The Relation Between Acausality and Interference in Quantum-Like Bayesian Networks

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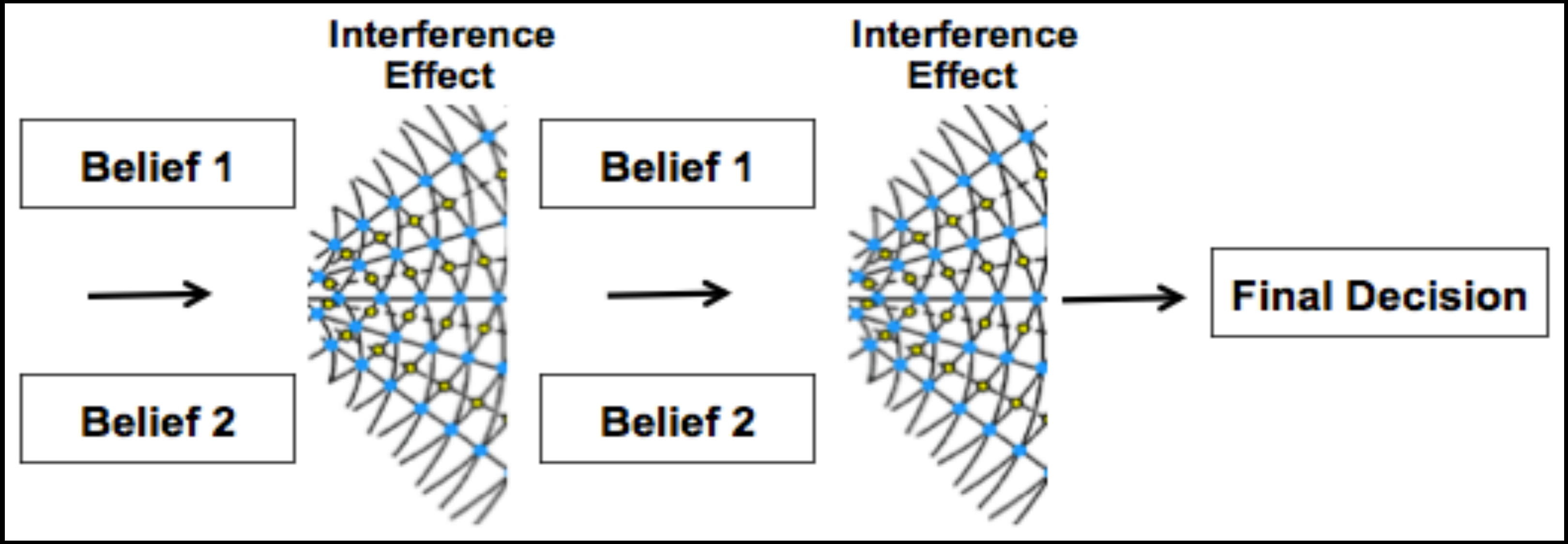
Motivation

Quantum probability and interference effects play an important role in explaining several inconsistencies in decision-making.



Moreira & Wichert (2014), Interference Effects in Quantum Belief Networks, Applied Soft Computing, 25, 64-85

Motivation

Current models of the literature have the following problems:

1. Require a manual parameter tuning to perform predictions;
2. Hard to scale for more complex decision scenarios;

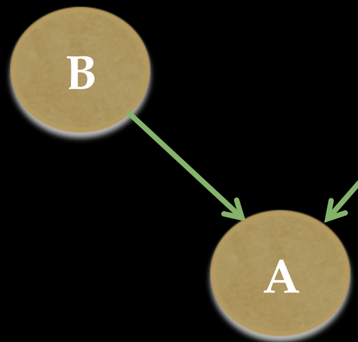
Research Question

Can we build a general quantum probabilistic model to make predictions in scenarios with high levels of uncertainty?

Bayesian Networks

Directed acyclic graph structure in which each **node** represents a random variable and each **edge** represents a direct influence from source node to the target node.

Pr(B = T) = 0.001
Pr(B = F) = 0.999



Pr(E = T) = 0.002
Pr(E = F) = 0.998

B E	Pr(A=T|B,E)	Pr(A=F|B,E)
T T	0.95	0.05
T F	0.94	0.06
F T	0.29	0.71
F F	0.01	0.99

Bayesian Networks

Inference is performed in two steps:

1. Computation of the **Full Joint Probability Distribution**
2. Computation of the **Marginal Probability**

Full Joint Probability Distribution:

$$Pr(X_1, \ldots, X_n) = \prod_{i=1}^{n} Pr(X_i | Parents(X_i))$$

Marginal Probability:

$$Pr_c(X|e) = \alpha Pr_c(X,e) = \alpha \left[\sum_{y \in Y} Pr_c(X,e,y) \right] \quad \text{Where} \quad \alpha = \frac{1}{\sum_{x \in X} Pr_c(X = x, e)}$$

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Bayes Assumption

Marginal Probability:

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Where $\alpha = \frac{1}{\sum_{x \in X} Pr_c(X = x, e)}$

Inference in Bayesian Networks

1. Compute the Full Joint Probability Distribution

$$Pr(X_1, \ldots, X_n) = \prod_{i=1}^{n} Pr(X_i | Parents(X_i))$$

B E A	Pr(A, B, E)
T T T	0.001 x 0.002 x 0.95 = 0.00000190
T T F	0.001 x 0.002 x 0.05 = 0.00000010
T F T	0.001 x 0.998 x 0.94 = 0.00093812
T F F	0.001 x 0.998 x 0.06 = 0.00005988
F T T	0.999 x 0.002 x 0.29 = 0.00057942
F T F	0.999 x 0.002 x 0.71 = 0.00141858
F F T	0.999 x 0.998 x 0.01 = 0.00997002
F F F	0.999 x 0.998 x 0.99 = 0.98703198

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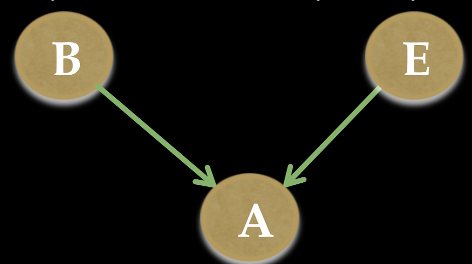

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Inference in Bayesian Networks

2. Compute Marginal Probability

$$Pr_c(X|e) = \alpha Pr_c(X,e) = \alpha \left[\sum_{y \in Y} Pr_c(X,e,y) \right] \quad \text{Where} \quad \alpha = \frac{1}{\sum_{x \in X} Pr_c(X=x,e)}$$

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$$Pr(B, A=T, E=T) = \alpha \sum_{b \in B} Pr(B=b, A=T, E=T)$$

$$Pr(B, A=F, E=T) = \alpha \sum_{b \in B} Pr(B=b, A=F, E=T)$$

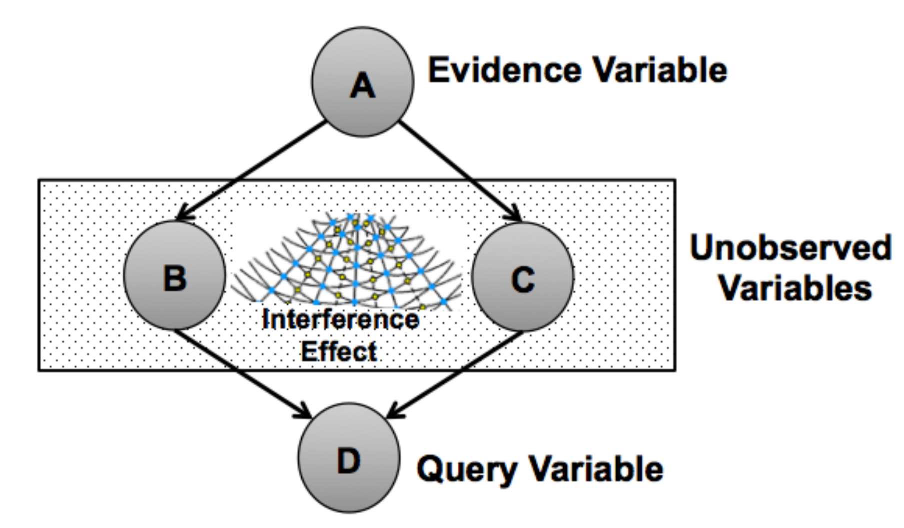
Research Question

How can we move from a **classical** Bayesian Network to a **Quantum-Like** paradigm?

Quantum-Like Bayesian Networks

General idea:

- Under **unobserved** events, the Quantum-Like Bayesian Network can use interference effects;
- Under **known** events, no interference is used, since there is no uncertainty.



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Interference Effects

Convert classical probabilities are converted into quantum amplitudes through Born's rule: squared magnitude quantum amplitudes.

For two dichotomous random variables:

- Classical Law of Total Probability:

$$Pr(B=t) = Pr(A=t) \cdot Pr(B=t|A=t) + Pr(A=f) \cdot Pr(B=t|A=f)$$

- Quantum Law of Total Probability:

$$Pr(B=t) = \left| \sum_{a \in A} \psi_{A=a} \psi_{B=t|A=a} \right|^2$$

Interference Effects

Quantum Law of Total Probability:

$$Pr(B=t) = \left| \sum_{a \in A} \psi_{A=a} \psi_{B=t|A=a} \right|^2$$

If we expand this term we obtain:

$$Pr(B=t) = \left| \psi_{A=t} \psi_{B=t|A=t} \right|^2 + \left| \psi_{A=f} \psi_{B=t|A=f} \right|^2 +$$

$$+ 2 \prod_{a \in A} \left| \psi_{A=a} \psi_{B=t|A=a} \right| \cos\left(\theta_A - \theta_B\right)$$

Interference Effects

Quantum Law of Total Probability for 2 random variables:

$$Pr(B=t) = \left| \sum_{a \in A} \psi_{A=a} \psi_{B=t|A=a} \right|^2$$

If we expand this term we obtain: **Classical Probability**

$$Pr(B=t) = \left| \psi_{A=t} \psi_{B=t|A=t} \right|^2 + \left| \psi_{A=f} \psi_{B=t|A=f} \right|^2 +$$

$$+ 2 \prod_{a \in A} \left| \psi_{A=a} \psi_{B=t|A=a} \right| \cos\left(\theta_A - \theta_B\right)$$

Quantum Interference

Quantum-Like Bayesian Networks

Convert classical probabilities are converted into quantum amplitudes through Born's rule: squared magnitude quantum amplitudes.

- Classical Full Joint Probability Distribution:

$$Pr(X_1, \ldots, X_n) = \prod_{i=1}^{n} Pr(X_i | Parents(X_i))$$

- Quantum Full Joint Probability Distribution:

$$Pr_q(X_1, \ldots, X_n) = \left| \prod_{i=1}^{n} QPr(X_i | Parents(X_i)) \right|^2$$

Quantum-Like Bayesian Networks

Convert classical probabilities are converted into quantum amplitudes through Born's rule: squared magnitude quantum amplitudes.

- Classical Marginal Probability Distribution:

$$Pr_c(X|e) = \alpha Pr_c(X,e) = \alpha \left[\sum_{y \in Y} Pr_c(X,e,y) \right]$$

- Quantum Marginal Probability Distribution:

$$Pr_q(X|e) = \gamma \left| \sum_{y} \prod_{x=1}^{N} QPr\left(X_x | Parents\left(X_x\right), e, y\right) \right|^2$$

Quantum-Like Bayesian Networks

- Quantum marginal probability;
- Extension of the Quantum-Like Approach (Khrennikov, 2009) for N random variables;

$$Pr_q(X|e) = \gamma \sum_{i=1}^{|Y|} \left| \prod_{x}^{N} QPr(X_x|Parents(X_x), e, y=i) \right|^2 + 2 \cdot Interference$$

$$Interference = \sum_{i=1}^{|Y|-1} \sum_{j=i+1}^{|Y|} \left| \prod_{x}^{N} QPr(X_x|Parents(X_x), e, y=i) \right| \left| \prod_{x}^{N} QPr(X_x|Parents(X_x), e, y=j) \right| \cos(\theta_i - \theta_j)$$

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Case Study

We studied the implications of the proposed Quantum-Like Bayesian Network in the literature

Quantum-Like Bayesian Networks

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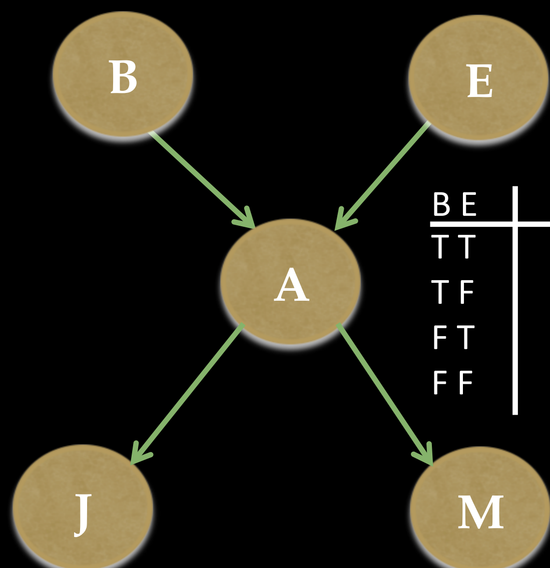

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J. Pearl. (1988). Probabilistic Reasoning in Intelligent Systems. Morgan Kaufmann Publishers Inc.

Quantum-Like Bayesian Networks

What happens if we try to compute the probability of A = t, given that we observed J = t?

Classical Probability:

$$Pr(A=t|J=t) = \gamma Pr(J=t|A=t) \sum_{b \in B} Pr(B=b) \sum_{e \in E} Pr(E=e) Pr(A=t|B=b,E=e) \sum_{m \in M} Pr(M=m|A=t)$$

Quantum Probability:

$$Pr(A=t|J=t) = \gamma \sum_{b \in B, e \in E, m \in M} \left| \psi_{J=t|A=t} \psi_{B=b} \psi_{E=e} \psi_{A=t|B=b,E=e} \psi_{M=m|A=t} \right|^2 + 2 \cdot Interference$$

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Quantum Probability:

Will generate 16 parameters

$$Pr(A=t|J=t) = \gamma \sum_{b \in B, e \in E, m \in M} \left| \psi_{J=t|A=t} \psi_{B=b} \psi_{E=e} \psi_{A=t|B=b,E=e} \psi_{M=m|A=t} \right|^2 + 2 \cdot Interference$$

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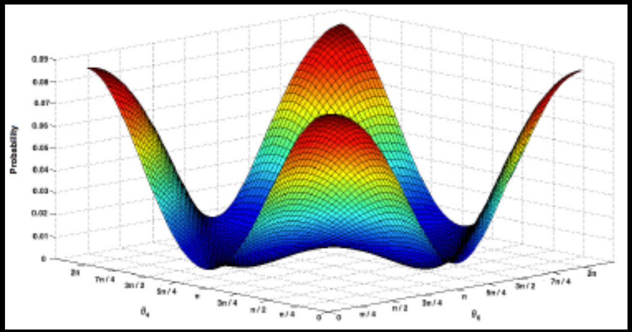
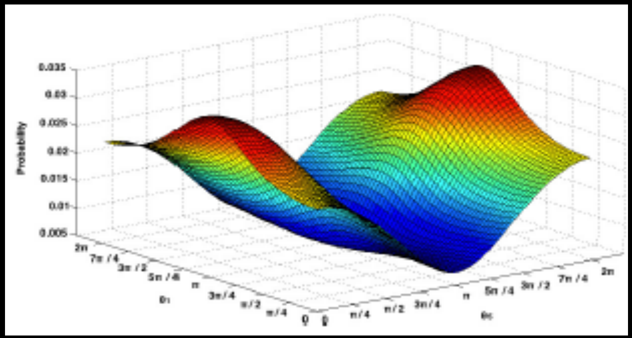
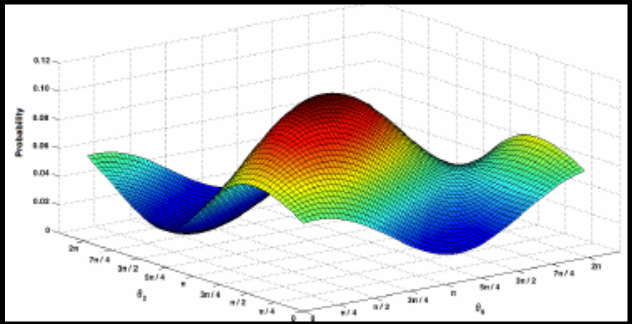
Problem!

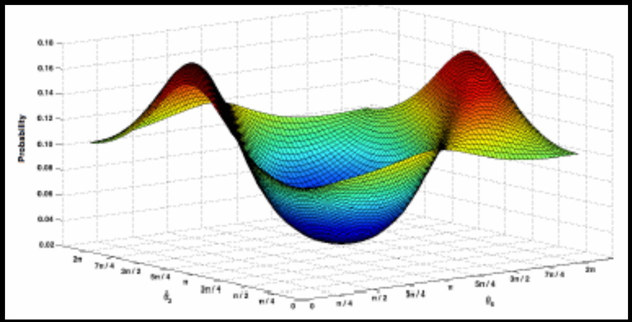
The number of parameters grows exponentially LARGE!

The final probabilities can be ANYTHING in some range!









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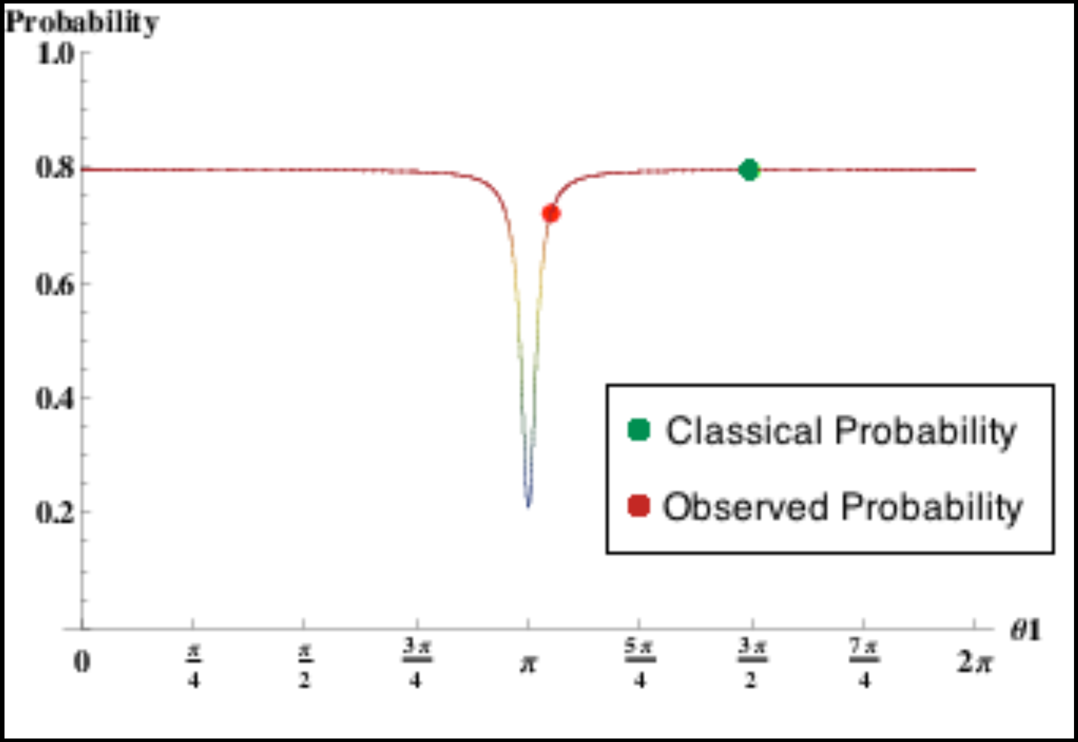
Problem!

Quantum parameters are very sensitive.

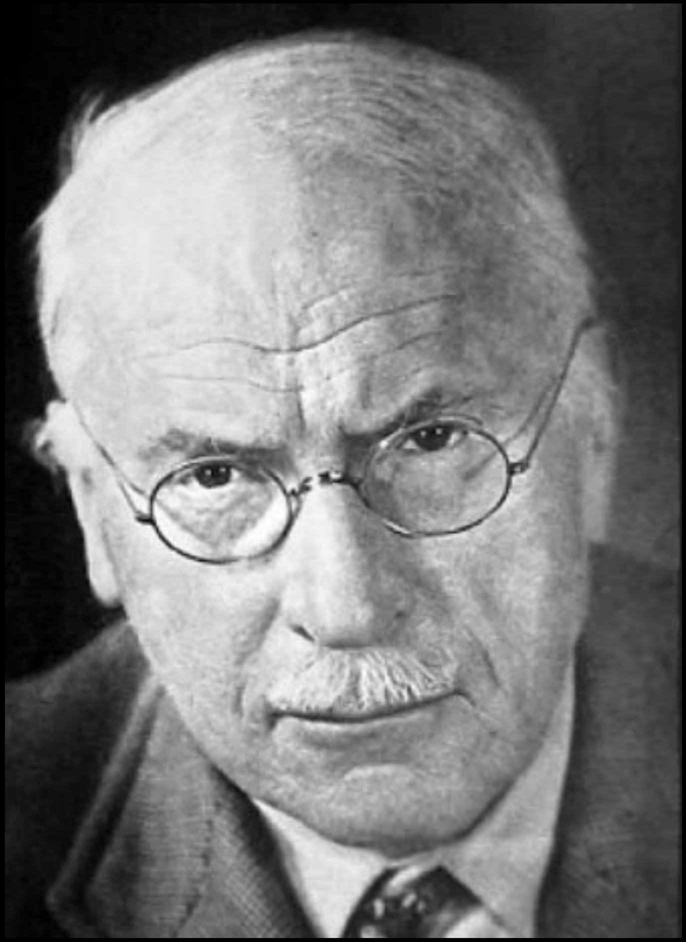
Small changes can lead to completely different probability values or can stabilize in a certain value!



Research Question

How can we deal automatically with an exponential number of quantum parameters?

The Synchronicity Principle



Synchronicity is an **acausal** principle and can be defined by **a meaningful coincidence** which appears between a mental state and an event occurring in the external world.

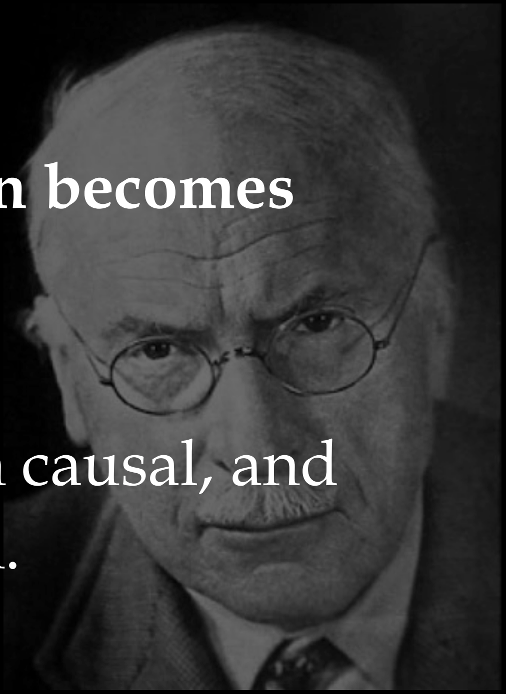
(Carl G. Jung, 1951)

The Synchronicity Principle

Natural laws are statistical truths. They are only valid when dealing with macrophysical quantities.

In the realm of very small quantities **prediction becomes uncertain**.

The connection of events may be other than causal, and requires an acausal principle of explanation.



Moreira & Wichert (2015), The Synchronicity Principle Under Quantum Probabilistic Inferences, NeuroQuantology, 13, 111-133

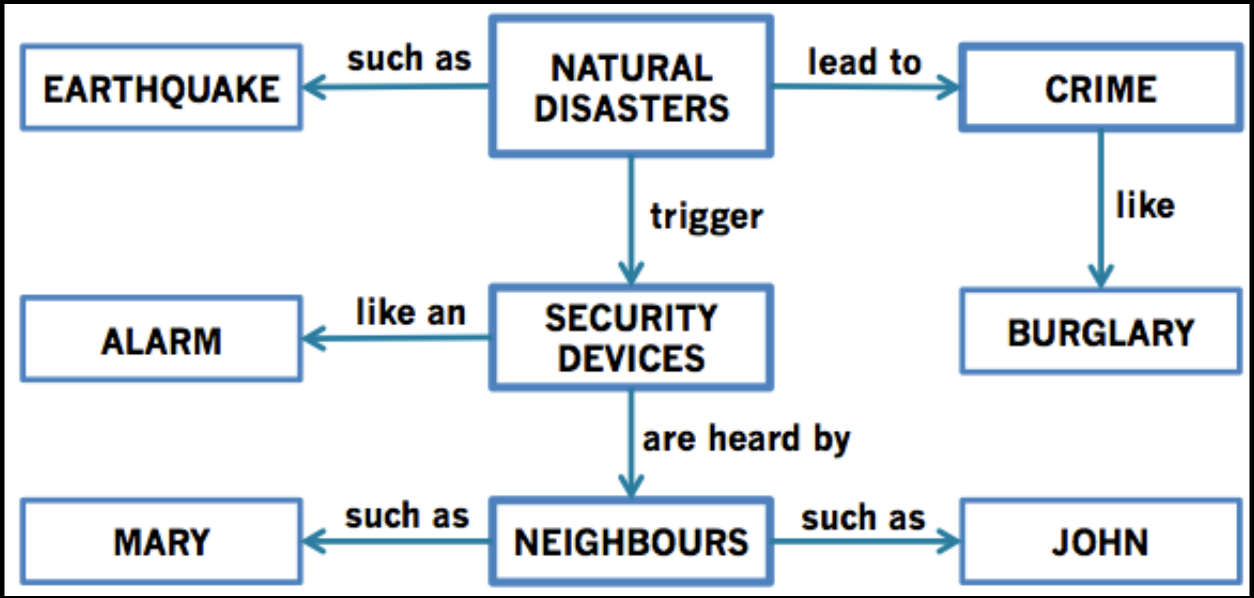
Research Question

How can we use the Synchronicity Principle in the Quantum-Like Bayesian Network and estimate quantum parameters?

Semantic Networks

Synchronicity Principle: defined by **a meaningful coincidence** between events.

Semantic Networks can help finding events that share a semantic meaning.



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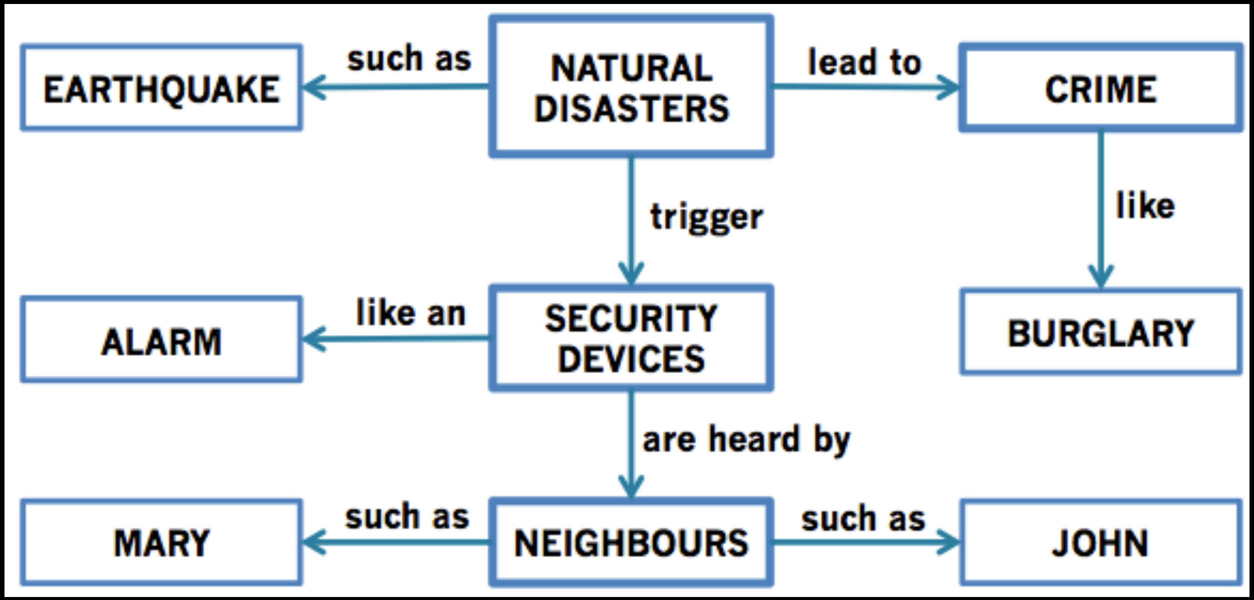

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Semantic Networks

!=

Causal Networks

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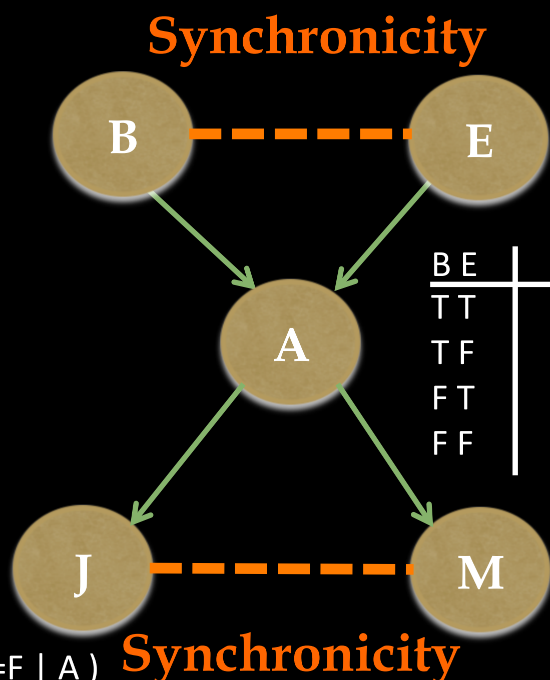
Quantum-Like Bayesian Network + Semantic Network



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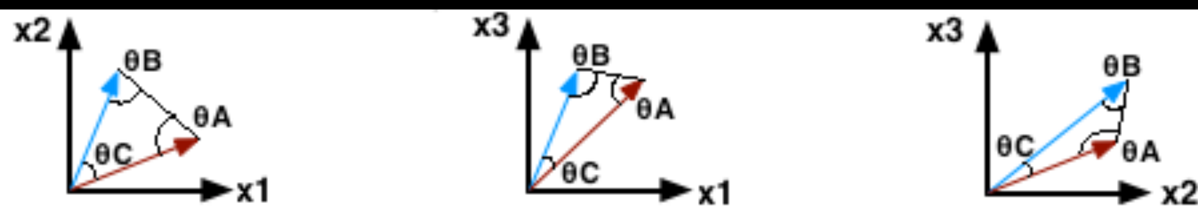
The Synchronicity Heuristic

The interference term is given as a sum of pairs of random variables.

$$interference = 2 \cdot \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} |\psi_i| \cdot |\psi_j| \cdot \cos(\theta_i - \theta_j)$$

Heuristic: parameters are calculated by computing different vector representations for each pair of random variables.



$$Pr(B) = \alpha \left[\sum_{i=1}^{N} |\psi_i|^2 + 2 \cdot |\psi_1| \cdot |\psi_2| \cdot \cos(\theta_1 - \theta_2) + 2 \cdot |\psi_1| \cdot |\psi_3| \cdot \cos(\theta_1 - \theta_3) + 2 \cdot |\psi_2| \cdot |\psi_3| \cdot \cos(\theta_2 - \theta_3) \right]$$

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The Synchronicity Heuristic

Since, in quantum cognition, the quantum parameters are seen as inner products, we represent each pair of random variables in 2-dimenional vectors.

$$\mathbf{a}(\mathbf{X}=\mathbf{T}) = \begin{bmatrix} \left|\psi_i \cdot e^{i\theta_i}\right|^2 \\ \left|\psi_i \cdot e^{i\theta_j}\right|^2 \end{bmatrix} \quad \mathbf{b}(\mathbf{X}=\mathbf{F}) = \begin{bmatrix} \left|\psi_i \cdot e^{i\theta_i}\right|^2 \\ \left|\psi_j \cdot e^{i\theta_j}\right|^2 \end{bmatrix}$$

We need to represent both assignments of the binary random variables

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The Synchronicity Heuristic

Using the semantic network, variables that did not share any dependence could be connected through their semantic meaning.

Variables that occur during the inference process should be more correlated than variables that do not occur. We use a quantum step phase angle of $\pi/4$ (Yukalov & Sornette, 2010).

Assignments of Variables		Angle
var_1 occurs	var_2 occurs	$\theta = 0$
var_1 occurs	var_2 not occurs	$\theta = \pi/4$
var_1 not occurs	var_2 occurs	$\theta = 3\pi/4$
var_1 not occurs	var_2 not occurs	$\theta = \pi$

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The Synchronicity Heuristic

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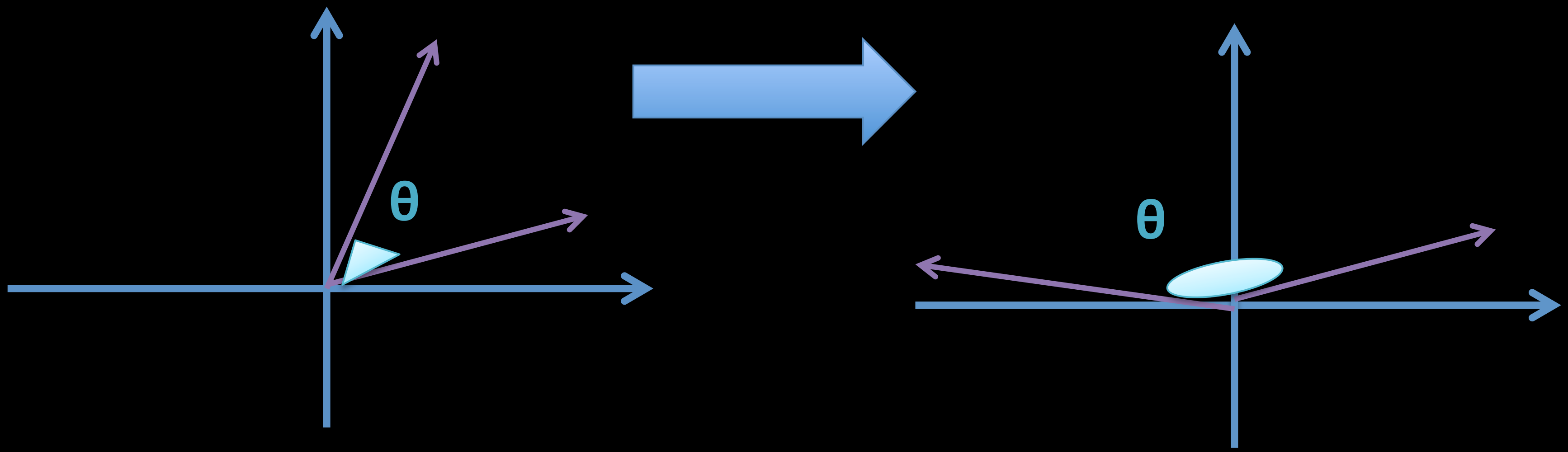

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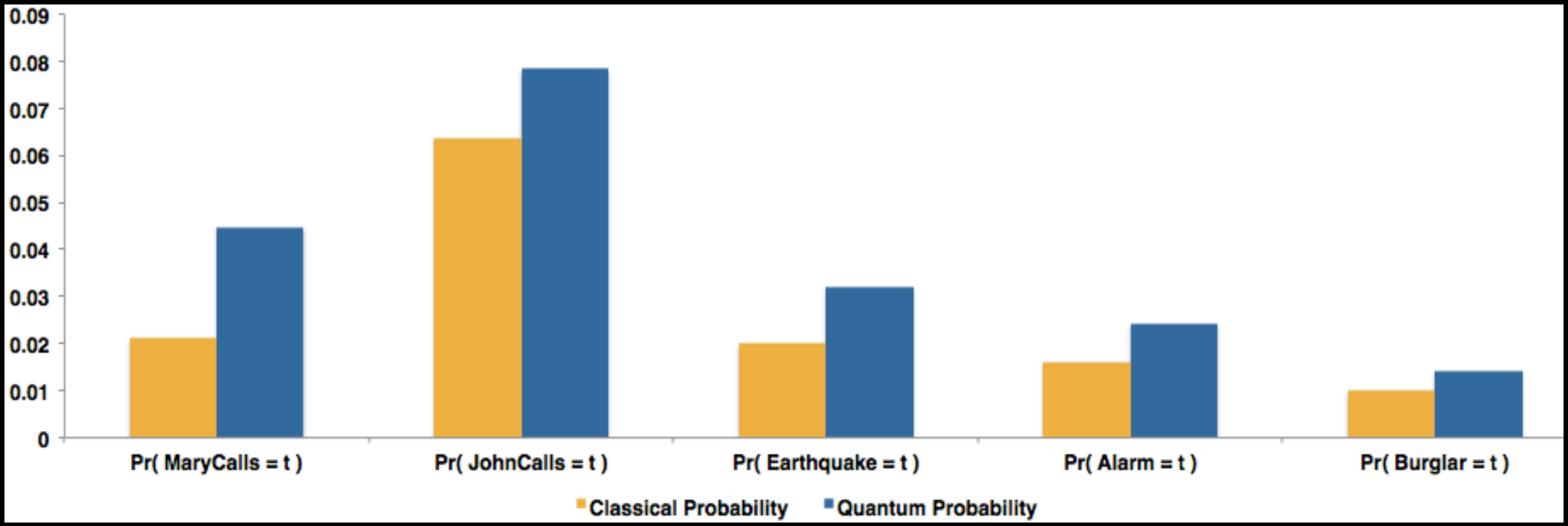
Research Question

How can an acausal connectionist theory affect quantum probabilistic inferences?

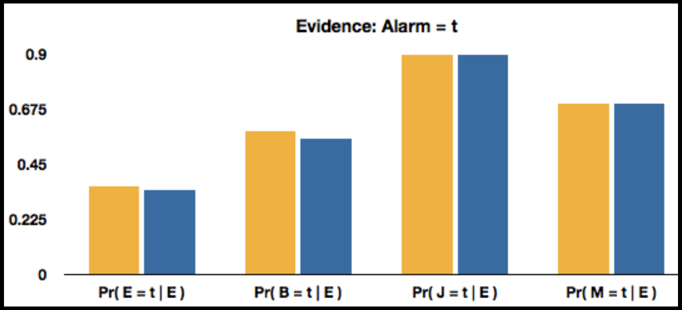
Classical vs Acausal Quantum Inferences

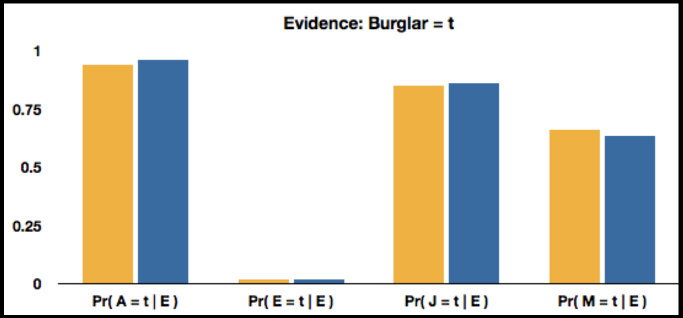
High levels of uncertainty during the inference process, lead to complete different results from classical theory.

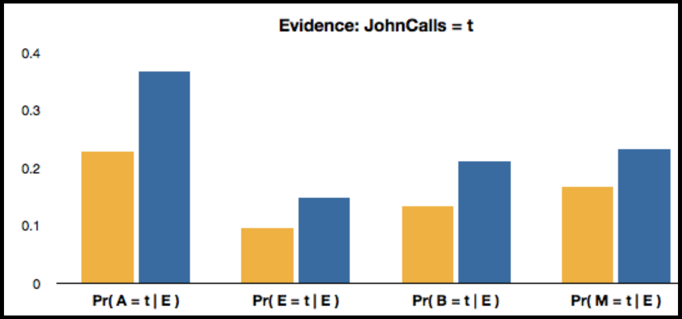


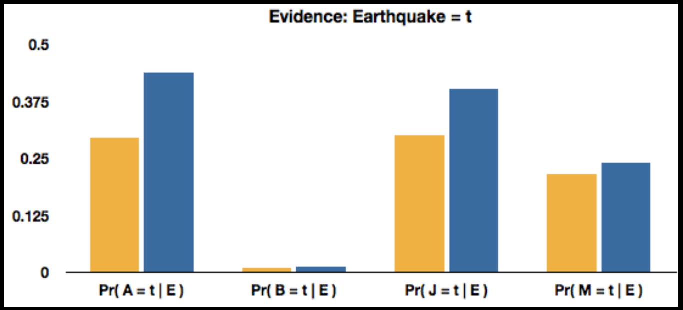
Classical vs Acausal Quantum Inferences

More evidence leads to lower uncertainty, which leads to an approximation to the classical inference.









Conclusions

1. Applied the mathematical formalisms of quantum theory to develop a Quantum-Like Bayesian Network;
2. Used a Semantic Network to find acausal relationships;
3. An heuristic was created to estimate quantum parameters;
4. Quantum probability is “stronger” with high levels of uncertainty;
5. With less uncertainty, the Quantum-Like network collapses to its classical counterpart;

Some Concluding Reflections

Can we validate this model for more complex decision problems?

Can we propose an experiment?