Técnico Lisboa
University of Lisbon

Quantum-Like Probabilistic Graphical Models

for Cognition and Decision

by Catarina Moreira

(Thesis Advisory Committee Report)

FCT
Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

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Motivation

Quantum physics uses **complex numbers** to represent probabilities.

Can **quantum probabilities** be used in areas outside of physics, such as **Decision-Making**?

What is the **impact** of a **Decision System** that makes use of **complex numbers** to deal with **uncertainty**?

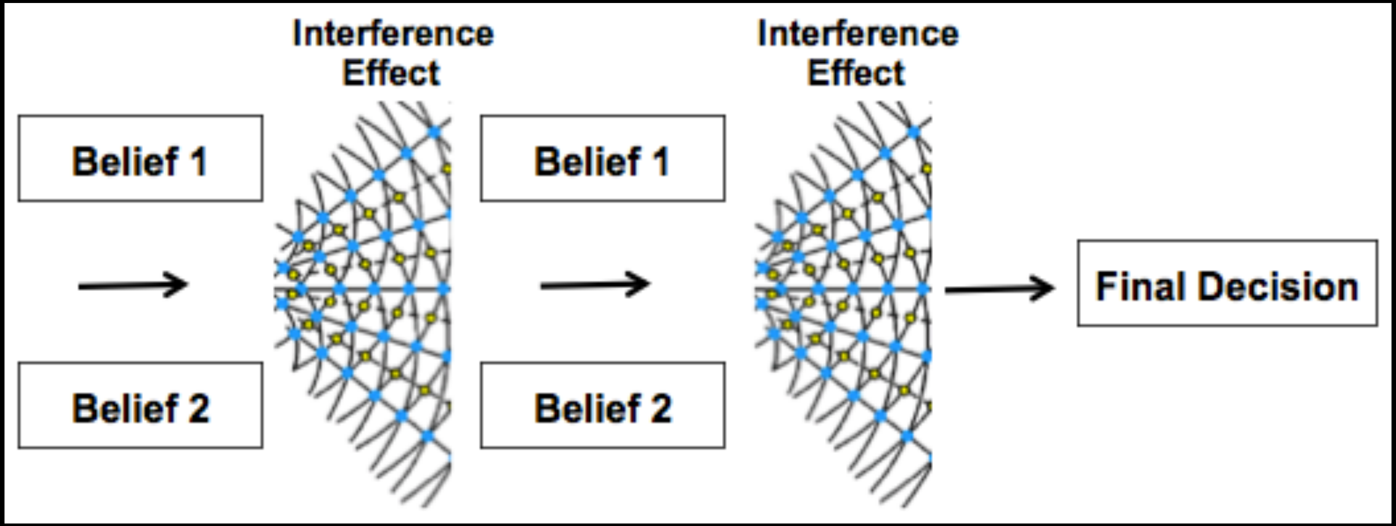
Quantum Cognition

- Research field that aims to build **cognitive models** using the mathematical principles of **quantum mechanics**.
- Mainly used to explain paradoxical empirical findings that **violate classical laws of probability theory and logic.**

C. Moreira and A. Wichert, The Relation Between Acausality and Interference in Quantum-Like Bayesian Networks, In Proceedings of the 9th International Conference on Quantum Interactions, 2015

Quantum Cognition

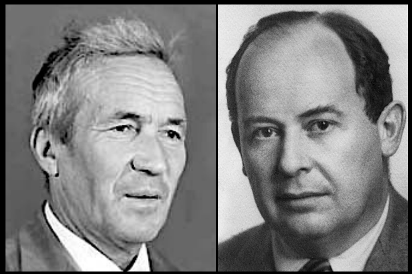
Quantum probability and interference effects play an important role in explaining several inconsistencies in decision-making.



Moreira & Wichert (2014), Interference Effects in Quantum Belief Networks, Applied Soft Computing, 25, 64-85

State of the Art

Kolmogorov / von Neumann



- **Foundations of:**
  Classical Probability (1933)
  Quantum Probability (1932)

Moreira & Wichert (2016), Quantum Probabilistic Models Revisited, Frontiers in Physics, (in press)

State of the Art

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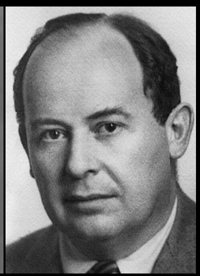

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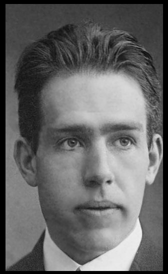

Quantum principles could be applied in other domains (1933-1967)

Niels Bohr

Moreira & Wichert (2016), Quantum Probabilistic Models Revisited, Frontiers in Physics, (in press)

State of the Art

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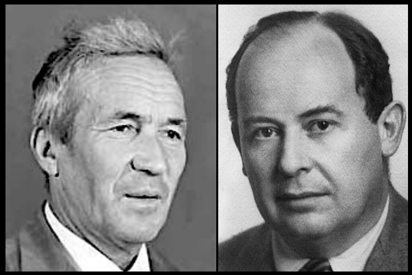

Sure Thing Principle (1954)
Maurice Savage

Expected Utility Theorem (1947)
von Neumann-Morgenstern

Foundations of:
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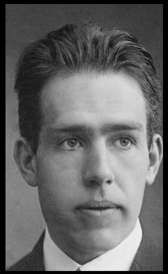

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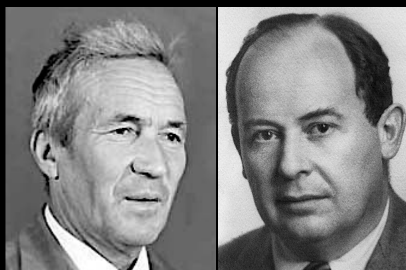

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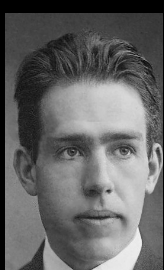

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Violations of Expected Utility (1974 – 1992)

Khaneman
Tversky
Shafir

Moreira & Wichert (2016), Quantum Probabilistic Models Revisited, Frontiers in Physics, (in press)

State of the Art

Violations to the Utility Theorem (1974 – 1992)

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State of the Art

D. Aerts

ε-Model (1994)

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Moreira & Wichert (2016), Quantum Probabilistic Models Revisited, Frontiers in Physics, (in press)

State of the Art

Quantum Prospect Decision Theory (2010)

V. Yukalov & D. Sornett

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State of the Art

Model	State representation	Quantum interference	Predictive	Comments
Quantum dynamical model	Superposition of subject's beliefs	Shröedinger's equation	No . requires manual fit	Enables time evolution
Quantum-Like approach	Contextual probabilities (observables/random variables)	Measure of supplementarity	No. requires manual fit	Can deal with hyperbolic spaces
Quantum prospect decision theory	Contextual probabilities (prospects/random variables)	Interference quarter law	Yes. Uses a static heuristic	It is predictive uses a heuristic
Quantum-Like Bayesian networks	Contextual probabilities (observables/random variables)	None	No. manual fit	Can easily scale to more complex scenarios

Moreira & Wichert (2016), Quantum Probabilistic Models Revisited, Frontiers in Physics, (in press)

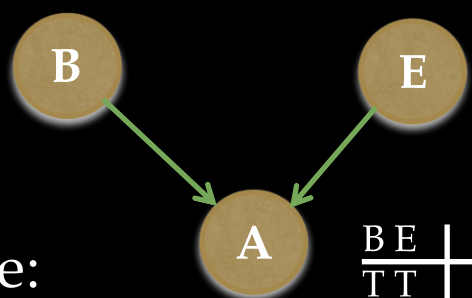
Research Question

Can we represent a network-based decision support system based on quantum probabilities?

Bayesian Networks

Directed acyclic graph structure in which each **node** represents a random variable and each **edge** represents a direct influence from source node to the target node.

Pr(B = T) = 0.001
Pr(B = F) = 0.999



Pr(E = T) = 0.002
Pr(E = F) = 0.998

B E	Pr(A=T | B,E)	Pr(A=F | B,E)
T T	0.95	0.05
T F	0.94	0.06
F T	0.29	0.71
F F	0.01	0.99

Bayesian Networks have:

- ⋆ Evidence variables (observed nodes)
- ⋆ Not observed nodes

Inferences in Bayesian Networks

Inference is performed in **two** steps:

1. Computation of the networks **full joint probability**;
2. Computation of the **marginal probability**;

Full joint probability for Bayesian Networks:

$$Pr(X_1, \ldots, X_n) = \prod_{i=1}^{n} Pr(X_i | Parents(X_i))$$

Marginal probability for Bayesian Networks:

$$Pr_c(X|e) = \alpha Pr_c(X,e) = \alpha \left[\sum_{y \in Y} Pr_c(X,e,y) \right]$$

Where $\alpha = \frac{1}{\sum_{x \in X} Pr_c(X = x, e)}$

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Bayes Assumption

Research Question

How can we move from a classical Bayesian network to a quantum paradigm?

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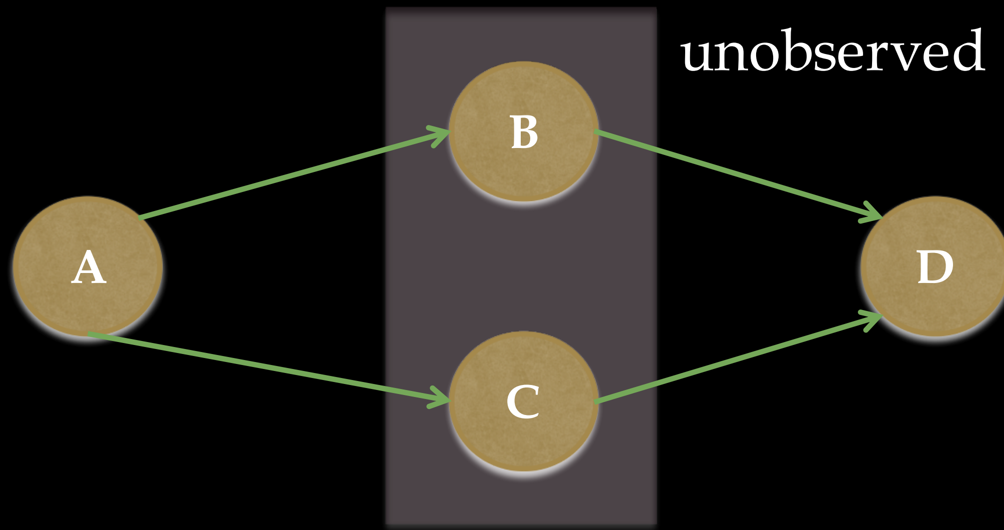
Feynman's Path Diagram Rules

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Feynman's Path Diagram Rules



$$Pr(A \to D) = |\psi_A \cdot \psi_{B|A} \cdot \psi_{D|B} + \psi_A \cdot \psi_{C|A} \cdot \psi_{D|C}|^2 =$$

$$= |\psi_A \cdot \psi_{B|A} \cdot \psi_{D|B}|^2 + |\psi_A \cdot \psi_{C|A} \cdot \psi_{D|C}|^2 + Interference$$

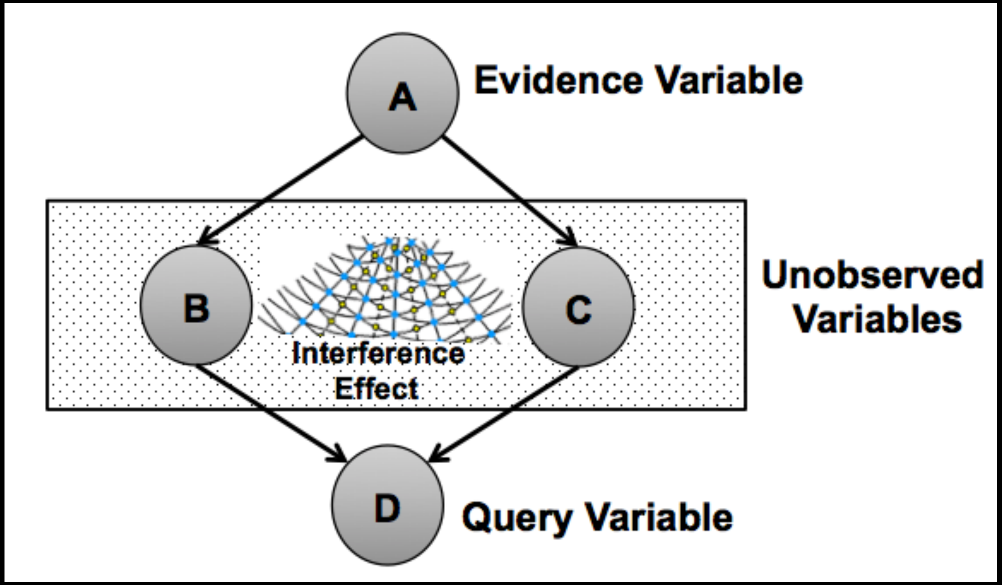
$$Interference = 2 \cdot |\psi_A \cdot \psi_{B|A} \cdot \psi_{D|B}| \cdot |\psi_A \cdot \psi_{C|A} \cdot \psi_{D|C}| \cos(\theta_1 - \theta_2)$$

Busemeyer & Bruza(2012), Quantum Models for Decision and Cognition, Cambridge University Press

Quantum-Like Bayesian Networks

- Under unknown events, the quantum-like Bayesian Networks can use interference effects.

- Under known events, no interference is used, since there is no uncertainty.



Moreira & Wichert (2014), Interference Effects in Quantum Belief Networks, Applied Soft Computing, 25, 64-85

Quantum-Like Bayesian Networks

- Convert classical probabilities into quantum amplitudes through Born's rule: squared magnitude quantum amplitudes.

- Classical full joint probability distribution

$$Pr(X_1, \ldots, X_n) = \prod_{i=1}^{n} Pr(X_i | Parents(X_i))$$

- Quantum full joint probability distribution

$$Pr_q(X_1, \ldots, X_n) = \left| \prod_{i=1}^{n} QPr(X_i | Parents(X_i)) \right|^2$$

Moreira & Wichert (2014), Interference Effects in Quantum Belief Networks, Applied Soft Computing, 25, 64-85

Quantum-Like Bayesian Networks

- Convert classical probabilities into quantum amplitudes through Born's rule: squared magnitude quantum amplitudes.

- Classical marginal probability distribution

$$Pr_c(X|e) = \alpha Pr_c(X,e) = \alpha \left[\sum_{y \in Y} Pr_c(X,e,y) \right]$$

- Quantum marginal probability distribution

$$Pr_q(X|e) = \gamma \left| \sum_{y} \prod_{x=1}^{N} QPr(X_x | Parents(X_x), e, y) \right|^2$$

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum-Like Bayesian Networks

* Quantum marginal probability distribution
* Extension of the Quantum-Like approach (Khrennikov, 2009) for N Random Variables

$$Pr_q(X|e) = \gamma \sum_{i=1}^{|Y|} \left| \prod_{x}^{N} QPr(X_x | Parents(X_x), e, y = i) \right|^2 + 2 \cdot Interference$$

$$Interference = \sum_{i=1}^{|Y|-1} \sum_{j=i+1}^{|Y|} \left| \prod_{x}^{N} QPr(X_x | Parents(X_x), e, y = i) \right| \left| \prod_{x}^{N} QPr(X_x | Parents(X_x), e, y = j) \right| \cos(\theta_i - \theta_j)$$

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum-Like Bayesian Networks

* Quantum marginal probability distribution

* Extension of the Quantum-Like approach (Khrennikov, 2009) for N Random Variables

CLASSICAL PROBABILITY

$$Pr_q(X|e) = \gamma \sum_{i=1}^{|Y|} \left| \prod_{x}^{N} QPr(X_x|Parents(X_x), e, y=i) \right|^2 + 2 \cdot Interference$$

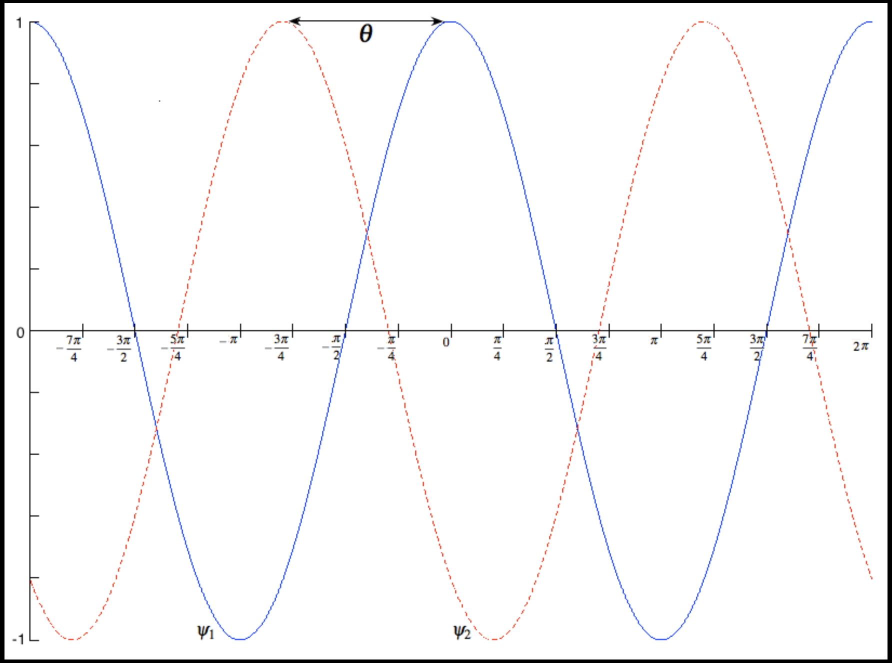
QUANTUM INTERFERENCE

$$Interference = \sum_{i=1}^{|Y|-1} \sum_{j=i+1}^{|Y|} \left| \prod_{x}^{N} QPr(X_x|Parents(X_x), e, y=i) \right| \left| \prod_{x}^{N} QPr(X_x|Parents(X_x), e, y=j) \right| \cos(\theta_i - \theta_j)$$

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum Parameters

In Quantum Mechanics, the quantum parameter θ represents the shift of energy waves.



Moreira & Wichert (2015), The Synchronicity Principle Under Quantum Probabilistic Inferences, NeuroQuantology, 13, 111-133

Quantum Parameters

In Quantum Cognition, the quantum parameter θ represents the inner product between two random variables

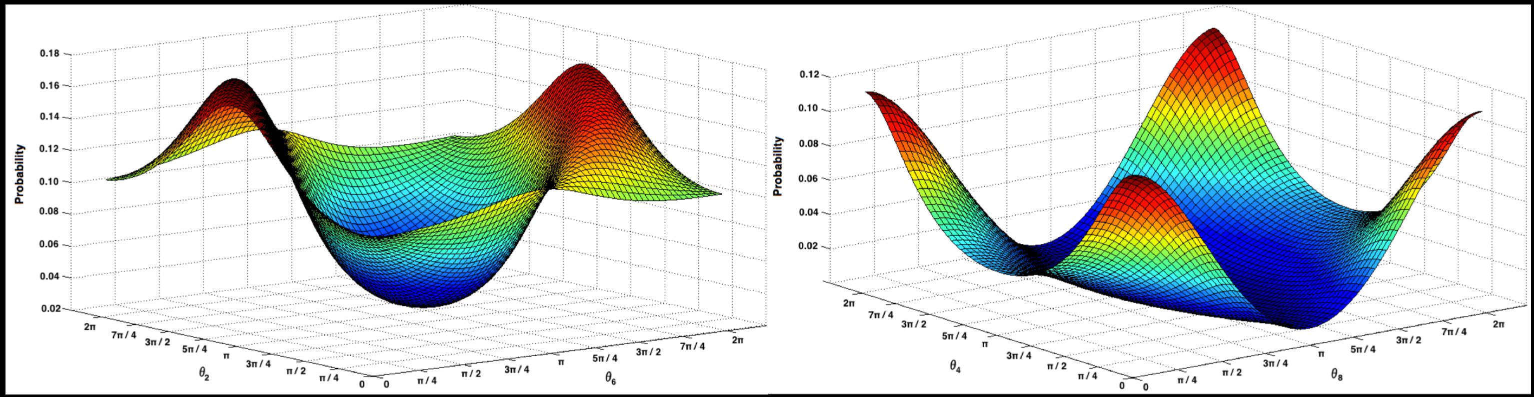
Busemeyer & Bruza(2012), Quantum Models for Decision and Cognition, Cambridge University Press

Problem: Quantum Parameters

- The number of parameters grows exponentially large!

$$Interference = \sum_{i=1}^{|Y|-1} \sum_{j=i+1}^{|Y|} \left| \prod_{x}^{N} QPr(X_x | Parents(X_x), e, y=i) \right| \cdot \left| \prod_{x}^{N} QPr(X_x | Parents(X_x), e, y=j) \right| \cdot \cos(\theta_i - \theta_j)$$

- The final probabilities can be **ANYTHING** in some range of probabilities!



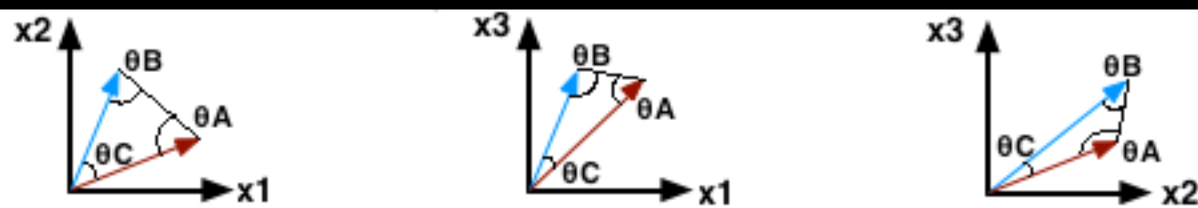
Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Similarity Heuristic

* The interference term is given as a sum of pairs of random variables.

$$interference = 2 \cdot \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} |\psi_i| \cdot |\psi_j| \cdot \cos\left(\theta_i - \theta_j\right)$$

* Heuristic: parameters are calculated by computing different vector representations for each pair of random variables.

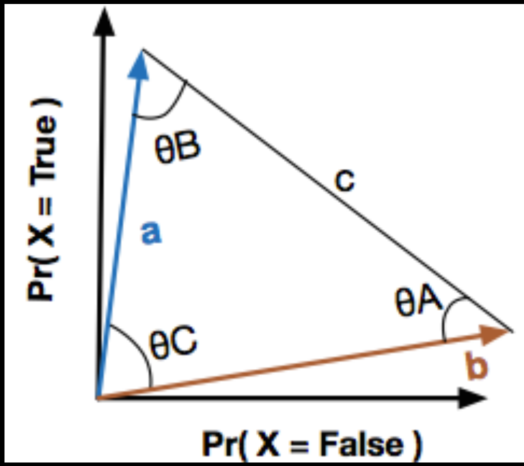


$$Pr(B) = \alpha \left[\sum_{i=1}^{N} |\psi_i|^2 + 2 \cdot |\psi_1| \cdot |\psi_2| \cdot \cos\left(\theta_1 - \theta_2\right) + 2 \cdot |\psi_1| \cdot |\psi_3| \cdot \cos\left(\theta_1 - \theta_3\right) + 2 \cdot |\psi_2| \cdot |\psi_3| \cdot \cos\left(\theta_2 - \theta_3\right) \right]$$

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Similarity Heuristic

* **What do Bayesian Networks represent?**
  * Correlations / Causal Relationships
* **What is similarity?**
  * Real-valued function that quantifies the degree of similarity between two vectors.
* **What do vectors represent?**
  * Relations between Random Variables
  * Semantical information
* **How do we measure similarity?**
  * Cosine Similarity
  * Combining different sources of information

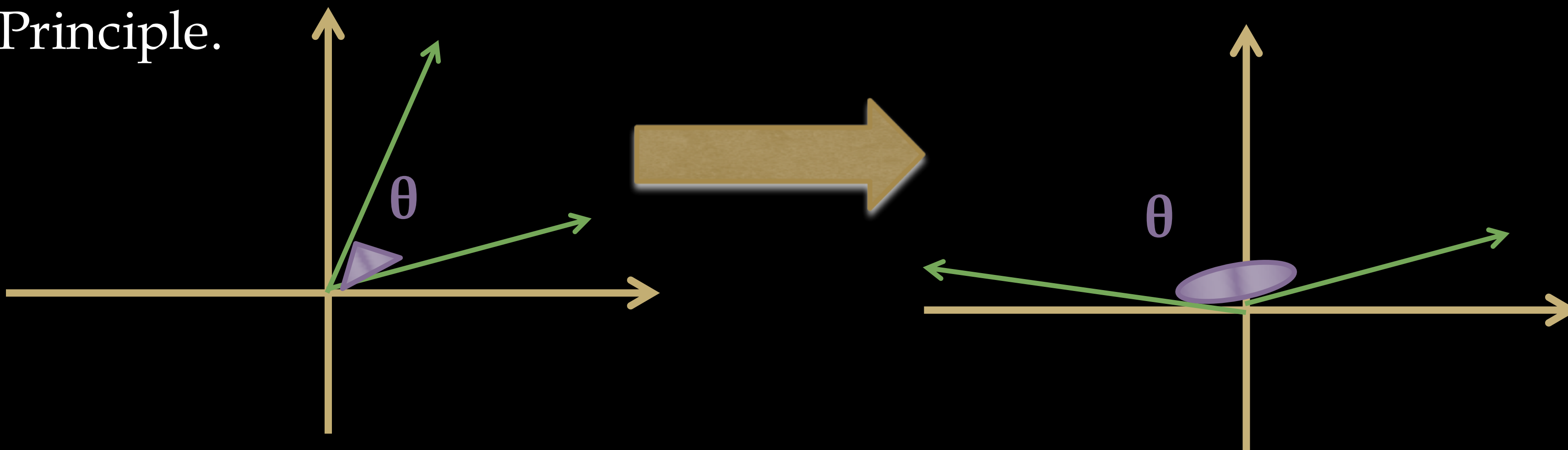


Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Similarity Heuristic

The vector representation of the random variables will always be positive.

We need to separate these vectors in order to obtain an interference term that can explain violations to the Sure Thing Principle.



Configuration extracted From Random Variables

Desired Configuration For Predictions

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Similarity Heuristic

* Vectors that are very close to each other are separated by setting their inner angle to π (minimum cosine value).
* When vectors are already separated, we just penalize a little the angle that they share.

$$h\left(a,\ b\right) = \begin{cases} \pi & \text{if } \phi < 0 \\ \pi - \theta_C/2 & \text{if } \phi > 0.2 \\ \pi - \theta_C & \text{otherwise} \end{cases}$$

$$\phi = \frac{\theta_C}{\theta_A} - \frac{\theta_B}{\theta_A}$$

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Validation

We validated the proposed Quantum-Like Bayesian Network and heuristic in several experiments that show **violations** to the **Sure Thing Principle**:

- the Prisoner's Dilemma Game;
- the Two Stage Gambling Game;

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The Prisoner's Dilemma Game

Two players who are in separate rooms with no means of speaking to the other. Each player is offered an agreement: they have the opportunity either to betray the other (**Defect**) or to **Cooperate** with the other by remaining silent.

.

Three conditions were verified:

- Player was **informed** that the other chose to **Defect;**
- Player was **informed** that the other chose to **Cooperate;**
- **Player was not informed of the other player's actions**;

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

The Prisoner's Dilemma Game

Several experiments in the literature show violations of the Sure Thing Principle under the Prisoner's Dilemma Game.

Literature	Known to Defect	Known to Collaborate	Unknown	Classical probability
Shafir and Tversky, 1992	0.9700	0.8400	0.6300	0.9050
Croson, 1999[a]	0.6700	0.3200	0.3000	0.4950
Li and Taplin, 2002[b]	0.8200	0.7700	0.7200	0.7950
Busemeyer et al., 2006a	0.9100	0.8400	0.6600	0.8750
Hristova and Grinberg, 2008	0.9700	0.9300	0.8800	0.9500
Average	0.8700	0.7400	0.6400	0.8050

$$Pr(P_2 = Defect \mid P_1 = Defect) \geq Pr(P_2 = Defect) \geq Pr(P_2 = Defect \mid P_1 = Cooperate)$$

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum-Like Bayesian Network Predictions

We applied the proposed heuristic and tried to predict the probability of the player choosing to *Defect* under the Prisoner's Dilemma Game.

$\psi_{P1 = defect} = \sqrt{0.5}e^{i\theta 1}$

$\psi_{P1 = cooperate} = \sqrt{0.5}e^{i\theta 2}$

P1	ψ P2 = defect	ψ P2 = cooperate
defect	$\sqrt{0.87}e^{i\theta 3}$	$\sqrt{0.13}e^{i\theta 4}$
cooperate	$\sqrt{0.74}e^{i\theta 5}$	$\sqrt{0.26}e^{i\theta 6}$

FIGURE 4 | Bayesian Network representation of the Average of the results reported in the literature. The random variables, which were considered, are P1 and P2, corresponding to the actions chosen by the first participant and second participant, respectively.

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum-Like Bayesian Network Predictions

We also applied the proposed model to predict the results obtained for the Prisoner's Dilemma game for several works in the literature.

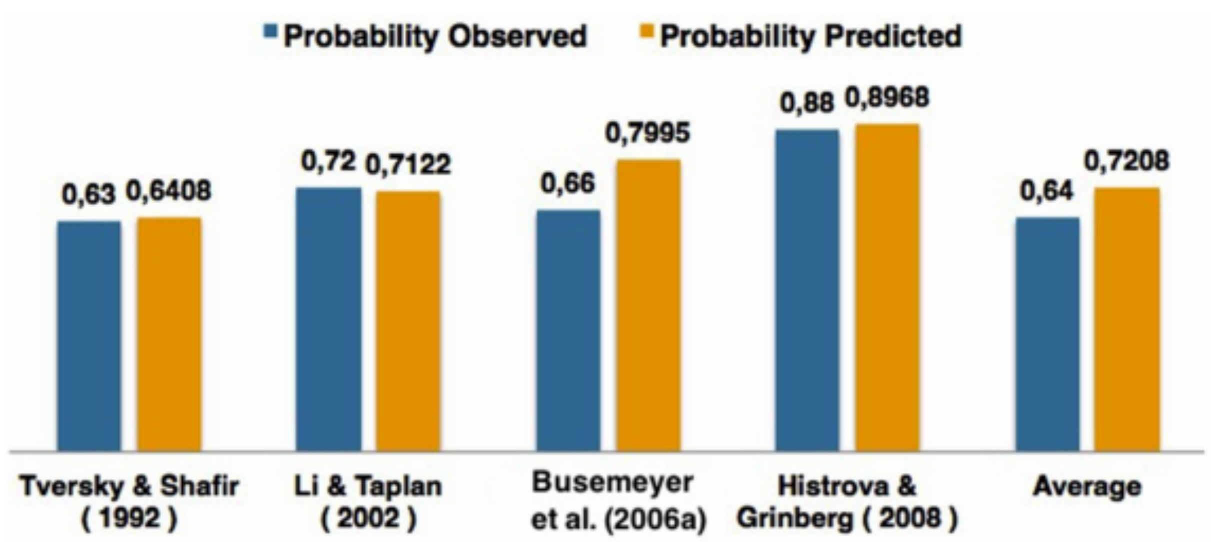

FIGURE 6 | Comparison of the results obtained for different works of the literature concerned with the Prisoner's Dilemma game.

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum-Like Bayesian Network Predictions

* We also applied the proposed model to predict the results obtained for the Prisoner's Dilemma game for several works in the literature.

Overall error percentage: 5.11 %

FIGURE 6 | Comparison of the results obtained for different works of the literature concerned with the Prisoner's Dilemma game.

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

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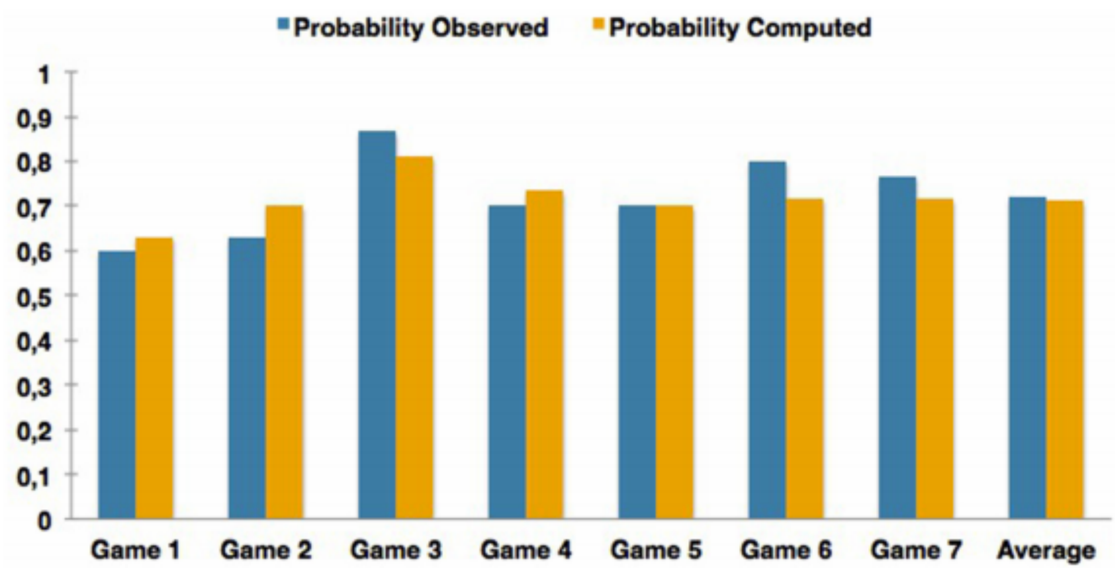


FIGURE 8 | Comparison of the results obtained for different experiments reported in the work of Li and Taplin (2002) in the context of the Prisoner's Dilemma game.

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum-Like Bayesian Network Predictions

- We applied the proposed model to predict the results obtained for the Prisoner's Dilemma game for several works in the literature.

Overall error percentage: 5.74%

FIGURE 8 | Comparison of the results obtained for different experiments reported in the work of Li and Taplin (2002) in the context of the Prisoner's Dilemma game.

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

The Two Stage Gambling

Participants were asked to **play a gambling game** that has an equal chance of **winning** $200 or **losing** $100. **Three** conditions were verified:

- **Informed** that they **won** the 1st gamble;
- **Informed** that they **lost** the 1st gamble;
- **Did not know** if they won or lost the 1st gamble;

The Two Stage Gambling

- Several experiments in the literature show violations of the Sure Thing Principle under the Two Stage Gambling game.

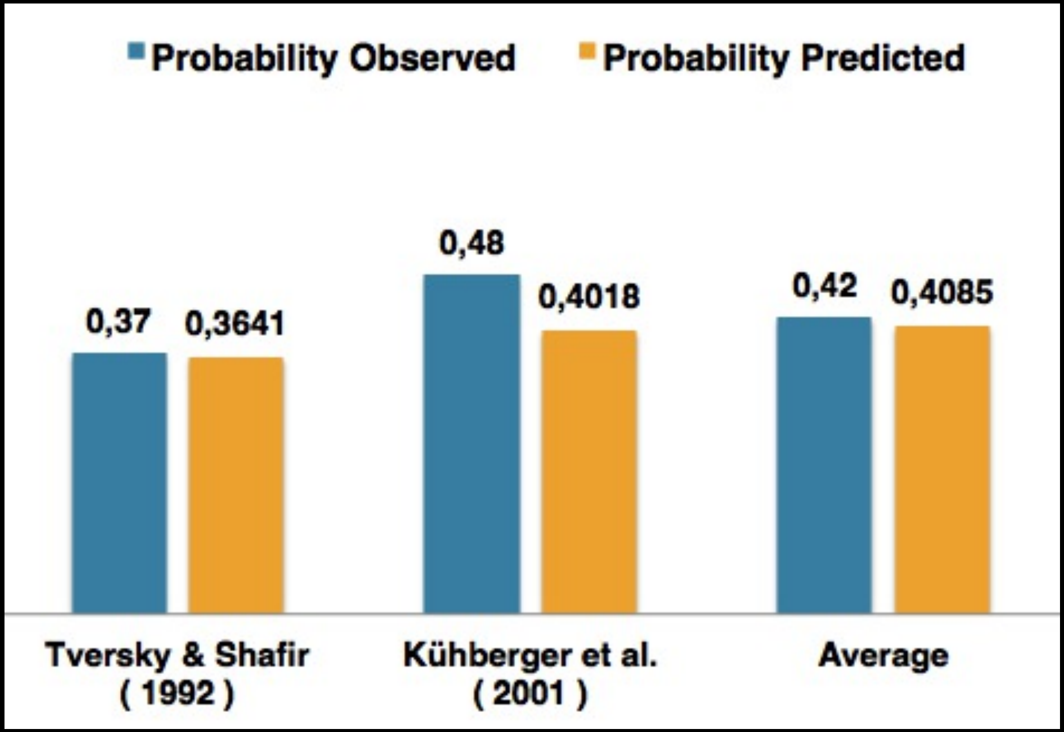
Literature	Known to Win	Known to Loose	Unknown	Classical Probability
Tversky and Shafir (1992)	0.69	0.58	0.37	0.6350
Kuhberger et al. (2001)	0.72	0.47	0.48	0.5950
Lambdin and Burdsal (2007)	0.63	0.45	0.41	0.5400
Average	0.68	0.50	0.42	0.5900

Table 7: Experimental results of the Two-Stage Gambling game reported in different works of the literature.

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum-Like Bayesian Network Predictions

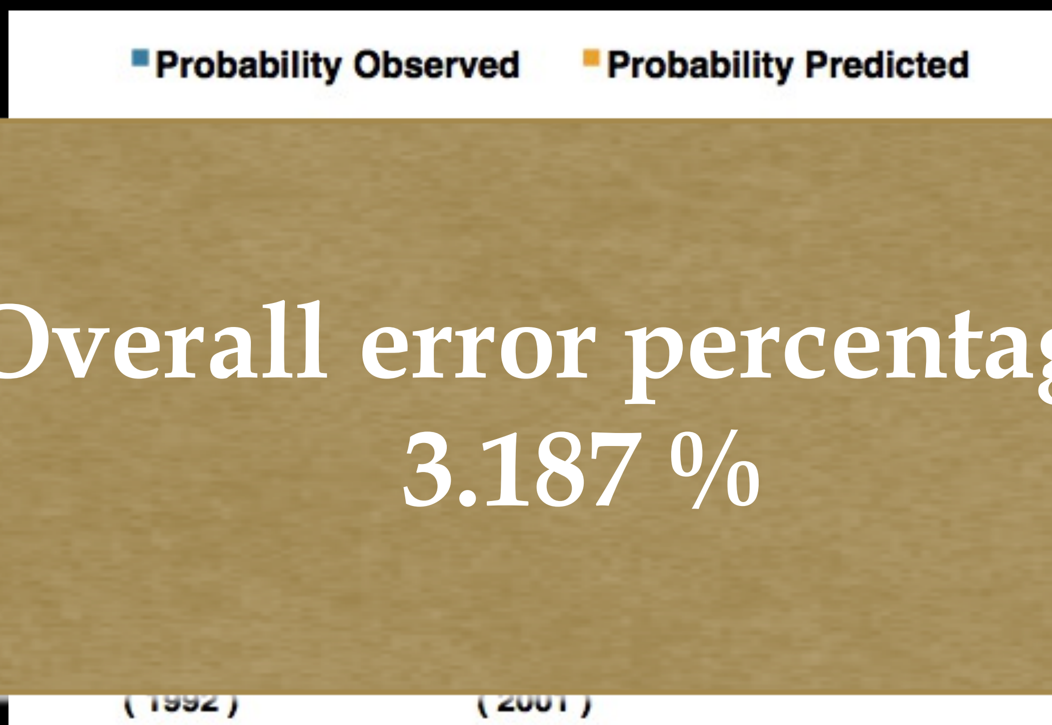
* We applied the proposed heuristic and tried to predict the probability of the player choosing to *play* the 2nd gamble.



Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Quantum-Like Bayesian Network Predictions

- We applied the proposed heuristic and tried to predict the probability of the player choosing to *play* the 2^{nd} gamble.



Overall error percentage:
3.187 %

Moreira & Wichert (2016), Quantum-Like Bayesian Networks for Modelling Decision Making, Frontiers in Psychology: Cognition, 7, 1-20

Conclusions

* Literature overview and comparison of the different models proposed in the literature;
* Applied the mathematical formalisms of quantum theory and Feynman's Path Diagrams to develop a Quantum-Like Bayesian Network;
* Developed an heuristic based on vector similarities in order to automatically tune quantum parameters;
* The proposed heuristic managed to make predictions for several experiments of the literature;

List of Publications

* **Journal Publications:**
  1. **Catarina Moreira** and Andreas Wichert, **Interference Effects in Quantum Belief Networks**, Journal of Applied Soft Computing, 25, 64-85, 2014
  2. **Catarina Moreira** and Andreas Wichert, **The Synchronicity Principle Under Quantum Probabilistic Inferences**, NeuroQuantology, 13,111-133, 2015
  3. **Catarina Moreira** and Andreas Wichert, **Quantum-Like Bayesian Networks for Modeling Decision Making**, Frontiers in Psychology: Cognition, 7, 2016
  4. **Catarina Moreira** and Andreas Wichert, **Quantum Probabilistic Models Revisited: the Case of Disjunction Effects in Cognition**, Frontiers in Physics: Interdisciplinary Physics, in press, 2016.

List of Publications

* **Conference Publications:**
  1. **Catarina Moreira** and Andreas Wichert, **Application of Quantum-Like Bayesian Networks in Social Sciences**, In Proceedings of the 4th **Champalimaud** NeuroScience Symposium, Champalimaud Center of the Unknown, 2015 **(poster presentation)**
  2. **Catarina Moreira** and Andreas Wichert, **The Relation Between Acausality and Interference in Quantum-Like Bayesian Networks**, In Proceedings of the 9th International Conference on Quantum Interactions, 2015
  3. **Catarina Moreira** and Andreas Wichert, **Quantum-Like Bayesian Networks using Feynman's Path Diagram Rules**, In Proceedings of the 16th Växjö Conference on Quantum Theory: from Foundations to Technologies, 2015 **(extended abstract)**
  4. **Catarina Moreira** and Andreas Wichert, **When to use Quantum Probabilities in Quantum Cognition: a Discussion**, In Proceedings of the 13h Biennual Meeting of the International Quantum Structure Association, 2016 **(extended abstract)**