each application, we consider a planning problem involving an initial setup and a goal.

Pots of water: Imagine that in the initial situation, we have two pots, a 5-litre one filled with water, and an empty 2-litre one. Our goal is to obtain 1 litre of water in the 2-litre pot.

1. Write a sentence of the situation calculus of the form $\exists x.\alpha$ which asserts the existence of the final goal situation.
2. Write a ground situation term e (that is, a term that is either $S0$ or of the form $do(a, e')$ where a is a ground action term and e' is itself a ground situation term) such that e denotes the desired goal situation.
3. Explain how you could use Resolution to automatically solve the problem for any initial state: how would you generate the clauses, and assuming the process stops, how would you extract the necessary moves? Explain why you need to use the successor state axioms, and not just effect axioms.
4. Suppose we were interested in formalizing the problem using a STRIPS representation. Decide what the operators should be, and then write the precondition, add list, and delete list for each operator. You may change the language as necessary.

5. Consider the database corresponding to the initial state of the problem. For each STRIPS operator, and each binding of its variables such that the precondition is satisfied, state what the database progressed through this operator would be.

6. Consider the final goal state of the problem. For each STRIPS operator, describe the bindings of its variables for which the operator can be the final action of a plan, and in those cases, what the goal regressed through the operator would be.